Intern. J. Fuzzy Mathematical Archive
Vol. 14, No. 1, 2017, 131-137
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v14n1a16

International Journal of
Fuzzy Mathematical
Archive

A New Approach for Type–2 Fuzzy Shortest Path Problem Based on Statistical Beta Distribution

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Received 1 November 2017; accepted 8 December 2017

Abstract. This paper presents a new type of fuzzy shortest path problem on a network by ranking function. The proposed algorithm gives the fuzzy shortest path and fuzzy shortest path length in which type-2 trapezoidal fuzzy number is assigned to each arc length in a network. In this the ranking function is based on the statistical beta distribution. An illustrative example also included to demonstrate our proposed algorithm.

Keywords: Type-2 fuzzy number, type-2 trapezoidal fuzzy number, fuzzy network, beta distribution.

AMS Mathematics Subject Classification (2010): 94D05

1. Introduction

The shortest path problem was one of the first network problem studied in terms of graph theory. A directed acyclic network is a network consists of a finite set of nodes and a set of directed acyclic arcs. Consider the edge weight of the network as uncertain: which means that is either imprecise or unknown.

Fuzzy set was introduced by Zadeh in 1965. Zadeh [10] proposed type-2 fuzzy sets as an extension of (type-1) fuzzy sets whose membership values are fuzzy sets in the interval [0,1]. The fuzzy shortest path problem was first analyzed by Dubois and Prade [4]. Okada and Soper [6] developed an algorithm based on the multiple labeling approach, by which a number of non-dominated paths can be generated.

The foundation of conventional mathematics is based on real numbers and the process of ranking fuzzy quantifies-such as color or quality of goods-plays a significant role in data analysis, economics and industrial systems.

In 1980, Yager [8] proposed a method of ranking fuzzy numbers based on their corresponding centroid index. In 2000, Yao and Wu [9] used the decomposition principle and the crisp ranking system on R to construct a ranking system for fuzzy numbers. In 2006, Asady and Zendehnam [2] presented a method for ranking fuzzy number by distance minimization. The fuzzy number ranking method proposed in 2008 by Chen and Wang[3] used α cuts for this purpose. In 2016, Rahmani [7], proposed the defuzzification process of a fuzzy number by obtaining the ranking value of its corresponding Beta distribution.

This paper is organized as follows: Section 2 presents some required basic concepts. Sec 3 gives an algorithm to find shortest path and shortest path length using ranking function based on distribution. Section 4 gives network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5. Section 6 presents conclusion.

2. Preliminaries

In this section some basic concepts of ype-2 fuzzy set and ranking function are reviewed.

2.1. Type-2 fuzzy set

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where x $\in$ X and u $\in$ $J_x \subseteq [0,1]$.

ie., $\tilde{A}$ = { ((x,u), $\mu_{\tilde{A}}(x,u)$) / $\forall$ x $\in$ X, $\forall$ u $\in$ $J_x \subseteq [0,1]$ } in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$.

$\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union

2.2. Interval type-2 fuzzy set

Interval type-2 fuzzy set is defined to be a T2FS where all its secondary grade are of unity for all $f_x(u) = 1$.

2.3. Footprint of uncertainty

Uncertainty in the primary membership of a type-2 fuzzy set, $\tilde{A}$, consists of a bounded region that we call the footprint of uncertainty (FOU). It is the union of all primary membership.

ie., FOU($\tilde{A}$) = $\int_{x\in X} J_x$

The FOU can be described in terms of its upper and lower membership function.

$$\text{FOU}(\tilde{A}) = \int_{x\in X} [\mu_{\tilde{A}}(x)^l, \mu_{\tilde{A}}(x)^u].$$

2.4. Principal membership function

The principal membership function defined as the union of all the primary membership having secondary grades equal to 1.

ie.,P_r ($\tilde{A}$) = $\int_{x\in X} u/x \,|\, f_x(u) = 1$.

