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On Statistical Hypothesis Testing Based on Interval Type-2 Hexagonal Fuzzy Numbers

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Abstract. The classical procedures for testing hypotheses are not appropriate for dealing with imprecise data. After the inception of the notion of fuzzy set theory, there have been attempts to analyze the problem of testing hypothesis for dealing with such imprecise data. In this paper, we consider the fuzzy data instead of crisp ones, and introduce a procedure for testing of hypothesis for imprecise data based on interval type -2 generalized hexagonal fuzzy numbers.

Keywords: Testing of hypothesis, interval type–2 generalized hexagonal fuzzy number

AMS Mathematics Subject Classification (2010): 62F03

1. Introduction

In classical approaches to test statistical hypothesis, it is assumed that both the underlying hypothesis are the available data are crisp. Moreover, statistical hypothesis testing is very important tool for finding decisions in real life problems. Usually, the underlying data are assumed to be precise numbers but it is much more realistic in general to consider fuzzy values which are non – precise numbers. In this case the test statistic will also yield a non – precise numbers. The statistical hypothesis testing under fuzzy environments has been studied by many authors using the fuzzy set theory concepts which was introduced by Zadeh [19] in the year 1965.

Casals et al. [5] and Casals and Gil [6, 7] discussed statistical testing based on a model represented by fuzzy events. Saade and Schwarslander [14] developed fuzzy hypothesis testing for hybrid data under which one hypothesis is a mixture of a random and a fuzzy component. Watanabe and Imaizumi [17] introduced a testing method of a fuzzy hypothesis for random data. Ramer and Kandel [13] investigated the impacts of vauge data on the statistical task of hypothesis testing. Arnold [1, 2, 3] presented an approach how to test fuzzily formulated hypothesis with crisp data. Taheri and Behboodian [15] formulated the problem of fuzzy hypothesis testing when the hypotheses are fuzzy and the observations are crisp. Montenegro et al. [12] studied two –

sample hypothesis tests based on generalized metric for fuzzy numbers. Viertl [16] investigated some methods to construct confidence intervals and statistical tests for fuzzy data. Wu [18] proposed some approaches to construct fuzzy intervals for the unknown fuzzy parameter. A new approach to the problem of testing statistical hypothesis is introduced by Chachi et al [8]. Asady [4] introduced a method to obtain the nearest trapezoidal approximation of fuzzy numbers. Kalpanapriya and Pandian [11] proposed a method for one sample t- test based on interval number using triangular fuzzy numbers. Gajivaradhan and Parthiban [9, 10] analyzed one sample and two sample t–test based on alpha cut method using trapezoidal fuzzy numbers. In this paper, we introduce a statistical fuzzy hypothesis testing of one–sample t–test in which the designated samples are in terms of interval type 2 hexagonal fuzzy number (IT2HFN) data.

The rest of the paper is organized as follows: Section 2 introduces basic concepts and definitions of interval type 2 hexagonal fuzzy number and its α-cut. Section 3 proposes a new procedure for single sample t–test for special interval data with an example. Section 4 proposes a new approach for single sample t–test for interval type 2 hexagonal fuzzy number data based on the procedure proposed in section 3 using α - cut of interval type 2 hexagonal fuzzy numbers (IT2HFN). Moreover, the section gives an example for demonstrate the proposed procedure. Finally, the paper ends with conclusion in section 5.

2. Preliminaries

Definition 2.1. (Fuzzy Number) A Fuzzy set $\tilde{A}$ is defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.

(i) $\tilde{A}$ is convex, i.e., $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) = \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}$, for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

(ii) $\tilde{A}$ is normal, i.e., there exists an $x_0 \in R$ such that $\tilde{A}(x_0) = 1$

(iii) $\tilde{A}$ is piecewise continuous

Definition 2.2. (Hexagonal Fuzzy Number) A fuzzy number $\tilde{A}$ is a hexagonal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$, where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6$ are real numbers satisfying $a_2 - a_1 \le a_3 - a_2$ and $a_5 - a_4 \le a_6 - a_5$, if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & a_2 \le x \le a_3 \\ 1 & a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & a_5 \le x \le a_6 \\ 0 & x > a_6 \end{cases}$$

