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Matching in Fuzzy Labeling Graph

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Abstract. In this paper, a new concept of matching in fuzzy labeling graph is introduced. A graph is said to be a fuzzy labeling graph if it has fuzzy labeling. Matching, Perfect Matching, M - alternating path, an M-augmenting path, saturated vertex and bipartite fuzzy graph have been discussed.

Keywords: Matching, M-alternating path, M-augmenting path.

AMS Mathematics Subject Classification (2010): 05C72, 54E50, 03F55

1. Introduction

In 1736, Euler introduced the concept of graph theory. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology,operations research and computer science.

Uncertainty in real life can be managed by fuzzy sets applications. Fuzzy set was first introduced by Zadeh in the year 1965. Rosenfeld (1975) introduced the notion of fuzzy graph. Graph labeling was first introduced in the late 1960's. In the intervening years dozens of graph labeling techniques have been studied in over 1000 papers.

Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or not related to each other. Mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes the values from [0,1].

In crisp graph, the labeling of graph is a mapping from the graph elements to numbers. The domain of labeling is the set of vertices and edges which is always mapped to integer. That is $f : V(G) \cup E(G) \rightarrow N$ such labeling is called total labeling.

In the mathematical discipline of graph theory, a matching or independent edge set of in a graph is a set of edges without common vertices. A vertex is matched (or saturated) if it is an end point of one of the edges in the matching.

2. Preliminaries

Definition 2.1. Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of U X V. A fuzzy graph $G = (\alpha, \beta)$ is a pair of functions $\alpha : V \rightarrow [0, 1]$ and $\beta: V \times V \rightarrow [0,1]$ where for all $u,v \in V$,we have $\beta(u,v) \leq \min\{\alpha(u), \alpha(v)\}$.

Definition 2.2. A graph G = (α , β) is said to be a *fuzzy labeling graph* if α : V $\rightarrow$[0, 1] and β: V XV $\rightarrow$ [0,1] is a bijective such that the membership value of edges and vertices are distinct and β (u ,v) < min { α(u), α (v) for all u ,v $\in$V.

Example 2.1.

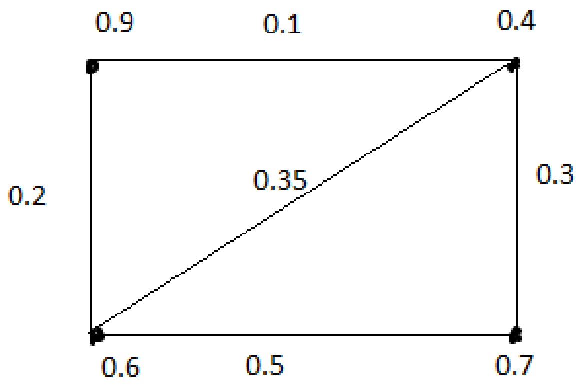

Figure 1: Fuzzy labeling graph

Definition 2.3. A *path* p in a fuzzy graph is a sequence of distinct points $v_1, v_2, ..., v_n$ such that $\beta(v_i ,v_{i+1}) > 0$ ($1\leq i \leq n-1$) here $n \geq 1$ is called the length of the path P. The consecutive pairs (v_i ,v_{i+1}) are called the edges of the path P.

Definition 2.4. A fuzzy graph G = (α , β) is said to be *complete* if β (u ,v) = min {α(u), α (v)} for all u ,v $\in$V.

Example 2.2.

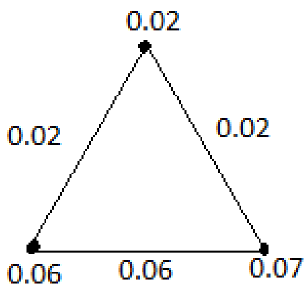

Figure 2:

Definition 2.5. A *star* in a fuzzy graph consists of two node sets V and U with $|V| = 1$ and $|U| \geq 1$ such that β (v, u_i) > 0 and β (u_i ,u_{i+1}) = 0 $1\leq i\leq$ n. It is denoted by $S_{1,n}$.

