Intern. J. Fuzzy Mathematical Archive
Vol. 14, No. 1, 2017, 47-57
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v14n1a7

International Journal of
Fuzzy Mathematical Archive

Fuzzy Colouring of Interval-Valued Fuzzy Graph

***K.Kalaiarasi*[1] and *R.Divya*[2]**

PG and research Department of Mathematics, Cauvery College for Women
Trichy-18, Tamil Nadu, India.
[1]email: kalaishruthi12@gmail.com; [2]email: rdivyamat@gmail.com

Received 1 November 2017; accepted 4 December 2017

Abstract. Fuzzy graphs have revolutionized the analysis of problematic data to arrive at a better decision making power are different kinds. Among them, the interval-valued fuzzy graph in the simplest and generalized once. The main purpose of this paper is to introduce the chromatic number of an interval-valued fuzzy graph. Here working rule of an interval-valued fuzzy graph, power cut graph are discussed.

Keywords: Interval-valued graph, chromatic numbers, power cut graph, independent power full, independent power less.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

In our daily life, the colouring of a graph is the most significant component of research in optimization technology and is used for various applications, viz. administrative sciences, wiring printed circuits, resource allocation [4], arrangement problems, and so on. These problems are represented by proper crisp graphs and are analysed by colouring these graphs. In the usual graph colouring problem, nodes receive the minimum number of colours such that two adjacent nodes do not have the same colour. A few studies discuss this point [6,7,11,14] . An interval- valued fuzzy graph representation is better than a crisp graph version. Interval-valued fuzzy graphs suitably represent every event.

Interval-valued fuzzy graph theory has a broad number of areas. Interval-valued fuzzy set notations and their properties was introduced by Zadeh in 1975 [2]. Interval-valued fuzzy sets are more advanced than fuzzy sets and more completely eliminate doubt. At present, Akram uses interval valued fuzzy graphs .

In graph theory, an intersection graph is a graph which represent the intersection of sets. An interval graph is the intersection of multiset of intervals on real line. Interval graphs are useful in resource allocation problem in operations research. Besides, interval graphs are used extensively in mathematical modeling, archaeology, developmental psychology, ecological modeling, mathematical sociology and organization theory.

In the present paper, a new idea to colour an interval-valued fuzzy graph is presented. Here, “colouring of interval-valued fuzzy graph” is defined. An interval-valued fuzzy graph is coloured by the range-valued fuzzy colour depending on the power of a branch incident to a node. Then, fuzzy colouring of interval-valued fuzzy graph is demonstrated.

2. Preliminaries

Definition 2.1. A graph G is a triplet that contains a set $X \neq \phi$, a branch set D and a connection that links each branch by two nodes (not as a particular matter of course) called its end points.

Definition 2.2. Node colouring and chromatic number
The node colouring of a graph G is the consign- ment of labels or colours to each node of a graph such that no branch links two similarly coloured nodes. The general type of node colouring search minimizes the number of colours for a graph. This type of colouring is known as least node colouring, and the lowest number of colours with which the nodes of a graph G can be coloured is called the chromatic number. The chromatic number of a graph G is denoted by $\chi(G)$.

Definition 2.3. Fuzzy set
Let P be a universal set. Then, the fuzzy set K on S is indicated by function η: S $\rightarrow [0,1]$, which is called the membership function. A fuzzy set is represented by $K = (S, \eta)$.

Definition 2.4. Fuzzy graph
A fuzzy graph $\delta = (X, \psi, \rho)$ is a set $X \neq \phi$ together with the functions $\psi : X \rightarrow [0,1]$ and $\rho : X \times X \rightarrow [0,1]$ such that for all

$$m, n \in X, \rho(m,n) \leq \min\{\psi(m), \psi(n)\},$$

where $\psi(m)$ and $\rho(m,n)$ denote the membership values of the node m and branch (m,n) in δ, respectively.

Definition 2.5. Path in a fuzzy graph
A path in a fuzzy graph is an arrangement of different nodes $p_0, p_1, ...p_n$ such that $\rho(p_{i-1}, p_i) > 0, 1 \leq i \leq n.$ The fuzzy path is said to be a fuzzy cycle if p_0 and p_n overlap.