2.5. Type-2 fuzzy number

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.6. Type-2 trapezoidal fuzzy number

Let A = (a_1, a_2, a_3, a_4) be a trapezoidal fuzzy number. A normal type-2 trapezoidal fuzzy number $\tilde{A}$ = { (x, $\mu_A^L(x), \mu_A^M(x), \mu_A^N(x), \mu_A^U(x)$)}, x Є R and

$\mu_A^L(x) \le \mu_A^M(x) \le \mu_A^N(x) \le \mu_A^U(x)$, for all x Є R. Denote $\tilde{A}$ = (A^L, A^M,A^N,A^U), where

$\tilde{A}$ = ((a^L_1, a^L_2, a^L_3, a^L_4; λ_A), (a^M_1, a^M_2, a^M_3, a^M_4), (a^N_1, a^N_2, a^N_3, a^N_4), (a^U_1, a^U_2, a^U_3, a^U_4))

2.7. Perfectly normal type-2 trapezoidal fuzzy number

In a type-2 trapezoidal fuzzy number, if λ_A = 1, then

$\tilde{A}$ = (A^L, A^M,A^N,A^U)
= ((a^L_1, a^L_2, a^L_3, a^L_4), (a^M_1, a^M_2, a^M_3, a^M_4), (a^N_1, a^N_2, a^N_3, a^N_4), (a^U_1, a^U_2, a^U_3, a^U_4)) is a perfectly normal type-2 fuzzy number.

2.8. Ranking function of beta distribution for trapezoidal fuzzy number

The Beta distribution R(A) corresponding to the trapezoidal fuzzy number A = (a_1, b_1, c_1, d_1) is obtained by the following relation:

$$R(A) = \frac{1}{18}(2a_1 + 7b_1 + 7c_1 + 2d_1)$$

2.9. Ranking function on beta distribution for type-2 trapezoidal fuzzy number

Let $\tilde{A}$ = ((a^L_1, a^L_2, a^L_3, a^L_4), (a^M_1, a^M_2, a^M_3, a^M_4), (a^N_1, a^N_2, a^N_3, a^N_4), (a^U_1, a^U_2, a^U_3, a^U_4)) be a type-2 normal trapezoidal fuzzy number, then the ranking function is defined as

$$R(\tilde{A}) = \frac{1}{324}\{2(2a_1 + 7b_1 + 7c_1 + 2d_1) + 7(2a_2 + 7b_2 + 7c_2 + 2d_2) + 7(2a_3 + 7b_3 + 7c_3 + 2d_3) + 2(2a_4 + 7b_4 + 7c_4 + 2d_4)\}$$

3. Algorithm

Step 1 : Form the possible paths from starting node to destination node.

Step 2 : Compute the path length $\tilde{L}_i$, for the possible paths, i = 1,2, . .n.

Step 3 : Compute the ranking function of beta distribution corresponding to all possible path lengths

R($\tilde{L}_i$)

$$=\frac{1}{324}\{2(2a_1 + 7b_1 + 7c_1 + 2d_1) + 7(2a_2 + 7b_2 + 7c_2 + 2d_2) + 7(2a_3 + 7b_3 + 7c_3 + 2d_3) + 2(2a_4 + 7b_4 + 7c_4 + 2d_4)\}$$

where $\tilde{L}_i$ = {(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3), (a_4, b_4, c_4, d_4)}

Step 4 : Decide the shortest path with minimum rank and that corresponding path Length is the fuzzy shortest path length.

4. Network terminology

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where i,j$\in$ V and i$\neq$j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence P_{ij} = {i = i_1, (i_1,i_2),i_2,. . . ., i_{l-1}, (i_{l-1},i_l), i_l = j} of alternating nodes and edges. The existence of at least one path P_{si} in G(V,E) is assume for every node i$\in$ V – {s}.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j\in P)} \tilde{d}_{ij}$

5. Numerical example

The problem is to find the shortest path and shortest path length between source node and destination node in the network comprising 5 vertices and 7 edges with type-2 trapezoidal fuzzy number.