Definition 2.3. (Interval Type 2 Hexagonal Fuzzy Number - IT2HFN)
A fuzzy number $\tilde{A}$ is a type 2 hexagonal fuzzy number denoted by $\tilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right] = \left[(a_1^L, a_2^L, a_3^L, a_4^L, a_5^L, a_6^L), (a_1^U, a_2^U, a_3^U, a_4^U, a_5^U, a_6^U)\right]$, where $(a_1^L \le a_2^L \le a_3^L \le a_4^L \le a_5^L \le a_6^L), (a_1^U \le a_2^U \le a_3^U \le a_4^U \le a_5^U \le a_6^U)$ are real numbers satisfying $a_2^L - a_1^L \le a_3^L - a_2^L$, $a_5^L - a_4^L \ge a_6^L - a_5^L$, $a_2^U - a_1^U \le a_3^U - a_2^U$ and $a_5^U - a_4^U \ge a_6^U - a_5^U$, if its membership functions $\mu_{\tilde{A}^L}(x)$ and $\mu_{\tilde{A}^U}(x)$ are given by

$$\mu_{\tilde{A}^L}(x) = \begin{cases} 0 & x < a_1^L \\ \frac{1}{2}\left(\frac{x - a_1^L}{a_2^L - a_1^L}\right) & a_1^L \le x \le a_2^L \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2^L}{a_3^L - a_2^L}\right) & a_2^L \le x \le a_3^L \\ 1 & a_3^L \le x \le a_4^L \\ 1 - \frac{1}{2}\left(\frac{x - a_4^L}{a_5^L - a_4^L}\right) & a_4^L \le x \le a_5^L \\ \frac{1}{2}\left(\frac{a_6^L - x}{a_6^L - a_5^L}\right) & a_5^L \le x \le a_6^L \\ 0 & x > a_6^L \end{cases} \qquad \mu_{\tilde{A}^U}(x) = \begin{cases} 0 & x < a_1^U \\ \frac{1}{2}\left(\frac{x - a_1^U}{a_2^U - a_1^U}\right) & a_1^U \le x \le a_2^U \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2^U}{a_3^U - a_2^U}\right) & a_2^U \le x \le a_3^U \\ 1 & a_3^U \le x \le a_4^U \\ 1 - \frac{1}{2}\left(\frac{x - a_4^U}{a_5^U - a_4^U}\right) & a_4^U \le x \le a_5^U \\ \frac{1}{2}\left(\frac{a_6^U - x}{a_6^U - a_5^U}\right) & a_5^U \le x \le a_6^U \\ 0 & x > a_6^U \end{cases}$$

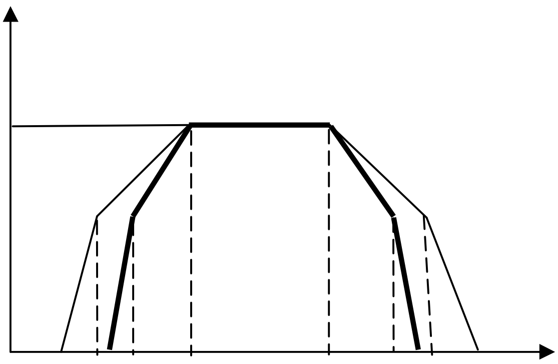

Figure 1: Interval Type 2 Hexagonal Fuzzy Number

Definition 2.4. (α - cut of an IT2HFN)
The α - cut of an Interval type 2 hexagonal fuzzy set is of the form:

$${}^{\alpha}\tilde{A} = \left[\begin{Bmatrix} \left[\tilde{A}_\alpha^U, \tilde{A}_\alpha^L\right], \alpha \in [0, \lambda) \\ \left[\tilde{A}_\alpha^U, \tilde{A}_\alpha^L\right], \alpha \in [\lambda, 1] \end{Bmatrix}, \begin{Bmatrix} \left[\tilde{A}_\alpha^L, \tilde{A}_\alpha^U\right], \alpha \in [\lambda, 1] \\ \left[\tilde{A}_\alpha^L, \tilde{A}_\alpha^U\right], \alpha \in [0, \lambda) \end{Bmatrix}\right]$$

$$
{}^{\alpha}\tilde{A} = \left[\begin{array}{l} \left\{ \begin{array}{l} \left[2\alpha(a_2^U - a_1^U) + a_1^U, 2\alpha(a_2^L - a_1^L) + a_1^L\right], \alpha \in [0,\lambda) \\ \left[2\alpha(a_3^U - a_2^U) - a_3^U + 2a_2^U, 2\alpha(a_3^L - a_2^L) - a_3^L + 2a_2^L\right], \alpha \in [\lambda,1] \end{array} \right\}, \\ \qquad \left\{ \begin{array}{l} \left[-2\alpha(a_5^L - a_4^L) + 2a_5^L - a_4^L, -2\alpha(a_5^U - a_4^U) + 2a_5^U - a_4^U\right], \alpha \in [0,\lambda) \\ \left[-2\alpha(a_6^L - a_5^L) + a_6^L, -2\alpha(a_6^U - a_5^U) + a_6^U\right], \alpha \in [\lambda,1] \end{array} \right\} \end{array} \right]
$$

