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A Novel Approach for Finding Shortest Path in Intuitionistic Fuzzy Network

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Abstract. This paper deals with finding the shortest path problem of a network with triangular intuitionistic fuzzy number (TIFN). It proposes a modified algorithm to find the shortest path with intuitionistic fuzzy arc lengths.

Keywords: Shortest path problem, triangular intuitionistic fuzzy number, magnitude measure, area measure, mean and centroid index, signed distance.

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1. Introduction

The fuzzy shortest path problem is an extension of fuzzy numbers and it has many real life applications in the field of communication, robotics, scheduling and transportation. Dubois [4] introduced the fuzzy shortest path problem for the first time. Klein [6] introduced a new model to solve the fuzzy shortest path problem for sub-modular functions. Lin and Chern [8] introduced a new design to find the fuzzy shortest path problem on single most vital arc length in a network by using dynamics programming approach. Li et.al. [9] solved the fuzzy shortest path problems by using neural network approach. Chuang et al. [3] used two steps to find the shortest path from origin to destination.

Yao and Lin [18] presented two new types of fuzzy shortest path network problems. The main results obtained from their studies were that the shortest path in the fuzzy sense corresponds to the actual paths in the network, and the fuzzy shortest path problem is an extension of the crisp case. Nayeem and Pal [12] have proposed an algorithm based on the acceptability index introduced by Sengupta and Pal [14] which gives a single fuzzy shortest path or a guideline for choosing the best fuzzy shortest path according to the decision maker’s viewpoint. Jain [5] was first introduced the fuzzy ranking index. Researches and scientists use this theory in many applications. Shortest path problem in fuzzy chain is one of the applications in which we used ranking index to find the shortest path between origins to destination node. Abbasbandy [1] proposed a new approach for ranking the trapezoidal fuzzy numbers.

This paper organized as follows: In section 2 some basic concepts on Intuitionistic fuzzy set theory have been introduced .Section 3 includes the modified algorithm for finding shortest path from source node to destination node. An illustrative example is provided in section 4. Finally, the paper is concluded in section 5.

2. Preliminaries

In this section, some basic definitions used throughout the paper are presented.

2.1. Fuzzy set [19]

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. In the pair (x, $\mu_A(x)$) the first element x belongs to the classical set A, the second element $\mu_A(x)$, belongs to the interval [0, 1], called Membership function. It can also be denoted by $\tilde{A} = \{\mu_A(x)/x : x \in A, \mu_A(x) \in [0,1]\}$

2.2. Fuzzy number [4]

The notion of fuzzy numbers was introduced by Dubois.D and Prade.H (1980). A fuzzy subset A of the real line R with membership function $\mu_A : R \to [0,1]$ is called a fuzzy number if

i. A is normal, i.e., there exists an element $x_0 \in A$ such that $\mu_A(x_0) = 1$

ii. A is fuzzy convex,

$i.e., \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2) \forall (x_1, x_2) \in R \,\&\, \forall \lambda \in [0,1]$

iii. μ_A is upper semi continuous

iv. Supp A is bounded where Supp $A = \{x \in R : \mu_A(x) > 0\}$

2.3. Triangular fuzzy number

A triangular fuzzy number $\tilde{A}$ is a fuzzy number fully specified by 3-tuples (a_1, a_2, a_3) such that $a_1 \leq a_2 \leq a_3$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, if a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, if a_2 \leq x \leq a_3 \\ 0, otherwise \end{cases}$$

2.4. Intuitionistic fuzzy set (IFS)

Let X be the universe of discourse, then an intuitionistic fuzzy set A in X is given by $A = \{x, \mu_A(x), \gamma_A(x)/ x \in X\}$ where μ_A (x) : X $\to$[0 , 1] and γ_A (x) : X $\to$[0 , 1] determine the degree of membership and non membership of the element x $\in$ X, respectively and for every $x \in X, 0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

2.5. Intuitionistic fuzzy number (IFN)

Let $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$ be an IFS, then we call the pair $(\mu_A(x), \gamma_A(x))$ an intuitionistic fuzzy number. We denote an intuitionistic fuzzy number by $(\langle a,b,c\rangle, \langle l,m,n\rangle)$, where $\langle a,b,c\rangle \in F(I), \langle l,m,n\rangle \in F(I), I = [0,1], 0 \le c+n \le 1.$

2.6. Triangular intuitionistic fuzzy number (TIFN) and its arithmetic

A TIFN 'A' is given by $A = \{(\mu_A, \gamma_A) / x \in R\}$, where μ_A and γ_A are triangular fuzzy numbers with $\gamma_A \le \mu_A^{\ C}$. So a triangular intuitionistic fuzzy number A is given by

A = $(\langle a,b,c\rangle, \langle e, f, g\rangle)$ with $\langle e, f, g\rangle \le \langle a,b,c\rangle^c$ i.e., either e ≥ band f ≥ cor f ≤ aand g ≤ b are membership and non-membership fuzzy numbers of A.