The original crisp graph of the fuzzy graph $\delta = (X, \psi, \rho)$, is denoted as $\delta^* = (X, \psi^*, \rho^*)$,
where $\psi^* = \{x \in X / \psi(x) > 0\}$ and $\rho^* = \{(x, y) \in X \times X \setminus \rho(x, y) > 0\}$.
For the original fuzzy graph, $\psi^* = X$.

Definition 2.6. Complete fuzzy graph
A fuzzy graph $\delta = (X, \psi, \rho)$ is said to be complete if $\rho(x, y) = \min\{\psi(x), \psi(y)\}$ for all x,y$\in X$, where (x,y) denotes the branch between nodes x and y.

Definition 2.7. Interval-valued fuzzy set
Let X= ϕ and $\psi^- : X \to [0,1]$ and $\psi^+ : X \to [0,1]$ be the mappings such that $\psi^-(x) \le \psi^+(x)$ for all $x \in X$. The interval-valued fuzzy set on X is denoted as $\left(X, [\psi^-, \psi^+]\right)$ and is defined as $\left(X, [\psi^-, \psi^+]\right) = \{(x, [\psi^-, \psi^+]) / x \in X\}$.

Definition 2.8. Interval-valued fuzzy graph
Interval-valued fuzzy graph $I_{FG} = \left(X, [\psi^-, \psi^+], [\rho^-, \rho^+]\right)$ (IVFG) is a set $X \ne \varphi$ together with the functions $\psi^- : X \to [0,1], \psi^+ : X \to [0,1]$ $\rho^- : X \times X \to [0,1]$ and $\rho^+ : X \times X \to [0,1]$ such that for all $u, v \in X$ $\rho^-(u,v) \le \min\{\psi^-(u), \psi^-(v)\}$ and $\rho^+(u,v) \le \min\{\psi^+(u), \psi^+(v)\}$ for every branch (u,v).

Definition 2.9. Complete interval-valued fuzzy graph
The complete interval-valued fuzzy graph $I_{FG} = \left(X, [\psi^-, \psi^+], [\rho^-, \rho^+]\right)$ (IVFG) is a set $X \ne \varphi$ together with the functions $\psi^- : X \to [0,1], \psi^+ : X \to [0,1]$ $\rho^- : X \times X \to [0,1]$ and $\rho^+ : X \times X \to [0,1]$ such that for all $u, v \in X$, $\rho^-(u,v) = \min\{\psi^-(u), \psi^-(v)\}$ and $\rho^+(u,v) = \min\{\psi^+(u), \psi^+(v)\}$ for every branch (u,v).

Definition 2.10. α-cut graph of interval-valued fuzzy graph
For $0 \le \alpha \le 1$, α-cut graph of interval-valued fuzzy graph $I_{FG} = \left(X, [\psi^-, \psi^+], [\rho^-, \rho^+]\right)$ is a crisp graph $I_{FG_\alpha} = (V_\alpha, E_\alpha)$ such that $V_\alpha = \{x \in X / [\psi^-(x), \psi^+(x)] \ge [\alpha, \alpha]\}, E_\alpha = \{(x,y), / [\rho^-(x,y), \rho^+(x,y)] \ge [\alpha, \alpha]\}$.

Definition 2.11. Interval-valued fuzzy neighbourhood
Interval-valued fuzzy neighbourhood of a node x of an interval-valued fuzzy graph $I_{FG} = \left(Y, [\psi^-, \psi^+], [\rho^-, \rho^+]\right)$ is an interval-valued fuzzy set $N(y) = (Y_y, m_y)$, where $Y_y = \{x / [\rho^-(y,x), \rho^+(y,x) > [0,0]]\}$ and $m_y : Y_y \to [0,1]$ is defined by $m_y(x) = \mu(y,x)$.

Definition 2.12. Interval-valued fuzzy star
A power neighbourhood of a node p is a node q such that (p,q) is a power full branch. An interval-valued fuzzy graph I_{FG} is said to be the interval-valued fuzzy star if each node of I_{FG} has precisely one power neighbourin I_{FG}.

3. Power cut graph of interval-valued fuzzy graph
In this part, the α-power cut graph of I_{FG} is defined with an example.