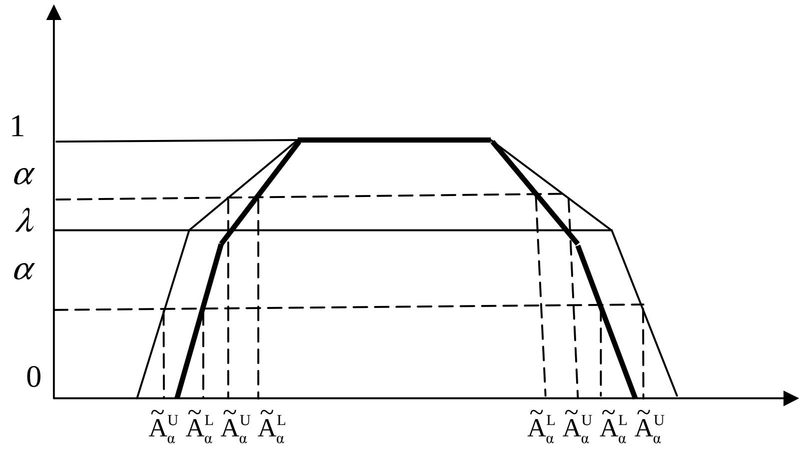

Figure 2: α - cut of an IT2HFN**3.**

Testing of hypothesis for special interval data

Let $\{[\{[x_{1i}, y_{1i}][x_{2i}, y_{2i}]\}, \{[x_{3i}, y_{3i}][x_{4i}, y_{4i}]\}], i = 1,2,......, n\}$ be a random sample with size n(<30) such that

$\{x_{1i}, i = 1,2,......, n\}, \{x_{2i}, i = 1,2,......, n\}, \{x_{3i}, i = 1,2,......, n\}, \{x_{4i}, i = 1,2,......, n\},$
$\{y_{1i}, i = 1,2,......, n\}, \{y_{2i}, i = 1,2,......, n\}, \{y_{3i}, i = 1,2,......, n\}, \{y_{4i}, i = 1,2,......, n\}$ are the eight random samples from a normal population and the population mean of the sample be $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}]$.

Now, the null hypothesis is going to be tested that the population mean of the given sample $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}]$ is equal to a specific interval $[\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$, that is,
$\eta_1 = \eta_{1o}, \eta_2 = \eta_{2o}, \eta_3 = \eta_{3o}, \eta_4 = \eta_{4o}$ and $\mu_1 = \mu_{1o}, \mu_2 = \mu_{2o}, \mu_3 = \mu_{3o}, \mu_4 = \mu_{4o}$.

Consider the random sample of lower values of the given special interval data,
$S_{L1} = \{x_{1i}, i = 1,2,......,n\}, S_{L2} = \{x_{2i}, i = 1,2,......, n\}, S_{L3} = \{x_{3i}, i = 1,2,......, n\},$
$S_{L4} = \{x_{4i}, i = 1,2,......, n\}, S_{U1} = \{y_{1i}, i = 1,2,......, n\}, S_{U2} = \{y_{2i}, i = 1,2,......, n\},$
$S_{U3} = \{y_{3i}, i = 1,2,......, n\}, S_{U4} = \{y_{4i}, i = 1,2,......, n\}.$

Now, the sample mean of $S_{L1}, S_{L2}, S_{L3}, S_{L4}, S_{U1}, S_{U2}, S_{U3}$ and S_{U4} are $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{y}_1, \bar{y}_2, \bar{y}_3$ and $\bar{y}_4$ respectively and sample standard deviation (S.D) of $S_{L1}, S_{L2}, S_{L3}, S_{L4}, S_{U1}, S_{U2}, S_{U3}$ and S_{U4} are $s_{x_1}, s_{x_2}, s_{x_3}, s_{x_4}, s_{y_1}, s_{y_2}, s_{y_3}, s_{y_4}$ and s_{y_4} respectively.