Example 2.3.

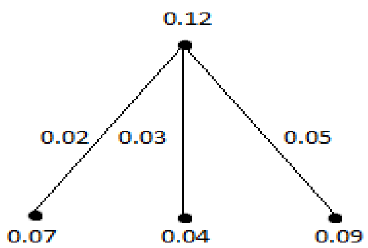

Figure :2 fuzzy star graph

Figure 3:

3. Main results

Definition 3.1. A subset M of β (v_i ,v_{i+1}) ,$1\leq i \leq n$ is called a *matching* in fuzzy graph if its elements are links and no two are adjacent in G. The two ends of an edge in M are said to be matched under M.

Example 2.4.

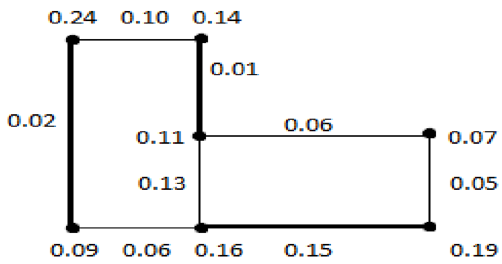

Figure 4:

Definition 3.2. A matching M saturates a vertex v then v is said to be *M-saturated.*

Example 2.5.

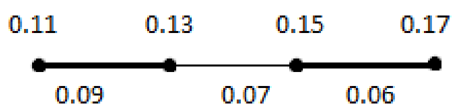

Figure 5:

The vertices {0.11 ,0.13,0.15,0.17 } are M- saturated.

Definition 3.3. If every vertex of fuzzy graph is M-saturated then the matching is said to be *perfect*.

Example 2.6.

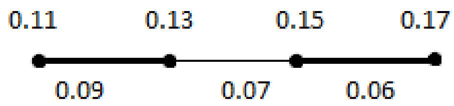

Figure 6: Perfect matching

Definition 3.4. Let M be a matching in fuzzy graph. An *M-alternating path* in G is a path whose edges alternatively in β –M and M.

Definition 3.5. An *M-Augmenting path* is an M-alternating path whose origin and terminal vertices are M-unsaturated.

Example 2.7.

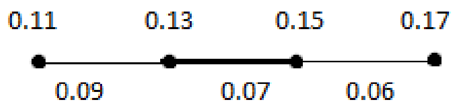

Figure 7:

Definition 3.6. A *bipartite fuzzy graph* consists of two node sets V and U with $|V| \geq m$ and $|U| \geq n$ such that $\beta(v, u_i) > 0$ and $\beta(u_i, u_{i+1}) = 0$, $1 \leq i \leq n$ and $m,n \in N$. It is denoted by $(f\text{-}g)_{m,n}$.

Definition 3.7. For any set of vertices in G , We define the *neighbour set* $N_G(S)$ to be the set of all vertices adjacent to the vertices in S.

Theorem 3.8. A matching M in G is maximum if and only if G contains no M-augmenting path.

Proof: Let M be maximum matching in a fuzzy labeling graph G. To prove G contains no M- Augmenting path.

Suppose we assume that G contains M- Augmenting path $v_0v_1v_2\ldots\ldots\ldots v_{2m+1}$, $1 \leq m \leq n$ and n is a positive integer.

Define M′ be proper or improper subset of $\beta(v_i, v_{i+1})$ by

$M' = \{ M / \{v_1v_2, v_4v_5\ldots\ldots\ldots v_{2m-1}v_{2m}\} \cup \{v_0v_1, v_2v_3\ldots\ldots\ldots v_{2m}v_{2m+1}\}\}$

Then M′ is a matching in a fuzzy labeling graph G and $|M'| = |M| + 1$. Thus M′ is amaximum matching. This gives a contradiction to our assumption.

Hence G contains no M- Augmenting path.

Conversely we assume that G contains no M- Augmenting path. To prove M be maximum matching in a fuzzy labeling graph G.