For an interval-valued fuzzy graph $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$, a branch $(m,n), m,n \in X$ is said to be independently powerful if $(0.5)\min\{\psi^+(m), \psi^-(n)\} \le \rho^-(m,n)$ and $(0.5)\min\{\psi^+(m), \psi^-(n)\} \le \rho^+(m,n)$. Otherwise, it is independent and powerless. The power of a branch (p,q) in an interval-valued fuzzy graph $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$ is denoted by $\tau_{(p,q)}$ and is defined as $\tau_{(p,q)} = \left[\tau^-{}_{(p,q)}, \tau^+{}_{(p,q)}\right]$, where $\tau^-{}_{(p,q)} = \dfrac{\rho^-(p,q)}{\min\{\psi^+(p), \psi^-(q)\}}$ and $\tau^+{}_{(p,q)} = \dfrac{\rho^+(p,q)}{\min\{\psi^+(p), \psi^-(q)\}}$. Again the power of a node w is denoted by τ_w and defined as $\tau_w = \left[\tau^-{}_w, \tau^+{}_w\right]$, where $\tau^-{}_w$ is the maximum value along its membership value $\psi^-(w)$,and the powers $\tau^-{}_{(w,x)}$ of branches (w,x) incident to w and $\tau^+{}_w$ are the maximum values along its membership value $\psi^+(w)$ and powers $\tau^+{}_{(w,x)}$ of branches (w,x) incident to w.

Definition 3.1. Let $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$ be an interval-valued fuzzy graph. For $0 \le \alpha \le 1$, the α-power cut graph of I_{FG} is defined to be the crisp graph $I_{FG}{}^{\alpha} = \left(X^{\alpha}, E^{\alpha}\right)$ such that $X^{\alpha} = \{p \in X / \tau_p \ge [\alpha, \alpha]\}$ and $E^{\alpha} = \{(p,q), p,q \in X / \tau_{(p,q)} \ge [\alpha, \alpha]\}$.

Theorem 3.1. Let I_{FG} be an interval-valued fuzzy graph. If $0 \le \alpha \le \beta \le \gamma \le 1$, then $I_{FG}{}^{\gamma} \subseteq I_{FG}{}^{\beta} \subseteq I_{FG}{}^{\alpha}$.

Proof: Suppose $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$ is an interval-valued fuzzy graph and $0 \le \alpha \le \beta \le \gamma \le 1$.

Now, $I_{FG}{}^{\alpha} = \left(X^{\alpha}, E^{\alpha}\right)$ such that $X^{\alpha} = \{p \in X / p \ge [\alpha, \alpha]\}$ and $E^{\alpha} = \{(p,q), p,q \in X / \tau_{(p,q)} \ge [\alpha, \alpha]\}$ and $I_{FG}{}^{\beta} = \left(X^{\beta}, E^{\beta}\right)$ such that $X^{\beta} = \{p \in X / p \ge [\beta, \beta]\}$ and $E^{\beta} = \{(p,q), p,q \in X / \tau_{(p,q)} \ge [\beta, \beta]\}$.

Similarly, $I_{FG}{}^{\gamma} = \left(X^{\gamma}, E^{\gamma}\right)$ such that $X^{\gamma} = \{p \in X / p \ge [\gamma, \gamma]\}$ and $E^{\gamma} = \{(p,q), p,q \in X / \tau_{(p,q)} \ge [\gamma, \gamma]\}$ and $I_{FG}{}^{\gamma} = \left(X^{\gamma}, E^{\gamma}\right)$ such that $X^{\gamma} = \{p \in X / p \ge [\gamma, \gamma]\}$ and $E^{\gamma} = \{(p,q), p,q \in X / \tau_{(p,q)} \ge [\gamma, \gamma]\}$.

Let $m \in X^{\gamma}$. Then $m \ge \gamma \ge \beta \ge \alpha$. Therefore, $m \in X^{\alpha}$. In the same way, for any element $(m,n) \in E^{\alpha}$. Therefore, $I_{FG}{}^{\gamma} \subseteq I_{FG}{}^{\beta} \subseteq I_{FG}{}^{\alpha}$.

The connection between α-cut graph I_{FG_α} of an interval-valued fuzzy graph I_{FG} and power cut graph $I_{FG}{}^{\alpha}$ of an interval-valued fuzzy graph Let I_{FG} is provided in the subsequent theorem.

Theorem 3.2. Let I_{FG} be an interval-valued fuzzy graph. If $0 \le \alpha \le \beta \le 1$, then $I_{FG_\alpha} \subseteq I_{FG^\alpha}$, $I_{FG_\beta} \subseteq I_{FG^\beta}$.