Test statistics:

$$t_{L_1} = \frac{(\bar{x}_1 - \eta_1)\sqrt{n}}{s_{x_1}}, t_{L_2} = \frac{(\bar{x}_2 - \eta_2)\sqrt{n}}{s_{x_2}}, t_{L_3} = \frac{(\bar{x}_3 - \eta_3)\sqrt{n}}{s_{x_3}}, t_{L_4} = \frac{(\bar{x}_4 - \eta_4)\sqrt{n}}{s_{x_4}} \text{ and}$$

$$t_{U_1} = \frac{(\bar{y}_1 - \mu_1)\sqrt{n}}{s_{y_1}}, t_{U_2} = \frac{(\bar{y}_2 - \mu_2)\sqrt{n}}{s_{y_2}}, t_{U_3} = \frac{(\bar{y}_3 - \mu_3)\sqrt{n}}{s_{y_3}}, t_{U_4} = \frac{(\bar{y}_4 - \mu_4)\sqrt{n}}{s_{y_4}}$$

Now, the rejection region of the alternative hypothesis for level α is given below:

Alternative Hypothesis	Rejection Region for level α test
H$_A$: $\left[\left\{[\eta_1, \mu_1][\eta_2, \mu_2]\right\}, \left\{[\eta_3, \mu_3][\eta_4, \mu_4]\right\}\right]$ $> \left[\left\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\right\}, \left\{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\right\}\right]$	$t_{L_1} \geq t_{\alpha,n-1}, t_{L_2} \geq t_{\alpha,n-1}$, $t_{L_3} \geq t_{\alpha,n-1}, t_{L_4} \geq t_{\alpha,n-1}$, $t_{U_1} \geq t_{\alpha,n-1}, t_{U_2} \geq t_{\alpha,n-1}$, $t_{U_3} \geq t_{\alpha,n-1}, t_{U_4} \geq t_{\alpha,n-1}$ (upper tailed test)
H$_A$: $\left[\left\{[\eta_1, \mu_1][\eta_2, \mu_2]\right\}, \left\{[\eta_3, \mu_3][\eta_4, \mu_4]\right\}\right]$ $< \left[\left\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\right\}, \left\{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\right\}\right]$	$t_{L_1} \leq -t_{\alpha,n-1}, t_{L_2} \leq -t_{\alpha,n-1}$, $t_{L_3} \leq -t_{\alpha,n-1}, t_{L_4} \leq -t_{\alpha,n-1}$, $t_{U_1} \leq -t_{\alpha,n-1}, t_{U_2} \leq -t_{\alpha,n-1}$, $t_{U_3} \leq -t_{\alpha,n-1}, t_{U_4} \leq -t_{\alpha,n-1}$ (lower tailed test)
H$_A$: $\left[\left\{[\eta_1, \mu_1][\eta_2, \mu_2]\right\}, \left\{[\eta_3, \mu_3][\eta_4, \mu_4]\right\}\right]$ $\neq \left[\left\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\right\}, \left\{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\right\}\right]$	$\lvert t_{L_1}\rvert \geq t_{\alpha/2,n-1}, \lvert t_{L_2}\rvert \geq t_{\alpha/2,n-1}$, $\lvert t_{L_3}\rvert \geq t_{\alpha/2,n-1}, \lvert t_{L_4}\rvert \geq t_{\alpha/2,n-1}$, $\lvert t_{U_1}\rvert \geq t_{\alpha/2,n-1}, \lvert t_{U_2}\rvert \geq t_{\alpha/2,n-1}$, $\lvert t_{U_3}\rvert \geq t_{\alpha/2,n-1}, \lvert t_{U_4}\rvert \geq t_{\alpha/2,n-1}$ (two tailed test)

Now, if $\lvert t_{L_i}\rvert < t_{\alpha,n-1}, i = 1,2,3,4$ (one tailed test) and $\lvert t_{U_i}\rvert < t_{\alpha,n-1}, i = 1,2,3,4$ (one tailed test), the difference between $\left[\left\{[\eta_1, \mu_1][\eta_2, \mu_2]\right\}, \left\{[\eta_3, \mu_3][\eta_4, \mu_4]\right\}\right]$ and $\left[\left\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\right\}, \left\{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\right\}\right]$ is not significant at α level. Then, the population mean corresponding to the given sample $\left[\left\{[\eta_1, \mu_1][\eta_2, \mu_2]\right\}, \left\{[\eta_3, \mu_3][\eta_4, \mu_4]\right\}\right]$ = $\left[\left\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\right\}, \left\{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\right\}\right]$. That is, the null hypothesis is accepted. Otherwise, the alternative hypothesis is accepted.

Now, if $\left|t_{L_i}\right| < t_{\alpha/2,n-1}, i = 1,2,3,4$ (two tailed test) and $\left|t_{U_i}\right| < t_{\alpha/2,n-1}, i = 1,2,3,4$ (two tailed test), the difference between $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}]$ and $[\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$ is not significant at α level. Then, the population mean corresponding to the given sample $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}]$ = $[\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$. That is, the null hypothesis is accepted. Otherwise, the alternative hypothesis is accepted.