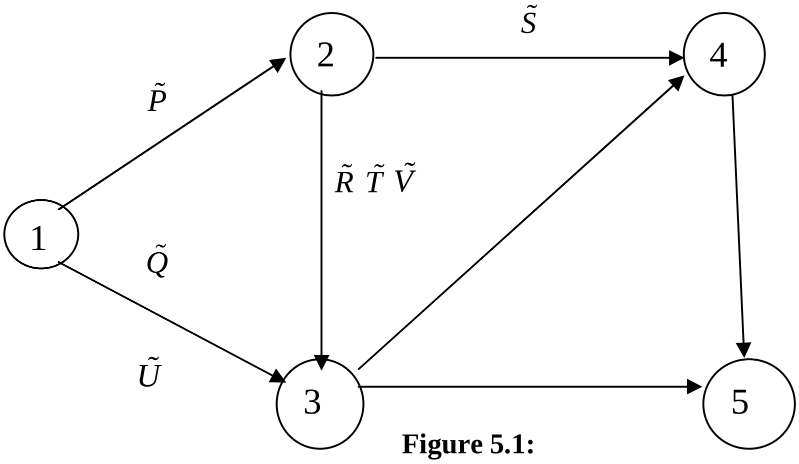

Figure 5.1:

In this network each edge has been assigned to type-2 trapezoidal fuzzy number as follows:

$\tilde{P}_1$ = ((0.5, 0.7, 0.8, 0.9; 0.4), (0.4, 0.6, 0.7, 0.9), (0.2, 0.3, 0.5, 0.8), (0.1, 0.5, 0.7. 1.0))

$\tilde{P}_2$ = ((0.4, 0.5, 0.7, 1.0; 0.5), (0.3, 0.5, 0.7, 0.9), (0.3, 0.5, 0.7, 0.8), (0.2, 0.4, 0.6, 0.9))

$\tilde{P}_3$ = ((0.6, 0.8, 0.9, 0.9; 0.7), (0.5, 0.6, 0.7, 0.9), (0.3, 0.5, 0.6, 0.8), (0.1, 0.2, 0.3, 0.4))

$\tilde{P}_4$ = ((0.4, 0.5, 0.5, 0.9; 0.6), (0.3, 0.5, 0.6, 0.9), (0.3, 0.4, 0.6, 0.8), (0.2, 0.4, 0.5, 0.7))

$\tilde{P}_5$ = ((0.6, 0.7, 0.7, 0.9; 0.3), (0.4, 0.6, 0.7, 0.9), (0.4, 0.6, 0.9, 1.0), (0.3, 0.5, 0.8, 0.9))

$\tilde{P}_6$ = ((0.7, 0.8, 0.8, 0.9; 0.4), (0.5, 0.7, 0.9, 1.0), (0.3, 0.5, 0.7, 0.9), (0.2, 0.3, 0.5, 0.8))

$\tilde{P}_7$ = ((0.5, 0.6, 0.8, 1.0; 0.2), (0.3, 0.5, 0.8, 1.0), (0.2, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6))

Solution :

Step 1 : Form the possible paths from starting node to destination node.

Possible paths are $\tilde{P}_1 = 1 – 2 – 4 – 5$

$\tilde{P}_2 = 1 – 2 – 3 - 4 – 5$

$\tilde{P}_3 = 1 – 2 – 3 – 5$

$\tilde{P}_4 = 1 – 3 – 5$,

$\tilde{P}_5 = 1 – 3 – 4 - 5$

Step 2 : Compute the path length $\tilde{L}_i$, for the possible paths, i = 1,2, . .n.

Path lengths are $\tilde{L}_1 = \tilde{P} + \tilde{S} + \tilde{V}$

$\tilde{L}_2 = \tilde{P} + \tilde{R} + \tilde{T} + \tilde{V}$

$\tilde{L}_3 = \tilde{P} + \tilde{R} + \tilde{U}$

$\tilde{L}_4 = \tilde{L}_5 + \tilde{U}$

$\tilde{L}_5 = \tilde{Q} + \tilde{T} + \tilde{V}$

$\tilde{L}_1$ = ((0.5+ 0.4 + 0.5, 0.7 + 0.5 + 0.6, 0.8 + 0.5 + 0.8, 0.9 + 0.9 + 1.0), (0.4 + 0.3 +0.3, 0.6 + 0.5 + 0.5, 0.7 + 0.6 + 0.8, 0.9 + 0.9 + 1.0), (0.2 + 0.3 + 0.2, 0.3 + 0.4 + 0.6, 0.5 + 0.6 + 0.8, 0.8 + 0.8 + 0.9), (0.1 + 0.2 + 0.2, 0.5 + 0.4 + 0.4, 0.7 + 0.5 + 0.5, 1.0 + 0.7 + 0.6)