Proof: Suppose $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$ is an interval-valued fuzzy graph. Then , $I_{FG_\alpha} = \left(X_\alpha, E_\alpha\right)$ such that $X_\alpha = \left\{p \in X / [\psi^-(p), \psi^+(p)] \ge [\alpha, \alpha]\right\}$ and $E_\alpha = \left\{(p,q), / [\rho^-(p,q), \rho^+(p,q)] \ge [\alpha, \alpha]\right\}$.

Similarly, $I_{FG_\beta} = \left(X_\beta, E_\beta\right)$ such that $X_\beta = \left\{p \in X / [\psi^-(p), \psi^+(p)] \ge [\beta, \beta]\right\}$and $E_\beta = \left\{(p,q), / [\rho^-(p,q), \rho^+(p,q)] \ge [\beta, \beta]\right\}$.

Again, $I_{FG}{}^{\alpha} = \left(X^\alpha, E^\alpha\right)$ such that $X^\alpha = \left\{p \in X / p \ge [\alpha, \alpha]\right\}$ and $E^\alpha = \left\{(p,q), p, q \in X / \tau_{(p,q)} \ge [\alpha, \alpha]\right\}$ .Let $p, q \in X_\alpha$ and $p, q \in E_\alpha$. Therefore, $[\psi^-(p), \psi^+(p)] \ge [\alpha, \alpha]$ and $[\rho^-(p,q), \rho^+(p,q)] \ge [\alpha, \alpha]$. These results along with $\alpha \le 1$.

And $I_{FG}{}^{\beta} = \left(X^\beta, E^\beta\right)$ such that $X^\beta = \left\{p \in X / p \ge [\beta, \beta]\right\}$and $E^\beta = \left\{(p,q), p, q \in X / \tau_{(p,q)} \ge [\beta, \beta]\right\}$. Let $p, q \in X_\beta$ and $p, q \in E_\beta$. Therefore, $[\psi^-(p), \psi^+(p)] \ge [\beta, \beta]$ and $[\rho^-(p,q), \rho^+(p,q)] \ge [\beta, \beta]$. These results along with $\beta \le 1$.

Give

$$\left[\frac{\rho^-(p,q)}{\min\left\{\psi^+(p), \psi^+(q)\right\}}, \frac{\rho^+(p,q)}{\min\left\{\psi^+(p), \psi^+(q)\right\}}\right] \ge [\alpha, \alpha]$$

$$\left[\frac{\rho^-(p,q)}{\min\left\{\psi^+(p), \psi^+(q)\right\}}, \frac{\rho^+(p,q)}{\min\left\{\psi^+(p), \psi^+(q)\right\}}\right] \ge [\beta, \beta]$$

And $\tau_{(x,y)} \ge [\alpha, \alpha]$; $\tau_{(x,y)} \ge [\beta, \beta]$; therefore $(p,q) \in E^\alpha$ and $(p,q) \in E^\beta$. Thus, for every branch I_{FG_α} and I_{FG_β} there is a branch I_{FG^α} and I_{FG^β}.

Now, clearly from the definition of power of nodes $X_\alpha \subseteq X^\alpha$, $X_\beta \subseteq X^\beta$. Hence, the result $I_{FG_\alpha} \subseteq I_{FG^\alpha}, I_{FG_\beta} \subseteq I_{FG^\beta}$ is true.

3.1. Working rule of an interval-valued fuzzy graph colouring

In crisp graph colouring, if any two nodes are adjacent, then these two nodes receive distinct colours; if not, the colours is the same. Here, an interval-valued fuzzy graph is coloured by the range-valued fuzzy colour. In this way, two nodes have distinct fundamental colours if they are adjacent to an independent power full branch. If not , they have distinct range-valued fuzzy colours.

Suppose $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$ is a connected interval-valued fuzzy graph and $P = \{p_1, p_{2,} .... p_k\}$ is a set of fundamental colours. We observed that there are two types of interval-valued fuzzy branches in interval-valued fuzzy graphs, viz. independent power Engineering Branches and independent Non power Engineering Branches. The independent power less branch is less significant than the independent power full branch. As a result, the relationship between the corresponding nodes is independent power less. Our latest colouring idea is like conventional graph colouring, which depends on independent power full and independent power less branches. The planed interval-valued fuzzy graph colouring can be divied into three groups that depend on the power of the branches.