An intuitionistic fuzzy number $(\langle a,b,c\rangle, \langle e, f, g\rangle)$ with e ≥ b and f≥ c is shown in the following figure:

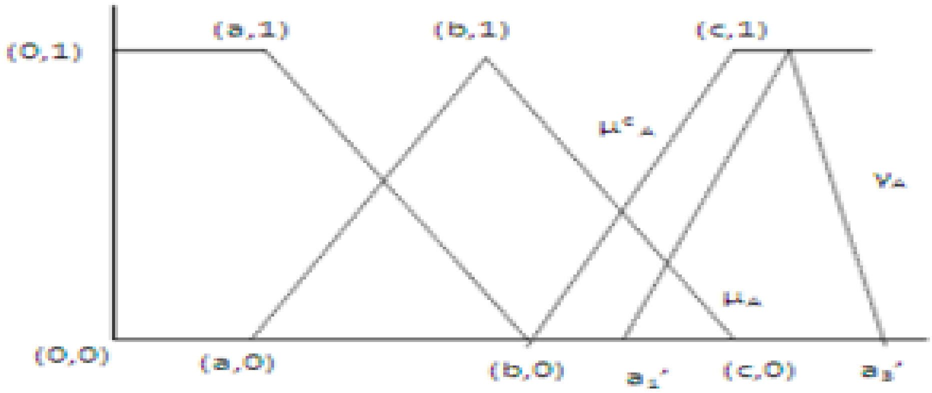

Figure 1: Triangular intuitionstic fuzzy number

Addition

For two triangular Intuitionistic fuzzy numbers

A = $(\langle a_1, b_1, c_1\rangle : \mu_A, \langle e_1, f_1, g_1\rangle : \gamma_A)$ and B = $(\langle a_2, b_2, c_2\rangle : \mu_B, \langle e_2, f_2, g_2\rangle : \gamma_B)$

with $\mu_A \ne \mu_B$ and $\gamma_A \ne \gamma_B$, define A+B =

$$(\langle a_1 + a_2, b_1 + b_2, c_1 + c_2\rangle : Min(\mu_A, \mu_B), \langle e_1 + e_2, f_1 + f_2, g_1 + g_2\rangle : Max(\gamma_A, \gamma_B))$$

Subtraction

A-B =

$$(\langle a_1 - a_2, b_1 - b_2, c_1 - c_2\rangle : Min(\mu_A, \mu_B), \langle e_1 - e_2, f_1 - f_2, g_1 - g_2\rangle : Max(\gamma_A, \gamma_B))$$

where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

2.7. Magnitude measure

The magnitude measure of the triangular intuitionistic fuzzy number

$\tilde{A} = (\langle a,b,c\rangle,\langle d,e,f\rangle)$, Mag ($\tilde{A}$) = $\dfrac{m+7l+n}{12}$, where l = a+d; m = b+e; n= c+f.

2.8. Mean and centroid index

The mean and centroid index are calculated as follows:

TIFN $_{mean}$= $\dfrac{2l + n - m}{2}$; TIFN $_{centroid}$ = $\dfrac{3l+n-m}{3}$; where l= a+d; m=c+f; n= b+e.

2.9. Area measure

Let $\tilde{A}$ = $(\langle a,b,c\rangle,\langle d,e,f\rangle)$ be a TIFN such that d$\geq$b and e$\geq$c then area measure of A= $\dfrac{1}{4}\left[(b+e)-(c+f)+(a+d)\right]$.

2.10, Signed distance

The signed distance from 0 to L_i is defined by d($L_{i,}$ 0) = $\dfrac{1}{4}\left[2(a,b,c)+(d,e,f)\right]$.

3. Proposed algorithm

3.1. Algorithm for triangular intuitionistic fuzzy shortest path problem based on Magnitude measure

Step 1: Construct a network G=(V,E) where V is the set of vertices E is the set of edges. Hence, G is an acyclic digraph and the arc length takes the triangular intuitionistic fuzzy numbers.

Step 2: From the possible paths p_i from source vertex to the destination vertex and compute the corresponding path lengths L_i ,i=1,2,3,4,5.

Step 3: Calculate the Magnitude measure using definition (2.7)

Step 4: The path having the minimum magnitude measure is identified as the shortest path and the corresponding path length is the shortest path length.

3.2. Algorithm for triangular intuitionistic fuzzy shortest path problem based on mean and centroid index and area measure

Step 1: Construct a network G=(V,E) where V is the set of vertices E is the set of edges. Hence, G is an acyclic digraph and the arc length takes the triangular intuitionistic fuzzy numbers.

Step 2: From the possible paths p_i from source vertex to the destination vertex and compute the corresponding path lengths L_i , i=1,2,3,4,5.

Step 3: Calculate the mean and centroid index using definition(2.8) and Area measure using definition (2.9).