$\tilde{L}_1$ = ((1.4, 1.8, 2.1, 2.8), (1.0, 1.6, 2.1, 2.8), (0.7, 1.3, 1.9, 2.5), (0.5, 1.3, 1.7, 2.3))

In the same way we get the path lengths are as follows:

$\tilde{L}_2$ = ((2.2, 2.8, 3.2, 3.7), (1.6, 2.3, 2.9, 3.7), (1.1, 2.0, 2.8, 3.5), (0.7, 1.6, 2.3, 2.9))

$\tilde{L}_3$ = ((1.8, 2.3, 2.5, 2.7), (1.4, 1.9, 2.3, 2.8), (0.8, 1.3, 1.8, 2.5), (0.4, 1.0, 1.5, 2.2))

$\tilde{L}_4$ = ((1.1, 1.3, 1.5, 1.9), (0.8, 1.2, 1.6, 1.9), (0.6, 1.0, 1.4, 1.7), (0.4, 0.7, 1.1, 1.7))

$\tilde{L}_5$ = ((1.5, 1.8, 2.2, 2.9), (1.0, 1.6, 2.2, 2.8), (0.9, 1.7, 2.4, 2.7), (0.7, 1.3, 1.9, 2.4))

Step 3 : Compute the ranking function of beta distribution corresponding to all possible path lengths

$R(\tilde{L}_i)=\frac{1}{324}\{2(2a_1+7b_1+7c_1+2d_1)+\ 7(2a_2+7b_2+7c_2+\ 2d_2)+\ 7(2a_3+7b_3+7c_3+2d_3)+\ 2(2a_4+7b_4+\ 7c_4+2d_4)\}$

where $\tilde{L}_i = \{(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3), (a_4, b_4, c_4, d_4)\}$

$R(\tilde{L}_1)= \frac{1}{324}\{ 2(2*1.4 + 7*1.8 + 7*2.1 + 2*2.8) + 7 (2*1.0 + 7*1.6 + 7*2.1 + 2*2.8)+ 7 (2*0.7 + 7*1.3 + 7*1.9 + 2*2.5) + 2 (2*0.5 + 7*1.3 + 7*1.7 + 2*2.3)\}$

$R(\tilde{L}_1)= 1.7392$

$R(\tilde{L}_2) = 2.4852$

$R(\tilde{L}_3) = 1.8312$

$R(\tilde{L}_4) = 1.2642$

$R(\tilde{L}_5)= 1.9182$

Step 4 : Decide the shortest path with minimum rank and that corresponding path length is the fuzzy shortest path length.

Path length $\tilde{L}_4$ is having the minimum rank ($R(\tilde{L}_4) = 1.2642$)

The shortest path length is
((1.1, 1.3, 1.5, 1.9), (0.8, 1.2, 1.6, 1.9), (0.6, 1.0, 1.4, 1.7), (0.4, 0.7, 1.1, 1.7))
And the corresponding shortest path is

1 – 3 – 5

Fuzzy shortest path of type-2 fuzzy number is 1 – 3- -5.

6. Conclusion

This paper presents a solution for fuzzy shortest path problem with type-2 trapezoidal fuzzy number. The main issue dealt with was developing a ranking function for type-2 trapezoidal fuzzy number with minimum number of process steps. This algorithm can be implemented using statistical Beta distribution chosen by the decision maker. This algorithm is executed for complement of type-2 trapezoidal fuzzy number and verified. It provides the same path as the shortest path in complement type-2 fuzzy number.

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