Viz. (i) All branches are independent power full.
(ii) Some branches are independent power full.
(iii) All branches are independent power less.

Case 1. The interval-valued fuzzy graph contains all indepent power full branches.
If an interval-valued fuzzy graph contains all independent power full branches, the colouring of this interval-valued fuzzy graph is identical to the colouring of crisp graphs, such that two nodes can be coloured by two different range-valued fuzzy fundamental colours if there is an independent power full branch between the nodes.

Case 2. The interval-valued fuzzy graph contains some independent power full branches.
Suppose u is a node and the set of all neighbourhoods of node u is $N(u) = \{u_j, j = 1,2,......n\}$. For simplicity, We assume that v_1, v_2 are two nodes such that $(u, v_1), (u, v_2)$ are only the two independent power full branches incident on u and all remaining branches $(u, v_i), i = 3,4...n$ are independent power less branches. In this case, we colour u by the colour $(c_1, [1,1])$ and v_1 by a different colour .Similarly, v_2 will achieve a distinct colour other than the colour of u.

Now, we consider node v_3 for colouring. Notice that (u, v_3) is an independent power less branch.

Sub case 2.1. None of the adjacent nodes of v_3 are coloured.

As (u, v_3) is an independent power less branch, v_3 has a range-valued fuzzy colour corresponding to the colour of u. If the colour of u is $(c_1, [1,1])$, the interval-valued fuzzy colour of v_3 is $\left(C, \left[g^-(c), g^+(c)\right]\right)$, where $\left[g^-(c), g^+(c)\right]$ can be calculated as $[g^-(c), g^+(c)] = [1 - \tau^+{}_{(u,v_3)}, 1 - \tau^-{}_{(u,v_3)}]$.

Sub case 2.2. All adjacent nodes of v_3 are coloured.

If a branch (v_3, z) incident to v_3 is independent power full, then of v_3 cannot be coloured by the colour of z. That is, if the colour of z is $\left(c_z, \left[g^-(c_z), g^+(c_z)\right]\right)$, v_3

cannot be coloured by any range-valued fuzzy colour of c_z. Suppose that v_3 has some independent power less incident branches, then $(v_3, z_i), i = 1,2....q.$ without a loss of generality, we assume that the colour of z_i is

$\left(y_i, \left[g^-(y_i), g^+(y_i)\right]\right)$, i=1,2,….q.

Now, to determine the colourof v_3, Calculate the lengths of the intervals $\left[1-\tau^+_{(v_3,z_1)}, 1-\tau^-_{(v_3,z_1)}\right]$, $\left[1-\tau^+_{(v_3,z_2)}, 1-\tau^-_{(v_3,z_2)}\right]$, ... $\left[1-\tau^+_{(v_3,z_q)}, 1-\tau^-_{(v_3,z_q)}\right]$, and intervals $\left[1-\tau^+_{(u,v_3)}, 1-\tau^-_{(u,v_3)}\right]$ to determine the maximum length of the interval and then take the interval as L, corresponding to the maximum length.

Let L contribute to the branch (v_3, z_p), i.e., L=$\left[1-\tau^+_{(v_3,z_q)}, 1-\tau^-_{(v_3,z_q)}\right]$. If the colour of z_p is $(y_p, g(y_p))$, then v_3 receives the colour (y_p, L). If the interval lengths are the same, an arbitrary choice can be made.

Sub case 2.3. Some of the adjacent nodes of v_3 are coloured.

The adjacent nodes that are not coloured do not affect the colouring of v_3. Then, the adjacent nodes that are coloured will be considered for the colouring of v_3. The process of colouring v_3 is comparable to the sub case 2.2.

After colouring v_3, all of the other nodes are coloured in the same manner.

Case 3. Interval-valued fuzzy graph contains all independent power less branches.

If I_{FG} contains all independent power less branches, only one fundamental colour is needed to colour the nodes of I_{FG}. Arbitrarily consider any node (say x) and assign the fundamental colour to this node $(c_x, [1,1])$; for the remaining nodes, assign the range-valued fuzzy colours corresponding to $(c_x, [1,1])$ similar to the above sub case 2.2.

Remark. If an interval-valued fuzzy graph has more than one component and each component is coloured by the above described method.

