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Application of Fuzzy Matrices in Medical Diagnosis

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Abstract. In this paper we define fuzzy matrices and an algorithm is developed for medical diagnosis. Example is illustrated to verify the procedure developed.

Keywords: Fuzzy matrix, complement of fuzzy matrix, sup i-composition, relativity function, medical diagnosis and decision making.

AMS Mathematics Subject Classification (2010): 15B15

1. Introduction

In 1965 [8], Zadeh introduced the notion of fuzzy set theory. The field of medicine is one of the most fruitful and interesting areas of applications for fuzzy set theory. Sanchez [6] formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases. Esogbue and Eder [3] utilized fuzzy cluster analysis to model medical diagnostic.

Meenakshi and Kaliraja [4] have extended Sanchez's approach for medical diagnosis using representation of a interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it.

Fuzzy set theory also plays a vital role in the field of decision making. Decision making is a most important scientific, social and economic endeavour. For decision making in fuzzy environment one may refer Bellman and Zadeh [1]. Most probably the fuzzy decision model in which over all ranking or ordering of different fuzzy sets are determined by using comparison matix introduced and developed by Shimura [7]. In 2010 Cagman et.al [2] defined fuzzy soft matrix theory and its application in decision making.

2. Preliminaries

Definition 2.1. An mxn matrix $A = (a_{ij})$ whose components are unit interval [0,1] is called a fuzzy matrix.

Definition 2.2. Let $A = (a_{ij})_{mxn}$ be a fuzzy matrix of order mxn then the complement of A is denoted by,

$A^c = (c_{ij})$ where $c_{ij} = 1 - a_{ij}$ for all i and j.

Definition 2.3. Let A = (a_{ij}) and B = (b_{ij}) be the fuzzy matrices of order mxn. Then the sup-i composition is defined as, $A \overset{i}{\circ} B$ sup i $[A(x, y)b(y, z)]$ where i is a t-norm.

Definition 2.4. Let x and y be variables defined on a universal set X. The relativity function is denoted by $f\left(\frac{x}{y}\right)$ where,

$$f\left(\frac{x}{y}\right) = \frac{\mu_y(x) - \mu_x(y)}{\max\{\mu_y(x), \mu_x(y)\}}$$

where $\mu_y(x)$ is the membership function of x with respect to y.

Definition 2.5. Let $A = \{x_1, x_2, ...x_{i-1}, x_i, x_{i+1}, ...x_n\}$ be a set of n variables defined on a universal set X. Form a matrix of relativity values $f\left(\frac{x_i}{y_i}\right)$ where x_i's (for i=1 to n) are n variables defined on an universe X. The matrix R = (r_{ij}) is a square matrix of order 'n' is called the comparison matrix with $f\left(\frac{x_i}{y_i}\right) = \mu_{x_j}(x_i)$.

3. Medical diagnosis and decision making

Fuzzy matrix frame work have been utilized in several different approaches to model the medical diagnostic process and decision making process.

A fuzzy decision making is a almost important scientific social and economic endeavour, there exist several major approaches within the theories of fuzzy decision making. The ranking order to deal with the vagueness in imprecise determination of preferences.

3.1. Procedure

Let S be the set of symptoms of certain diseases, D is a set of diseases and P is a set patients.

Step 1: Construct a symptom – disease fuzzy matrix $A = (a_{ij})$ of order mxn.

Step 2: Construct a patient – symptom fuzzy matrix $B = (b_{ij})$ of order mxn.

Step 3: Compute $C = A \overset{i}{\circ} B$ where o is sup – i composition.

Step 4: Form the complement matrix A^c and B^c of A and B of order mxn.

Step 5: Computing the composition matrix $A^c \overset{i}{\circ} B^c$ of A^c and B^c denoted by,

$D = A^c \overset{i}{\circ} B^c = \sup i\{A^c(x, y) B^c(y, z)\}$ where i = ab. And compute M = C (-) D.

where (-) denotes the min operator.

