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Coloring of Regular and Strong Arcs Fuzzy Graphs

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Abstract. In this paper, we introduce coloring of regular fuzzy graph G=(V,B). We can apply two different approaches to coloring of Regular fuzzy graph. The first approach is based on the odd regular coloring fuzzy graphs and the second approach is based on the even regular coloring fuzzy graph. We establish strong coloring of a fuzzy graph and we change $\delta - arc$ into $\alpha - arc$ or $\beta - arc$.

Keywords: Fuzzy graphs, coloring of fuzzy graphs, coloring of regular fuzzy graphs, coloring of Strong arcs.

AMS Mathematics Subject Classification (2010): 03E72, 05C15

1. Introduction

The origin of graph theory started with Konigsberg bridge problem in 1735. This problem led to the concept of Eulerian graph. Euler studied the Konigsberg problem and constructed a structure that solves the problem that is referred to as an Eulerian graph. In this paper we introduced the notation of the coloring regular fuzzy graphs and coloring of strong arcs fuzzy graph. Currently, concept of graph theory are highly utilize by computer science applications, especially in area of research, including data mining image segmentation, clustering and net working.

Graph Coloring is one of the most important problems of combinatorial optimization. Many Problems of practical interest can be modeled as coloring problems. Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph. Edge coloring is a function which assigns to the edges so that incident edges receive different colors. We know that graphs are simple model of relation. A graph is a convenient way of representing information involving relationship between objects. The objects is represented by vertices and relations by edges.

The first basic definitions of fuzzy graph was proposed by Ghorai and Pal [12]. In 1987, Bhattachary introduced the concept of some remarks on fuzzy graphs, pattern recognition letters and the important role of this paper concept is regular fuzzy graph it is introduced by Akaram and Dudek [1].

Also Kalaiarasi and Mahalakshmi [7,8] defined basic definitions and an introduction to fuzzy strong graphs and fuzzy soft graphs, complement of fuzzy strong and soft graph. Then they are also proposed to regular and irregular m-polar fuzzy graphs. In this paper we introduced some new concepts of fuzzy coloring. That is coloring of regular fuzzy graphs and also introduced strong arcs of coloring fuzzy graphs.

2. Preliminaries

In this section, some elementary aspects that are necessary for this paper are included.

Definition 2.1. Fuzzy graph

A fuzzy graph is an ordered triple $G(V,\sigma,\mu)$ where V is a set of vertices $\{u_1,u_2,.....,u_n\}$ and σ is a fuzzy subset of V that is $\sigma : V \to [0,1]$ and is denoted by $\sigma = \{(u_1,\sigma(u_1)),(u_2,\sigma(u_2)),......,(u_n,\sigma(u_n))\}$ and μ is a fuzzy relation on σ.That is $\mu(u,v) \le \sigma(u),\sigma(v).$

Definition 2.2. Coloring of fuzzy graph

A coloring of colors to its vertices so that no two adjacent vertices have the same color(also called proper coloring).The set of all vertices with any one color is independent and is called a color class.

A family $\Gamma\{\gamma_1,\gamma_2,.....,\gamma_k\}$ of fuzzy sets on a set V is called a $k-$fuzzy coloring of $G=(V,\sigma,\mu)$ if

(i) $\vee\Gamma = \sigma$

(ii) $\gamma_i \wedge \gamma_j = 0$

(iii) For every strong edge $(x,y)[(i.e)\mu(x,y)>0]$ of $G.\min\{\gamma_i(x),\gamma_i(y)\} = 0(1 \le i \le k)$

Definition 2.3. Chromatic number of fuzzy graph

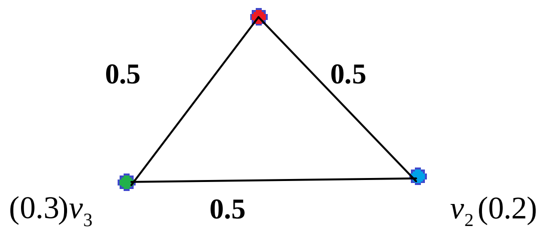
The Chromatic number of fuzzy graph $G:(V,\sigma,\mu)$ is defined as $\chi(G) = \max\{\chi_\alpha / \alpha \in L\}$ where $\chi_\alpha = \chi(G_\alpha).$

3. Coloring of regular, neighborly regular and totally regular fuzzy graph

Definition 3.1. Coloring of regular fuzzy graph

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^* : (V,E)$ .If $d_G(v) = k$ for all $v \in V.$ That is if each vertex has same degree k ,then G is said to be a regular fuzzy graph of degree k (or) a k-regular fuzzy graph no two adjacent vertices have the same color.