3.2. Chromatic number of an interval-valued fuzzy graph colouring

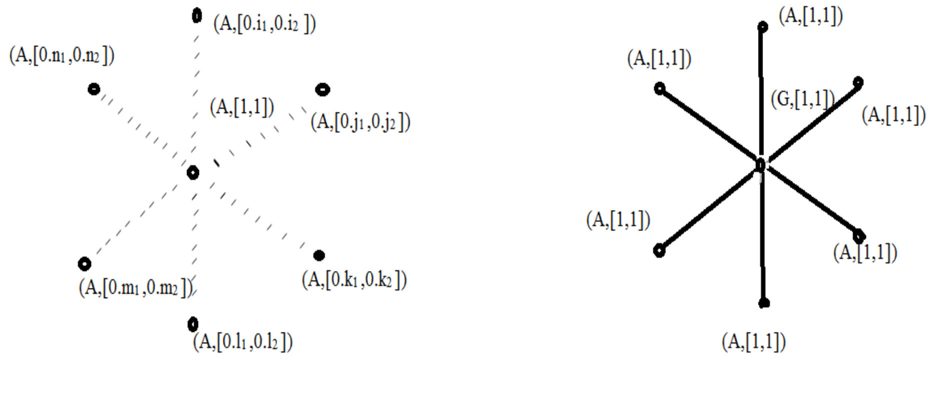
The lowest number of fundamental colours used to colour an interval-valued fuzzy graph is known as a fuzzy chromatic number. A fuzzy chromatic number of an interval-valued fuzzy graph I_{FG} is denoted by $\gamma(I_{FG})$. Here, we give two examples for various fuzzy chromatic numbers.

Suppose we consider two interval-valued fuzzy graphs in which Fig.2(a) contains all independent power less branches are independent power less, One fundamental colour is sufficient to colour all of the nodes. Here the central node is coloured by $(A, [1,1])$ and the other nodes are coloured by different range-valued fuzzy colours $(A, [0.i_1, 0.i_2])$, $(A, [0.j_1, 0.j_2])$, $(A, [0.k_1, 0.k_2])$, $(A, [0.l_1, 0.l_2])$, $(A, [0.m_1, 0.m_2])$ and $(A, [0.n_1, 0.n_2])$ of G, where $i_1, i_2, j_1, j_2, k_1, k_2, l_1, l_2, m_1, m_2, n_1$ and n_2 are natural numbers.

Therefore, the fuzzy chromatic number of this interval-valued fuzzy graph is one. In Fig.2(b), all of the branches are independent power full.

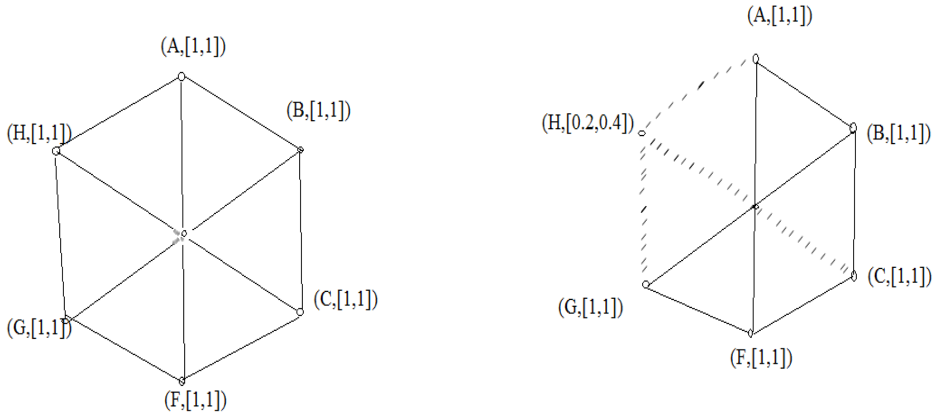
Suppose we take four interval-valued fuzzy graphs whose original graph is k_6. In Fig.3(a), an interval-valued fuzzy graph is measured such that all of its branches are independent powerfull. At that point, the interval-valued fuzzy graph is six chromatic as for k_6.

In Fig. 3(b), an interval-valued fuzzy graph is considered such that branches incident to exactly one node are independent power less and the remaining are power full. Again, colouring of one node depends on this type of independent power less incident branches whose other end nodes are coloured.