Step 6: Calculating the relativity values $f\left(\frac{p_i}{d_i}\right)$ and form the comparision matrix

$$R = \left(r_{ij}\right)_{mxn} = \left[f\left(\frac{p_i}{d_i}\right)\right]_{i=1,2,3.}$$

Then maximum value in each of the rows of the R – matrix will have the maximum possibility for ranking purposes. This gives solution to the required problem.
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3.2. Illustrative example

Suppose there are patients P_1, P_2 and P_3 in a hospital with symptoms high temperature, head ache and cough. These symptoms are represented by S_1, S_2 and S_3. Let the possible diseases relating to the above symptoms be Dengue, Viral fever and Malaria which are represented by d_1, d_2 and d_3 respectively.

Step 1: Consider a symptom – disease fuzzy matrix A of order 3x3 such that

$$A = \begin{matrix} & d_1 & d_2 & d_3 \\ s_1 & 0.5 & 0.6 & 0.4 \\ s_2 & 0.2 & 0.7 & 0.3 \\ s_3 & 0.6 & 0.4 & 0.7 \end{matrix}$$

Step 2: Consider a patient – symptom fuzzy matrix B as,

$$B = \begin{matrix} & s_1 & s_2 & s_3 \\ p_1 & 0.4 & 0.2 & 0.7 \\ p_2 & 0.3 & 0.6 & 0.9 \\ p_3 & 0.7 & 0.5 & 0.8 \end{matrix}$$

Step 3: Compute $C = A \overset{i}{\circ} B$ where o is sup-i composition

$$A \overset{i}{\circ} B = \begin{bmatrix} 0.5 & 0.6 & 0.4 \\ 0.2 & 0.7 & 0.3 \\ 0.6 & 0.4 & 0.7 \end{bmatrix} \overset{i}{\circ} \begin{bmatrix} 0.4 & 0.2 & 0.7 \\ 0.3 & 0.6 & 0.9 \\ 0.7 & 0.5 & 0.8 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.3 & 0.4 & 0.5 \\ 0.2 & 0.4 & 0.6 \\ 0.5 & 0.4 & 0.6 \end{bmatrix}$$

Step 4: Form the complement matrix A^c and B^c of A and B,

$$A^c = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.8 & 0.3 & 0.7 \\ 0.4 & 0.6 & 0.3 \end{bmatrix}, \quad B^c = \begin{bmatrix} 0.6 & 0.8 & 0.3 \\ 0.7 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}$$

Step 5: Computing the composition matrix $A^c \overset{i}{\circ} B^c$ denoted by D.

$$D = A^c \overset{i}{\circ} B^c$$

$$D = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.8 & 0.3 & 0.7 \\ 0.4 & 0.6 & 0.3 \end{bmatrix} \overset{i}{\circ} \begin{bmatrix} 0.6 & 0.8 & 0.3 \\ 0.7 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.5 & 0.6 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix}$$

and M = C (-) D

$$= \begin{bmatrix} 0.3 & 0.4 & 0.5 \\ 0.2 & 0.4 & 0.6 \\ 0.5 & 0.4 & 0.6 \end{bmatrix} (-) \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.5 & 0.6 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix}$$

$$M = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \overset{\begin{matrix} d_1 & \quad d_2 & \quad d_3 \end{matrix}}{\begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix}}$$

represents the relation between the patients and the diseases.