Suppose we assume that M is not a maximum matching in G and let M′ be maximum matching in G. Then $|M'| > |M|$ (A)

Set H =G [M∆ M′],where M∆ M′ = (M-M′) U(M′ -M).Then each vertex of H has degree either one or two in H, since it can be incident with at most one edge of M and one edge of M′.

Thus each component of H is either an even cycle with edges alternatively in M and M′ or else a path with edges alternatively in M and M′.

By equation (A), H contains more edges of M′ that of M and therefore some path component P of H must start and end with edges of M′.

The origin and terminal vertices are M′-saturated in H and are M-unsaturated in G. Thus P is an M-augmenting path in G which gives contradiction.

Therefore, M be a maximum matching in G.

Theorem 3.9. Let G be a fuzzy bipartite graph with bipartition (U,V).Then G contains a matching that saturates every vertex in U if and only if $|N(S)| \geq |S|$for all S contained in U.

Proof: Suppose that G contains a matching that saturates every vertex in U and let S be the subset of U.

Since the vertices of S are matched under M with distinct vertices in N (S),then clearly we get $|N(S)| \geq |S|$ for all S contained in U.

Conversely, we assume that$|N(S)| \geq |S|$for all S contained in U.

To prove G contains a matching that saturates every vertex in U.

Suppose that G contains no matching that saturates every vertex in U. Then $|N(S)| \geq |S|$ for all S contained in U not satisfied.

Hence G contains a matching that saturates every vertex in U.

Definition 3.10. Let $G_1 : (\alpha_1, \beta_1)$ and $G_2 : (\alpha_2, \beta_2)$ be two fuzzy labeling graphs. Then the union of two fuzzy graphs G_1 and G_2 is a fuzzy graphs $G = G_1 \cup G_2 : (\alpha_1 \cup \alpha_2, , \beta_1 \cup \beta 2)$ defined by

$$(\alpha_1 \cup \alpha_2)\, u = \begin{cases} \alpha_2(u) \text{if } u \in V_1 - V_2, \\ \alpha_1(u) \text{if } u \in V_2 - V_1 \end{cases}$$

and

$$(\beta_1 \cup \beta_2)(uv) = \begin{cases} \beta_1(u) \text{if } uv \in E_1 - E_2, \\ \beta_2\,(u) \text{if } uv \in E_2 - E_1 \end{cases}$$

Preposition 3.11. Let G_1 be a fuzzy labeling graph with perfect matching M_1 and G_2 be a fuzzy labeling graph with perfect matching M_2 then $G_1 \cup G_2$ need not have perfect matching.

Proof: Given G_1 and G_2 be a fuzzy labeling graph in which every vertex is saturated by M_1 and M_2.

But in $G_1 \cup G_2$ some vertices are not saturated.

Hence $G_1 \cup G_2$ need not have perfect matching.

Definition 3.12. Consider the join $G^* = G^*1 + G^*2 = (\alpha_1 \cup \alpha_2, , \beta_1 \cup \beta_2 \cup \beta')$ of graphs where β' is the set of all arcs joining the nodes of V_1 and V_2 where we assume that $V_1 \cap V_2 = \emptyset$.

Then the join of two fuzzy graphs G_1 and G_2 is a fuzzy graphs $G = G_1 + G_2 : (\alpha_1+ \alpha_2, \beta_1+\beta_2)$ defined by

$(\alpha_1+ \alpha_2)(u) = (\alpha_1 \cup \alpha_2)(u)$, $u \in V_1 \cup V_2$ and

$$(\beta_1+\beta_2)(uv) = \begin{cases} (\beta1 \cup \beta2)(uv) & \text{if } uv \in E1 \cup E2 \\ \min(\beta1(u), \beta2(v)) & \text{if } uv \in E' \end{cases}$$

Note. The above result is true for join and intersection also.

4. Conclusion

Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. In this paper, the concept of matching in fuzzy labeling graph have been introduced. We plan to extend our research work to matching in bipartite graph.

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