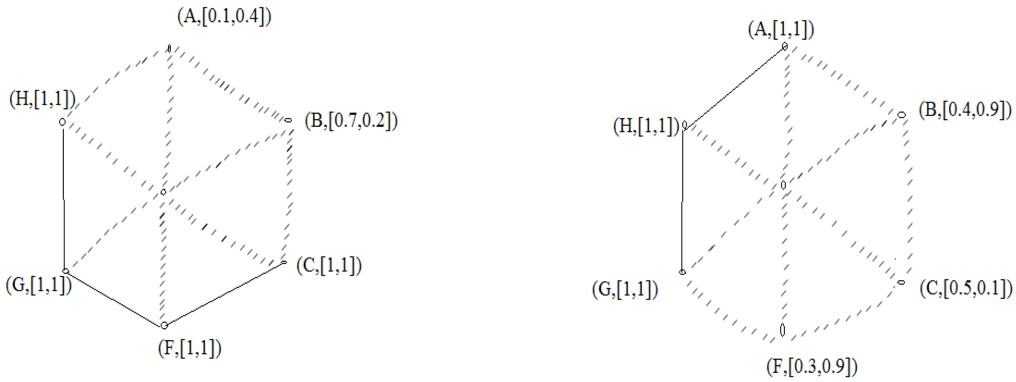


(a) Fuzzy chromatic number 1 (b) Fuzzy chromatic number 2

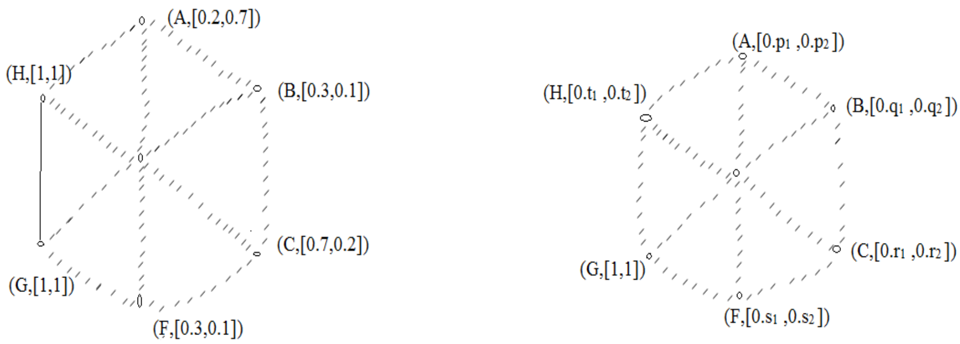
Figure 2: Fuzzy chromatic number graphs.



a) $\gamma(F_G) = 6$ b) $\gamma(F_G) = 5$



c) $\gamma(F_G) = 4$ d) $\gamma(F_G) = 3$



e) $\gamma(F_G) = 2$ f) $\gamma(F_G) = 1$

Figure 3: Interval-valued fuzzy graph.

That node is coloured by a range-valued fuzzy colour that is determined by measuring the powers of such independent power less branches.

Let the minimum of the powers be [m,n] (here m=0.8,n=0.6), corresponding to the branch (u,v) and v is coloured by (G,[1,1]). Then, node u will be coloured by the range-valued fuzzy colour (G,[1-n,1-m]) i.e.,(G,[0.2,0.4]). Similarly, four other interval-valued fuzzy graphs are coloured(see Fig.3(c),(d),(e) and (f).

Theorem 3.2.1. If I_{FG} is an interval-valued fuzzy graph, then $\chi\left(I_{FG}{}^{0.2}\right) = \gamma(I_{FG})$

Proof. Let $I_{FG} = \left(X, \left[\psi^-, \psi^+\right], \left[\rho^-, \rho^+\right]\right)$ be an interval-valued fuzzy graph. Now, $I_{FG}{}^{0.2} = (V^{0.2}, E^{0.2})$, where $V^{0.2} = \left\{p \in V / \tau_p \geq [0.2,0.2]\right\}$ and $E^{0.2} = \left\{(p,q) / \tau_{(p,q)} \geq [0.2,0.2]\right\}$.

All independent power full branches of I_{FG} are branches of $I_{FG}{}^{0.5}$. Thus, these is a one to one connection between the independent power full branches in I_{FG}

and the branches in $I_{FG}{}^{0.2}$, i.e., there is one branch of $I_{FG}{}^{0.2}$ for all independent power full branch in I_{FG}. Thus, $\chi\left(I_{FG}{}^{0.2}\right)=\gamma(I_{FG})$.