Step 6: Calculate the relativity values of $f\left(\frac{p_i}{d_i}\right)$ and form the comparison matrix

$$R = (r_{ij}), \; = \; f\left(\frac{p_i}{d_i}\right)_{i=1,2,3}$$

$$f\left(\frac{p_i}{d_i}\right) = \frac{\mu_{d_i}(p_i) - \mu_{p_i}(d_i)}{\max\left\{\mu_{d_i}(p_i),\ \mu_{p_i}(d_i)\right\}}$$

$$f\left(\frac{p_1}{d_1}\right) = \frac{\mu_{d_1}(p_1) - \mu_{p_1}(d_1)}{\max\left\{\mu_{d_1}(p_1),\ \mu_{p_1}(d_1)\right\}} = \frac{0.3 - 0.3}{\max\ (0.3, 0.3)} = \frac{0}{0.3} = 0.$$

$$f\left(\frac{p_1}{d_2}\right) = \frac{\mu_{d_2}(p_1) - \mu_{p_1}(d_2)}{\max\left\{\mu_{d_2}(p_1),\ \mu_{p_1}(d_2)\right\}}$$

$$= \frac{0.4 - 0.2}{\max\{0.4, 0.2\}} = \frac{0.2}{0.4} = 0.5$$

$$f\left(\frac{p_1}{d_3}\right) = \frac{\mu_{d_3}(p_1) - \mu_{p_1}(d_3)}{\max\left\{\mu_{d_3}(p_1),\ \mu_{p_1}(d_3)\right\}}$$

$$= \frac{0.2 - 0.4}{\max\{0.2, 0.4\}} = \frac{-0.2}{0.4} = -0.5$$

$$f\left(\frac{p_2}{d_1}\right) = \frac{\mu_{d_1}(p_2) - \mu_{p_2}(d_1)}{\max\left\{\mu_{d_1}(p_2),\ \mu_{p_2}(d_1)\right\}}$$

$$= \frac{0.2 - 0.4}{\max\{0.2, 0.4\}} = \frac{-0.2}{0.4} = -0.5$$

$$f\left(\frac{p_2}{d_2}\right) = \frac{\mu_{d_2}(p_2) - \mu_{p_2}(d_2)}{\max\left\{\mu_{d_2}(p_2),\ \mu_{p_2}(d_2)\right\}}$$

$$= \frac{0.4 - 0.4}{\max\{0.4, 0.4\}} = \frac{0}{0.4} = 0$$

$$f\left(\frac{p_2}{d_3}\right)=\frac{\mu_{d_3}(p_1)-\mu_{p_2}(d_3)}{\max\{\mu_{d_3}(p_2),\ \mu_{p_2}(d_3)\}}$$

$$=\frac{0.2-0.3}{\max 3}=\frac{-0.2}{0.3}=-0.6$$

$$f\left(\frac{p_3}{d_1}\right)=\frac{\mu_{d_1}(p_3)-\mu_{p_3}(d_1)}{\max\{\mu_{d_1}(p_3),\ \mu_{p_3}(d_1)\}}$$

$$=\frac{0.4-0.2}{\max\{0.4,0.2\}}=\frac{0.2}{0.4}=0.5$$

$$f\left(\frac{p_3}{d_2}\right)=\frac{\mu_{d_2}(p_3)-\mu_{p_3}(d_2)}{\max\{\mu_{d_2}(p_3),\ \mu_{p_3}(d_2)\}}$$

$$=\frac{0.3-0.2}{\max\{0.3,0.2\}}=\frac{0.1}{0.3}=0.3$$

$$f\left(\frac{p_3}{d_3}\right)=\frac{\mu_{d_2}(p_3)-\mu_{p_3}(d_2)}{\max\{\mu_{d_3}(p_3),\ \mu_{p_3}(d_2)\}}$$

$$=\frac{0.1-0.1}{\max\{0.1,0.1\}}=0$$

$\therefore$ The comparison matrix $R=\begin{bmatrix} 0 & 0.5 & -0.5 \\ -0.5 & 0 & -0.6 \\ 0.5 & 0.3 & 0 \end{bmatrix}$

Maximum of ith row $=\begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix}$

From the ranking of the problem, conclude that patient P_1 is easily affected by Viral fever, P_2 is also affected by Viral fever and patient P_3 is affected by Dengue.

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