Now, the 100(1-α)% confidence limit for the population mean $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}]$ corresponding to the given below:

$$\left[\left\{\begin{matrix}\left[\bar{x}_1 - t_{\alpha/2,n-1}\left(\frac{s_{x_1}}{\sqrt{n}}\right), \bar{y}_1 - t_{\alpha/2,n-1}\left(\frac{s_{y_1}}{\sqrt{n}}\right)\right] \\ \left[\bar{x}_2 - t_{\alpha/2,n-1}\left(\frac{s_{x_2}}{\sqrt{n}}\right), \bar{y}_2 - t_{\alpha/2,n-1}\left(\frac{s_{y_2}}{\sqrt{n}}\right)\right]\end{matrix}\right\}, \left\{\begin{matrix}\left[\bar{x}_3 - t_{\alpha/2,n-1}\left(\frac{s_{x_3}}{\sqrt{n}}\right), \bar{y}_3 - t_{\alpha/2,n-1}\left(\frac{s_{y_3}}{\sqrt{n}}\right)\right] \\ \left[\bar{x}_4 - t_{\alpha/2,n-1}\left(\frac{s_{x_4}}{\sqrt{n}}\right), \bar{y}_4 - t_{\alpha/2,n-1}\left(\frac{s_{y_4}}{\sqrt{n}}\right)\right]\end{matrix}\right\}\right]$$

$$< [\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}] <$$

$$\left[\left\{\begin{matrix}\left[\bar{x}_1 + t_{\alpha/2,n-1}\left(\frac{s_{x_1}}{\sqrt{n}}\right), \bar{y}_1 + t_{\alpha/2,n-1}\left(\frac{s_{y_1}}{\sqrt{n}}\right)\right] \\ \left[\bar{x}_2 + t_{\alpha/2,n-1}\left(\frac{s_{x_2}}{\sqrt{n}}\right), \bar{y}_2 + t_{\alpha/2,n-1}\left(\frac{s_{y_2}}{\sqrt{n}}\right)\right]\end{matrix}\right\}, \left\{\begin{matrix}\left[\bar{x}_3 + t_{\alpha/2,n-1}\left(\frac{s_{x_3}}{\sqrt{n}}\right), \bar{y}_3 + t_{\alpha/2,n-1}\left(\frac{s_{y_3}}{\sqrt{n}}\right)\right] \\ \left[\bar{x}_4 + t_{\alpha/2,n-1}\left(\frac{s_{x_4}}{\sqrt{n}}\right), \bar{y}_4 + t_{\alpha/2,n-1}\left(\frac{s_{y_4}}{\sqrt{n}}\right)\right]\end{matrix}\right\}\right]$$

The solution procedure can be illustrated using the following numerical example.

Example 1. The Director of School Education claims that the passing percent of school students in the particular district of his state is between {[40, 62][46, 65]} and {[74, 92][77, 95]} that is, slow learner boys and girls between 40 to 62 percent and 46 to 65 percent respectively as well as fast learner boys and girls between 74 to 92 and 77 to 95 respectively.

Only 10 schools in the district of the state of Tamil Nadu are selected at random. Then, the passing percentages of slow learner boys and girls and fast learner boys and girls in each school are represented in the special interval form as follows:

[{[35, 58][44, 67]}, {[71, 90][75, 93]}],	[{[41, 62][51, 64]}, {[74, 95][73, 90]}]
[{[42, 64][49, 70]}, {[78, 92][78, 91]}],	[{[37, 59][48, 65]}, {[80, 92][74, 89]}]
[{[47, 68][50, 69]}, {[73, 88][80, 98]}],	[{[40, 63][46, 61]}, {[81, 89][79, 91]}]
[{[38, 60][49, 68]}, {[79, 89][78, 94]}],	[{[43, 61][42, 64]}, {[77, 91][82, 97]}]
[{[45, 65][47, 61]}, {[69, 85][75, 97]}],	[{[44, 66][43, 68]}, {[70, 87][85, 96]}]

Now, we are going to consider the testing hypothesis

H_0 : $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}] = [\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$

H_A : $[\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}] \neq [\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$

Now the size of the sample, n = 10 and the population mean is $[\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$ = [{[40, 62][46, 65]}, {[74, 92][77, 95]}] with unknown sample S.D.. We use 5% level of significance.