In Odd regular graph the vertex have the different color, even regular graph having the vertex has same color.

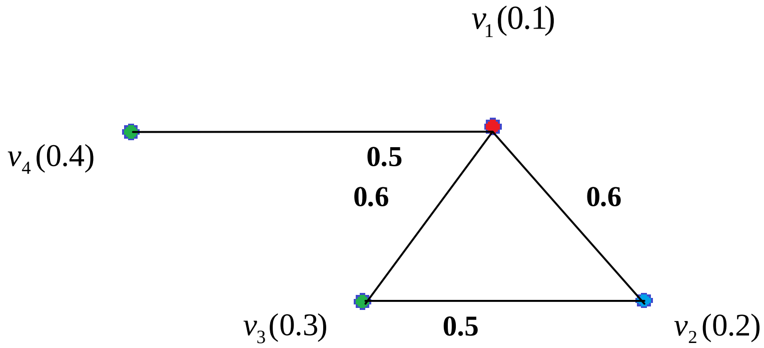


Example of odd-Regular graph:

No. of Vertices	Adjacent Vertices
V_1	V_2, V_3
V_2	V_1, V_3
V_3	V_1, V_2

The vertex have the different color for each because each vertex is adjacent.
$\therefore$ The odd regular graphs always get the distinct color in each vertex. Here $\chi(G) = 3$.
$d(v_1) = 1.0, d(v_2) = 1.0, d(v_3) = 1.0.$
$\therefore$ The graph is odd regular fuzzy graph.

Example of even-regular graph:



In even regular graph the alternative edges are equal.The adjacent vertex have distinct color otherwise it has the same color.Here,

No.of .Vertices	Adjacent Vertices	Not Adjacent Vertices
V_1	V_2, V_3, V_4	-
V_2	V_1, V_3	V_4
V_3	V_1, V_2	V_4
V_4	V_1	V_2, V_3

By our definition the adjacent vertex has distinct color otherwise it has the same color. Here we use three colors.

$\therefore\ \chi(G) = 3$.

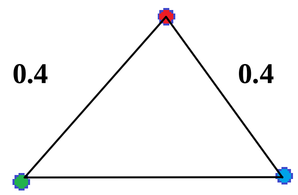
Definition 3.2. Coloring of totally regular graph

Let $G:(\sigma,\mu)$ be a fuzzy graph. The total degree of a vertex $u \in V$ is defined by $td_G(u) = \sum_{u \neq v} \mu(u,v) + \sigma(u) = d_G(u) + \sigma(u)$ .If each vertex of G has the same degree k ,then G is said to be a totally regular fuzzy graph of total degree k (or) k-totally regular fuzzy graph.

The vertex have different color because each vertex is adjacent.

$\therefore$ Totally regular graph always get the different color in each vertex.

$v_1(0.1)$



$v_3(0.3)$ $v_2(0.2)$

$$td(u) = \sum \mu(u,v) + \sigma(u)$$
$$= \mu(u,v) + \mu(v,w) + \mu(w,u) + \sigma(u) + \sigma(v) + \sigma(w)$$
$$=0.4+0.4+0.4+0.1+0.2+0.3$$
$$td(u) = 1.8$$

Definition 3.3. Coloring of neighborly and neighborly totally regular fuzzy graph

Let G be a connected fuzzy graph. Then G is called neighborly totally regular fuzzy graph. If for every two adjacent vertices of G have same total degree.