Step 4: The path having the minimum mean &centroid index and minimum area measure is identified as the shortest path and the corresponding path length is the shortest path length.

3.3. Algorithm for triangular intuitionistic fuzzy shortest path problem based on signed distance

Step 1: Construct a network G=(V,E) where V is the set of vertices E is the set of edges. Hence, G is an acyclic digraph and the arc length takes the triangular intuitionistic fuzzy numbers.

Step 2: From the possible paths p_i from source vertex to the destination vertex and compute the corresponding path lengths L_i , i=1,2,3,4,5.

Step 3: Calculate the signed distance using definition(2.10).

Step 4: The path having the minimum signed distance is identified as the shortest path and the corresponding path length is the shortest path length.

4. Illustrative example

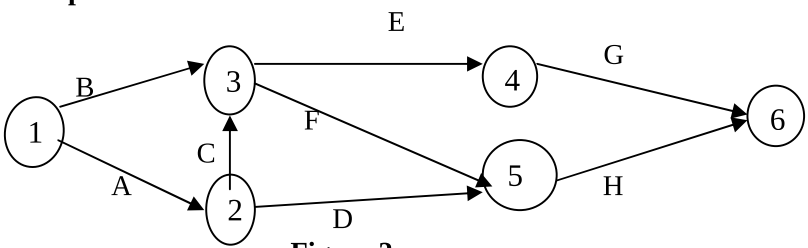

Figure 2:

Consider a network with triangular intuitionistic fuzzy arc lengths as shown below. The arc lengths are assumed to be

$A(1-2) = (\langle 2,3,4\rangle, \langle 4,5,6\rangle);\quad B(1-3) = (\langle 5,6,7\rangle, \langle 8,9,10\rangle);$

$C(2-3) = (\langle 1,2,3\rangle, \langle 3,4,5\rangle)$

$D(2-5) = (\langle 6,7,8\rangle, \langle 9,10,12\rangle);\ E(3-4) = (\langle 6,7,8\rangle, \langle 8,9,10,\rangle);$

$F(3-5) = (\langle 2,3,4\rangle, \langle 4,5,6\rangle)$

$G(4-6) = (\langle 7,8,9\rangle, \langle 9,10,11\rangle);\ H(5-6) = (\langle 1,2,3\rangle, \langle 4,5,6\rangle)$

The possible paths and its corresponding path lengths are as follows:

Table 1:

Paths(p_i)	Path lengths(Li)	Ranking
P_1 : 1-2-5-6	$(\langle 9,12,15\rangle,\langle 17,20,24\rangle)$	3
P_2: 1-2-3-4-6	$(\langle 16,22,24\rangle,\langle 24,28,32\rangle)$	4
P_3: 1-2-3-5-6	$(\langle 6,10,14\rangle,\langle 15,19,23\rangle)$	1
P_4:1-3-4-6	$(\langle 18,21,24\rangle,\langle 25,28,31\rangle)$	5
P_5:1-3-5-6	$(\langle 8,11,14\rangle,\langle 16,19,22\rangle)$	2

The shortest path of the same network given in the above figure , can also be found by using magnitude measure, Mean and centroid index , Area measure and Signed distance . The following table show the results and in all the cases P_3 is identified as the shortest path.

Table 2: Results of the network based on magnitude measure

Path (P_i)	Magnitude Measure(L_i)	Ranking
P_1 : 1-2-5-6	21.08	3
P_2: 1-2-3-4-6	32.17	4
P_3: 1-2-3-5-6	17.75	1
P_4:1-3-4-6	33.75	5
P_5:1-3-5-6	19.5	2

From the table , the path P_3 : 1-2-3-5-6 is identified as the shortest path in Magnitude measure.

Table 3: Results of the network based on mean and centroid index and area measure

Path (P_i)	Mean index	Centroid index	Area measure	Ranking
P_1 : 1-2-5-6	22.5	35.5	6.25	3
P_2: 1-2-3-4-6	37	57	8.5	4
P_3: 1-2-3-5-6	17	27.5	3.25	1
P_4:1-3-4-6	35	61.5	9.25	5
P_5:1-3-5-6	21	33	4.5	2

Here , the path P_3 : 1-2-3-5-6 is identified as the shortest path inMean and centroid index and Area measure.

Table 4: Results of the network based on signed distance

Path (P_i)	Signed Distance	Ranking
P_1 : 1-2-5-6	33.3	3
P_2: 1-2-3-4-6	52	4
P_3: 1-2-3-5-6	29.25	1
P_4:1-3-4-6	52.5	5
P_5:1-3-5-6	30.75	2

From the table, the path P_3 : 1-2-3-5-6 is identified as the shortest path in signed distance.

5. Conclusion

In this paper, an algorithm is developed for solving shortest path problem on a network with triangular intuitionistic fuzzy arc lengths, where the shortest path is identified using the concept of ranking function. We conclude that the algorithm developed in the current research are the simplest and is the alternative method for getting the shortest path intuitionistic fuzzy.

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