Now, the mean value of lower and upper interval values are $\bar{x}_1 = 41.2,\ \bar{x}_2 = 46.9,\ \bar{x}_3 = 75.2,\ \bar{x}_4 = 77.9$, $\bar{y}_1 = 62.6,\ \bar{y}_2 = 65.7,\ \bar{y}_3 = 89.8,\ \bar{y}_4 = 93.6$ respectively and the sample S.D. of the lower and upper interval values are

$s_{x_1} = 14.18, s_{x_2} = 9.43, s_{x_3} = 19.07, s_{x_4} = 14.32,$

$s_{y_1} = 10.27, s_{y_2} = 10.23, s_{y_3} = 8.17, s_{y_4} = 10.71.$

Now, the table value of t for 9 degrees of freedom at 5% level = 1.833

Now the lower values are $\eta_{1o} = 40, \eta_{2o} = 46, \eta_{3o} = 74, \eta_{4o} = 77$ and the upper values are $\mu_{1o} = 62, \mu_{2o} = 65, \mu_{3o} = 92, \mu_{4o} = 95$.

Test statistics:

$$t_{L_1} = \frac{(\bar{x}_1 - \eta_1)\sqrt{n}}{s_{x_1}} = 0.2676,\ t_{L_2} = \frac{(\bar{x}_2 - \eta_2)\sqrt{n}}{s_{x_2}} = 0.3018,\ t_{L_3} = \frac{(\bar{x}_3 - \eta_3)\sqrt{n}}{s_{x_3}} = 0.1990,$$

$$t_{L_4} = \frac{(\bar{x}_4 - \eta_4)\sqrt{n}}{s_{x_4}} = 0.1987,\ t_{U_1} = \frac{(\bar{y}_1 - \mu_1)\sqrt{n}}{s_{y_1}} = 0.1847,\ t_{U_2} = \frac{(\bar{y}_2 - \mu_2)\sqrt{n}}{s_{y_2}} = 0.2164,$$

$$t_{U_3} = \frac{(\bar{y}_3 - \mu_3)\sqrt{n}}{s_{y_3}} = -0.8515,\ t_{U_4} = \frac{(\bar{y}_4 - \mu_4)\sqrt{n}}{s_{y_4}} = -0.4134$$

Now, since

$|t_{L_1}| < t_{0.05}, |t_{L_2}| < t_{0.05}, |t_{L_3}| < t_{0.05}, |t_{L_4}| < t_{0.05}, |t_{U_1}| < t_{0.05}, |t_{U_2}| < t_{0.05}, |t_{U_3}| < t_{0.05}, |t_{U_4}| < t_{0.05}$, the null hypothesis H_0 is accepted and the 95% confidence limits for the population mean

$$[\{[32.98, 59.35][41.43, 59.77]\}, \{[64.15, 85.06][69.60, 87.39]\}]$$
$$< [\{[\eta_1, \mu_1][\eta_2, \mu_2]\}, \{[\eta_3, \mu_3][\eta_4, \mu_4]\}] <$$
$$[\{[49.42, 65.85][52.37, 71.63]\}, \{[86.25, 94.54][86.20, 99.81]\}]$$

4. Testing of hypothesis for IVT2HFN

Interval type 2 hexagonal fuzzy number $[(a_1^L, a_2^L, a_3^L, a_4^L, a_5^L, a_6^L), (a_1^U, a_2^U, a_3^U, a_4^U, a_5^U, a_6^U)]$ can be represented as a special interval number form by using the α- cut method as follows:

$$\left[(a_1^L, a_2^L, a_3^L, a_4^L, a_5^L, a_6^L), (a_1^U, a_2^U, a_3^U, a_4^U, a_5^U, a_6^U)\right]$$
$$= \left[\begin{array}{l} \left\{ \begin{array}{l} \left[2\alpha(a_2^U - a_1^U) + a_1^U, 2\alpha(a_2^L - a_1^L) + a_1^L\right], \alpha \in [0, \lambda) \\ \left[2\alpha(a_3^U - a_2^U) - a_3^U + 2a_2^U, 2\alpha(a_3^L - a_2^L) - a_3^L + 2a_2^L\right], \alpha \in [\lambda, 1] \end{array} \right\}, \\ \left\{ \begin{array}{l} \left[-2\alpha(a_5^L - a_4^L) + 2a_5^L - a_4^L, -2\alpha(a_5^U - a_4^U) + 2a_5^U - a_4^U\right], \alpha \in [0, \lambda) \\ \left[-2\alpha(a_6^L - a_5^L) + a_6^L, -2\alpha(a_6^U - a_5^U) + a_6^U\right], \alpha \in [\lambda, 1] \end{array} \right\} \end{array} \right] \text{-- (1)}$$

Suppose that the given sample is a type 2 fuzzy data that are valued type 2 hexagonal fuzzy numbers and we have to test the hypothesis about the population mean. Using the relation (1) and the proposed test procedure for special interval data, we can test the hypothesis by transferring the type 2 fuzzy data into special interval data. The solution procedure is illustrated with the help of the following example.