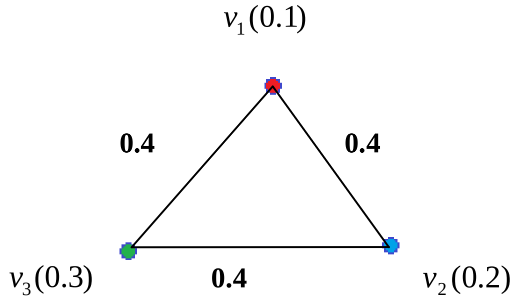
$$\chi(G) = \begin{cases} \text{Vertex are different color, if the degree is odd} \\ \text{Atleast to Vertex are same color, if the degree is even} \end{cases}$$

Theorem 3.1. If the graph $G = (A, B)$ is odd regular fuzzy graph iff each vertex has distinct color.

Proof: Let $G = (A, B)$ be a fuzzy graph on $G^* : (A, B)$ .If $d(v) = k \forall v \in V$. That is if each vertex has same degree k , then G is regular fuzzy graph.

Let $V = \{V_1, V_2, \ldots\ldots, V_n\}$. We assume that G is odd regular fuzzy graph.

To Prove: If all the vertex has distinct color.



If the adjacent vertices V_1 be $V_2, V_3, ...$ (where denote the odd number)with degree $d(V_1), d(V_2), ...d(V_n)$ respectively.

Every vertex is adjacent to other vertex. By our definition no vertex is adjacent to any vertex in that case the vertex has same color. Otherwise each vertex has distinct color. Conversely, we assume that the graph G has distinct color.

To Prove: The graph is odd regular fuzzy graph.

By our definition of coloring $\Gamma\{\gamma_1, \gamma_2, ..., \gamma_k\}$ of fuzzy sets on a set V is called a $k-$fuzzy coloring of $G = (V, \sigma, \mu)$ if

(i) $\vee \Gamma = \sigma$

(ii) $\gamma_i \wedge \gamma_j = 0$

(iii) For every strong edge $(x, y)[(i.e)\mu(x, y) > 0]$ of $G. \min\{\gamma_i(x), \gamma_i(y)\} = 0(1 \leq i \leq k)$

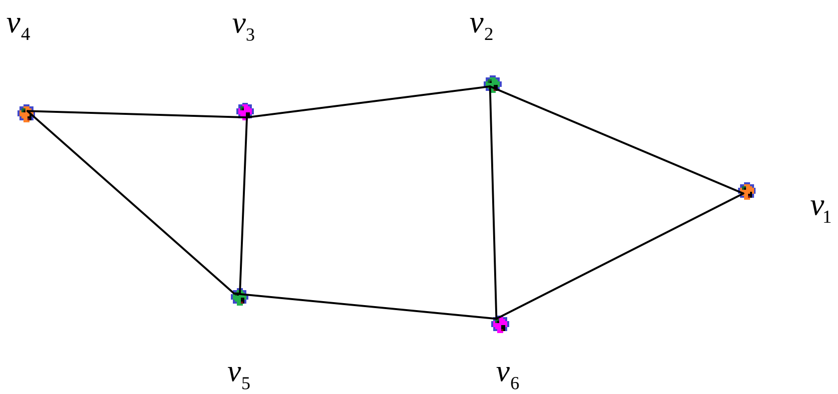
If each vertex is adjacent it has distinct color.Then the graph is odd regular fuzzy graph.

Theorem 3.2. If the graph $G = (A, B)$ is even regular fuzzy graph iff any two vertex are same color.

Proof: Let $G = (A, B)$ be a fuzzy graph on $G^* : (A, B)$ .If $d(v) = k,$ for all $v \in V$ that is if each vertex has same degree k then G is called regular fuzzy graph.

Let $v = \{v_1, v_2, ..., v_n\}$ we assume that G is an even regular fuzzy graph.

To prove: If atleast any two vertex has same color.



Number of vertex=6
The graph is even regular. In our graph we use 3 colors.
$\therefore \chi(G) = 3$.

No.of .Vertices	Adjacent Vertices	Not Adjacent Vertices
V_1	V_2, V_6	V_3, V_4, V_5
V_2	V_1, V_3, V_6	V_4, V_5
V_3	V_2, V_4, V_5	V_1, V_6
V_4	V_3, V_5	V_1, V_2, V_6
V_5	V_3, V_4, V_6	V_1, V_2
V_6	V_1, V_2, V_5	V_3, V_4

By our definition the adjacent vertex has distinct color. Otherwise it has the same color.
Conversely, we assume that the graph G has atleast two vertices has same color.
To Prove:
The graph is even regular graph.
Some vertices are adjacent. Hence any two vertex have same color. If the number of coloring is even then the graph strictly even regular graph.