Remark. We know that for complete crisp graph $k_n, \chi(k_n)$ is the number of its nodes. This result gives the following statement. If all of the branches of a complete interval valued fuzzy graph $I_{FG}=\left(X,[\psi^-,\psi^+],[\rho^-,\rho^+]\right)$ are independent power full, then $\gamma(I_{FG})$ is the number of nodes of I_{FG}.

4. Conclusion

This paper discussed interval-valued fuzzy graph with chromatic number of fuzzy graph colouring. There are several diverse ideas regarding the quality of a branch in an interval-valued fuzzy graph. The connection between the chromatic number of interval-valued fuzzy graph and its power cut graphs are also recognized.

REFERENCES

1. F.S.Al-Anzia, Y.N.Sotskov, A.Allahverdi, G.V.Andreev, Using mixed graph coloringto minimize total completion time in job shop scheduling, *Appl. Math. Comput.*, 182 (2) (2006) 1137-1148 .
2. L.A.Zadeh, Similarly relations and fuzzy orderings, *Inf. Sci.*, 3 (2) (1971) 177-200.
3. K.R.Bhutani and A.Rosenfeld, Strong arcs in fuzzy graphs, *Inf. Sci.*, 152 (2003) 319-322.
4. N. Christofides, Graph Theory: An Algorithmic Approach, Academic Press, 1975.
5. M.Gamache, A. Hertz, J.O. Ouellet, A graph colouring model for a feasibility problem in monthly crew scheduling with preferential bidding, *Comput. Operations Res.*, 34 (8) (2007) 2384-2395.
6. LT. Koczy, Fuzzy graphs in the evaluation and optimization of networks, *Fuzzy Sets Syst.*, 46 (1992) 307-319.
7. J.Korst, E.Aartsas, J.K. Lenstra and J. Wessels, Periodic assignment and graph colouring, *Discrete Appl. Math.*, 51 (1994) 291-305.
8. R.Lewis and J.Thompson, On the application of graph colouring techniques in round-robin sports scheduling, *Comput. Operations Res.*, 38 (1) (2011) 190-204.
9. C.L Liu, Introduction to Combinatorial Mathematics, McGraw-Hill, 1968.
10. J.N. Mordeson and P.S. Nair, *Fuzzy Graphs and Hypergraphs*, PhysicaVerlag, 2000.
11. S.K. Pal and S.S. Sarma, Graph coloring approach for hiding of information, *Procedia Technol.*, 4 (2012) 272-227.
12. S.Samanta and M. Pal, Fuzzy planar graph, *IEEE Trans. Fuzzy Syst.*, 23 (6) (2015) 1936-1942.
13. M. Akram and W.A. Dudek, Interval valued fuzzy graphs, *Comput. Math. Appl.*, 61 (2011) 288-289.
14. V.Yegnanarayanan, Graph colourings and partitions, *Theor. Comput. Sci.*, 263 (2001) 59-74.
15. R.Kalaiarasi and W.Ritha, Optimization of economic order quantity model on the boundaries of the fill rate, *International Mathematical Forum*, 6(63) (2011) 3101-3110.

16. S.Samanta and M.Pal, Fuzzy tolerance graphs, *Int. J. Latest Trends Math.*, 192 (2011) 57-67.
17. S.Samanta, T.Pramanik and M.Pal, Fuzzy colouring of fuzzy graphs, *Afrika Matematika*, 27(1-2) (2016) 37-50.
18. Sk. Md. Abu Nayeem and M.Pal, Genetic algorithm to solve the *p*-centre and *p*-radius problem on a network, *Intern. Journal of Computer Mathematics*, 82(5) (2005) 541--550.
19. Sk. Md. Abu Nayeem and M.Pal, Shortest path problem on a network with imprecise edge weight, *Fuzzy Optimization and Decision Making*, 4 (2005) 293-312.
20. S.Samanta and M.Pal, Fuzzy *k*-competition graphs and *p*-competition fuzzy graphs, *Fuzzy Inf. Eng.*, 5(2) (2013) 191-204.
21. S.Samanta, M. Akram and M. Pal, m-step fuzzy compitition graphs, J*ournal of Applied Mathematics and Computing*, 47 (2015) 461-472.