Example 2. The ministry of health department wants to claim that the minimum and maximum number of peoples affect by dengue fever in the country is known to be 3500 and 7200 respectively. Only 12 states of the country were tested because the data collection is laborious and take considerable time to complete. The situation was that the minimum and the maximum number of peoples affected the disease were not known exactly. The obtained minimum and maximum number of peoples affected by the disease was around a number. Therefore, the minimum and maximum numbers of peoples affected by the disease were taken to be interval type 2 hexagonal fuzzy numbers as follows.

[(3229, 3311, 3452, 3573, 3645, 3727), (6932, 7044, 7152, 7257, 7378, 7489)]
[(3026, 3106, 3246, 3365, 3439, 3519), (7015, 7125, 7237, 7347, 7476, 7584)]
[(3081, 3160, 3302, 3426, 3497, 3576), (7180, 7295, 7402, 7514, 7631, 7744)]
[(3408, 3483, 3618, 3736, 3805, 3880), (7111, 7220, 7335, 7438, 7564, 7681)]
[(3174, 3250, 3387, 3512, 3587, 3663), (6841, 6954, 7063, 7164, 7279, 7384)]
[(3385, 3468, 3599, 3716, 3784, 3867), (6778, 6885, 6991, 7106, 7228, 7342)]
[(4596, 3680, 3823, 3945, 4010, 4094), (7354, 7471, 7584, 7686, 7816, 7918)]
[(3512, 3599, 3746, 3862, 3935, 4022), (7274, 7382, 7487, 7596, 7714, 7829)]
[(3249, 3327, 3472, 3598, 3675, 3753), (6969, 7083, 7197, 7297, 7413, 7517)]
[(3463, 3543, 3681, 3795, 3862, 3942), (6670, 6776, 6892, 7003, 7134, 7252)]
[(3325, 3402, 3546, 3673, 3743, 3820), (7445, 7563, 7667, 7766, 7890, 7996)]
[(3097, 3182, 3321, 3436, 3501, 3586), (6685, 6790, 6902, 7006, 7126, 7242)]

Now, we are considering the testing hypotheses $\tilde{H}_o : \ \tilde{\mu} \approx \tilde{3500} \text{ to } \tilde{7200}$ and $\tilde{H}_A : \ \tilde{\mu} \succ \tilde{3500} \text{ to } \tilde{7200}$ where $\tilde{3500}$ *to* $\tilde{7200}$ is regarded as a linguistic data. Therefore, we can phrase the hypotheses

$\tilde{H}_o$: the minimum and maximum number of peoples affect by dengue fever is around 3500 to around 7200.

$\tilde{H}_A$: the minimum and maximum number of peoples affect by dengue fever is approximately greater than around 3500 to around 7200.

We may assume that the membership function of

$\tilde{3500}$ to $\tilde{7200}$
= [(3300, 3400, 3500, 3600, 3700, 3800), (7000, 7100, 7200, 7300, 7400, 7500)]

Now, the size of the sample, n = 12 and the population mean is
$\tilde{\mu}_o$ = [(3300, 3400, 3500, 3600, 3700, 3800), (7000, 7100, 7200, 7300, 7400, 7500)] with unknown sample standard deviation.

Now, using the relation (1), we convert the type 2 fuzzy interval data and then, we obtain that the mean value of the lower and upper interval values of the special interval are

$\bar{x}_1 = 222.33\alpha + 7021.17$, $\bar{y}_1 = 161\alpha + 3295.42$, $\bar{x}_2 = 220.17\alpha - 7242.42$,
$\bar{y}_2 = 280\alpha + 3235.75$, $\bar{x}_3 = -141\alpha + 3777.42$, $\bar{y}_3 = -244.83\alpha + 7593.17$,
$\bar{x}_4 = -161\alpha + 3787.42$, $\bar{y}_4 = -221.5\alpha + 7581.5$ respectively and the sample S.D. of the mean value of the lower and upper interval values are