(Or) If the vertex has the even number of color then the graph is strictly even regular graph.

Note: If the graph is odd (or) even regular graph then the vertex has odd number of color (or) even number of color.

Theorem 3.3. The coloring of a fuzzy regular graph is n, where n is the number of vertices of G.
Proof: Let G be a regular fuzzy graph. $G = (A, B)$ since $d(v_1) = d(v_2) = d(v_3)$.
Every pair of vertices are strongly adjacent. Degree of each vertex is n. Hence each vertex have distinct color. The number of each vertex color in n.
Otherwise it is not adjacent. Then any two vertex have same color. The number of each vertex color is $(n-1)$.

3.1. Coloring of strong arcs
An arc (u, v) is said to be strong if $\mu(u, v) \geq Conn(u, v)$.
The Strong arcs are classified as (i) $\alpha-$strong arc (ii) $\beta-$ strong arc (iii) $\delta-$strong arc.

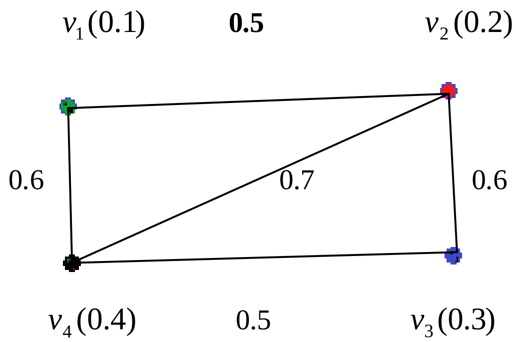
Definition 3.1.1. $\alpha-$ **strong arc**
An arc is said to be $\alpha-$ strong if $\mu(u,v) > Conn(u,v)$

Definition 3.1.2. $\beta-$ **strong arc**
An arc is said to be $\beta-$ strong if $\mu(u,v) = Conn(u,v)$

Definition 3.1.3. $\delta-$ **strong arc**
An arc is said to be $\delta-$ strong if $\mu(u,v) < Conn(u,v)$

Definition 3.1.4. The strength of connectedness between two nodes x and y, is defined as the maximum of strength of all paths between x and y, and it is denoted by $conn_G(x,y)$.

Example 3.1.1.



$\mu(v_1,v_2) = 0.5$

$conn_G(v_1,v_2) = \max\{\min[(v_1,v_4),(v_4,v_3),(v_3,v_2)], \min[(v_1,v_4),(v_{4,}v_2)]\}$

$= \max\{\min[0.6,0.5,0.6], \min[0.6,0.7]\}$

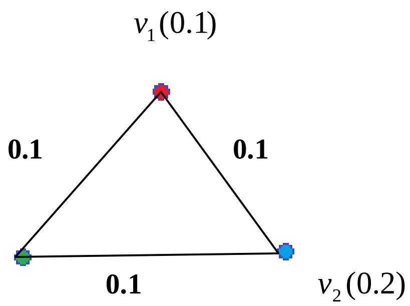
$= \max\{0.5,0.6\}$

$conn_G(v_1,v_2) = 0.6$

$\therefore$ The graph is $\delta-$arc.But by definition an arc is said to be strong if it is either $\alpha-$ arc or $\beta-$arc.

$\therefore$ The graph is not an example of strong graph.

Example 3.1.2.