$$s_{x_1} = \sqrt{75\alpha^2 + 3203\alpha + 66024}$$
$$s_{y_1} = \sqrt{56\alpha^2 + 729\alpha + 33287}\ , \qquad 0 \le \alpha < \lambda$$
$$s_{x_2} = \sqrt{67\alpha^2 + 1589\alpha + 66832}$$
$$s_{y_2} = \sqrt{81\alpha^2 + 177\alpha + 33556}\ , \qquad \lambda \le \alpha \le 1$$
$$s_{x_3} = \sqrt{60\alpha^2 + 939\alpha + 32450}$$
$$s_{y_3} = \sqrt{125\alpha^2 - 734\alpha + 66557}\ , \qquad 0 \le \alpha < \lambda$$
$$s_{x_4} = \sqrt{56\alpha^2 - 799\alpha + 33320}$$
$$s_{y_4} = \sqrt{123\alpha^2 + 2284\alpha + 65048}\ , \qquad \lambda \le \alpha \le 1$$

We take 5% level of significance and the table value of t for 11 degrees of freedom at 5% level, $t_{0.05} = 1.796$.

Now, the interval representation of $\tilde{\mu}_0$, $[\tilde{\mu}_0] = [\{[\eta_{1o}, \mu_{1o}][\eta_{2o}, \mu_{2o}]\}, \{[\eta_{3o}, \mu_{3o}][\eta_{4o}, \mu_{4o}]\}]$ where

$$\eta_{1o} = 200\alpha + 7000, \quad \mu_{1o} = 200\alpha + 3300$$
$$\eta_{2o} = 200\alpha - 7200, \quad \mu_{2o} = 200\alpha + 3300$$
$$\eta_{3o} = -200\alpha + 3800, \quad \mu_{3o} = -200\alpha + 7500$$
$$\eta_{4o} = -200\alpha + 3800, \quad \mu_{4o} = -200\alpha + 7500$$

Now, let $\lambda = 0.5$, we have the test statistics

$$t_{L_1} = \frac{(\bar{x}_1 - \eta_1)\sqrt{n}}{s_{x_1}} = \begin{cases} 0.2831, \alpha = 0 \\ 0.4255, \alpha = 0.49 \end{cases}, t_{L_2} = \frac{(\bar{x}_2 - \eta_2)\sqrt{n}}{s_{x_2}} = \begin{cases} 0.3987, \alpha = 0 \\ 0.0357, \alpha = 0.49 \end{cases},$$

$$t_{L_3} = \frac{(\bar{x}_3 - \eta_3)\sqrt{n}}{s_{x_3}} = \begin{cases} 0.4140, \alpha = 0.5 \\ 0.5449, \alpha = 1 \end{cases}, t_{L_4} = \frac{(\bar{x}_4 - \eta_4)\sqrt{n}}{s_{x_4}} = \begin{cases} 1.1545, \alpha = 0.5 \\ 1.7884, \alpha = 0.92 \\ 1.8034, \alpha = 0.93 \\ 1.9089, \alpha = 1 \end{cases},$$

$$t_{U_1} = \frac{(\bar{y}_1 - \mu_1)\sqrt{n}}{s_{y_1}} = \begin{cases} 0.4038, \alpha = 0 \\ 0.9526, \alpha = 0.49 \end{cases}, t_{U_2} = \frac{(\bar{y}_2 - \mu_2)\sqrt{n}}{s_{y_2}} = \begin{cases} 0.2820, \alpha = 0 \\ -0.0130, \alpha = 0.49 \end{cases}$$

$$, t_{U_3} = \frac{(\bar{y}_3 - \mu_3)\sqrt{n}}{s_{y_3}} = \begin{cases} 0.7729, \alpha = 0.5 \\ 1.1516, \alpha = 1 \end{cases}, t_{U_4} = \frac{(\bar{y}_4 - \mu_4)\sqrt{n}}{s_{y_4}} = \begin{cases} 0.1379, \alpha = 0.5 \\ -0.0067, \alpha = 1 \end{cases}$$

5. Conclusion

Since for α, $0 \le \alpha < 0.5$, $t_{L_1} < t_{0.05}, t_{L_2} < t_{0.05}, t_{U_1} < t_{0.05}$ and $t_{U_2} < t_{0.05}$ and for α, $0.5 \le \alpha \le 0.92$, $t_{L_3} < t_{0.05}, t_{L_4} < t_{0.05}$, $t_{U_3} < t_{0.05}$ and $t_{U_4} < t_{0.05}$. Therefore, $\tilde{\mathrm{H}}_o$: the minimum and maximum number of peoples affect by dengue fever is around 3500 to around 7200 is accepted based on the given fuzzy data with condition $0 \le \alpha \le 0.92$.

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