$\mu(v_1,v_2) = 0.1$

$conn(v_1, v_2) = \max\{\min(v_1, v_2), \min[(v_1, v_3), (v_3, v_2)]\}$

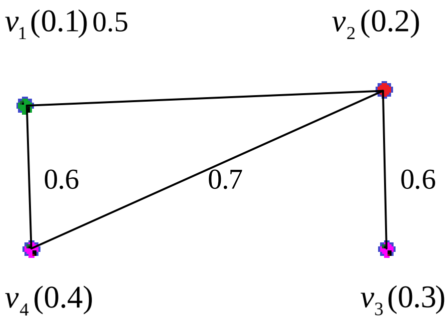
=max{min(0.1),min(0.1,0.1)}

=0.1

$\mu(v_1, v_2) = conn(v_1, v_2)$

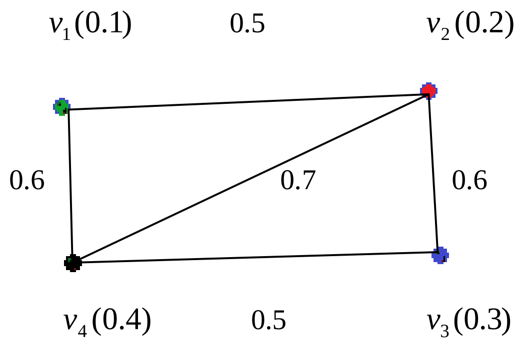
$\therefore$ The graph is $\beta -$ strong arc.

Example 3.1.3.



The graph is not connected. Therefore we does not find any strong arc of this graph.

Example 3.1.4.



$\mu(v_1, v_2) = 0.5$

$conn_G(v_1, v_2) = \max\{\min[(v_1, v_3), (v_3, v_2)], \min[(v_1, v_3), (v_3, v_4), (v_{4,} v_2)]\}$

$= \max\{\min[0.7, 0.4], \min[0.7, 0.5, 0.7]\}$

$= \max\{0.4, 0.5\}$

=0.5

$\mu(v_1, v_2) = conn(v_1, v_2)$

$\therefore$ The graph is $\beta -$ strong arc.

Note. Every odd regular (or) even regular fuzzy graph has a strong arc.That is the graph satisfy $\alpha -$ arc or $\beta -$ arc.

Theorem 3.1.1. If the graph $G=(A,B)$ is odd regular strong arc fuzzy graph iff each vertex has distinct color.

Proof: Let $G=(A,B)$ be a fuzzy graph on $G^*:(A,B)$.If $d(v)=k\forall v\in V$.That is if each vertex has same degree k,then G is regular fuzzy graph.If the number of vertex is odd number then the graph is said to be odd regular graphs otherwise is said to be even regular graphs.

An arc is said to be strong if $\mu(u,v)\geq Conn(u,v)$.Let $V=\{V_1,V_2,.....,V_n\}$.We assume that G is odd (or) even regular strong arc fuzzy graph.

To Prove:

If all the vertex has distinct color.

Every vertex is adjacent to other vertex. By our definition no vertex is adjacent to any vertex in that case the vertex has same color. Otherwise each vertex has distinct color.

Conversely, we assume that the graph G has distinct color.

To Prove:

The graph is odd (or) even regular strong arc fuzzy graph.

By our definition of coloring $\Gamma\{\gamma_1,\gamma_2,.....,\gamma_k\}$ of fuzzy sets on a set V is called a $k-$fuzzy coloring of $G=(V,\sigma,\mu)$ if

(i) $\vee\Gamma=\sigma$

(ii) $\gamma_i\wedge\gamma_j=0$

(iii) For every strong edge

$(x,y)[(i.e)\mu(x,y)>0]$ of $G.\min\{\gamma_i(x),\gamma_i(y)\}=0(1\leq i\leq k)$

If each vertex is adjacent it has distinct color and satisfy the condition $\mu(v_1,v_2)=conn(v_1,v_2)$ (or) $\mu(u,v)>Conn(u,v)$.Then the graph is odd regular strong arc fuzzy graph (or) if any two vertex is not adjacent it has same color in any two vertex also the graph satisfy the condition $\mu(v_1,v_2)=conn(v_1,v_2)$ (or) $\mu(u,v)>Conn(u,v)$. Then the graph is even regular strong arc fuzzy graph.

4. Conclusion

Graph theory is an extremely useful tool for solving different areas. Because research of modeling of real world problems often involve multi-agent, multi-object, multi-index, multi-polar information. In this paper we have described odd and even regular graphs and coloring of strong arcs in the fuzzy graphs. We plan to extend our research of fuzzification to the coloring of regular and irregular bipolar graphs, m-polar graphs and coloring of strong arcs of bipolar and m-polar graphs.

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