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A New Distance and Ranking Method for Triangular Fuzzy Numbers

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Abstract. This study presents an approximate approach for ranking fuzzy triangular numbers based on the distance method of a fuzzy triangular number and its area. The total approximate is determined by convex combining of fuzzy triangular number's relative and its area that based on decision maker's optimistic perspectives. The proposed approach is simple in terms of computational efforts and is efficient in ranking a large quantity of fuzzy triangular numbers. By a group of examples in [2] demonstrate the accuracy and applicability of the proposed approach. Finally we construct a new ranking system for fuzzy triangular number which is very realistic and also matching our intuition as the crisp ranking system on R.

Keywords: Fuzzy numbers, triangular number, approximate approach, ranking, distance.

AMS Mathematics Subject Classification (2010): 94D05, 90BXX

1. Introduction

The fuzzy set theory pioneered by Zadeh [5] has been extensively used. Fuzzy numbers or fuzzy subsets of the real line Rare applied to represent the imprecise numerical measurements of different alternatives. Therefore, comparing the different alternatives is actually comparing the resulting fuzzy numbers. Also in many applications of fuzzy set theory to decision making we need to know the best select from a collection of possible solution. This selection process may require that we rank or order fuzzy numbers. Several researchers presented ranking methods [1,2]. Various techniques are applied to compare the fuzzy numbers. Some of the exiting approaches are difficult to understand and have suffered from different plights, e.g., the lack of discrimination, producing counterintuitive orderings, and ultimately resulting in inconsistent ordering if a new fuzzy number is added. Also nearly all approaches should acquire membership functions of fuzzy numbers before the ranking is performed; however, this may be infeasible in real applications. Furthermore, accuracy and efficiency should be of priority concern in the ranking process if ranking a large amount of fuzzy numbers. In light of the above discussion, specially in [3] Chen and Lu has proposed an approximate approach for ranking fuzzy numbers, in that they worked with dominance. Also we studied here the approach is determined by convex combining the centroid point(x_0;y_0) *of* but just x_0 and areas of a fuzzy number that it performs simple arithmetic operations for the ranking purpose and it can be a rank the combination case of some fuzzy numbers and crisp numbers and the case of discrete fuzzy numbers and useful in ranking a large quantity of fuzzy numbers. Comparing the proposed approximate approach with the existing methods using both Bortolan and Degani's examples [2].

The methods of measuring of distance between fuzzy numbers have became important due to the significant applications in diverse fields like remote sensing, data mining, pattern recognition and multivariate data analysis and so on. Several distance measures for precise numbers are well established in the literature. Several researchers focused on computing the distance between fuzzy numbers [1, 2, 4]. Here we introduce a new distance between two trapezoidal fuzzy numbers by the new approach proposed for them. This paper is organized as following:

In Section 2, the basic concept of fuzzy number operation is brought. In Section 3, introduces the ranking approach, and presents some comparative examples which demonstrate the accuracy of the proposed approach over the exiting methods. A new distance measure between fuzzy numbers is defined in Section 4. Concluding remarks are finally made in section 5.

2. Preliminaries

Definition 2.1. A fuzzy number $\widetilde{A}$ in R is said to be a triangular fuzzy number if its membership function $\mu_{\widetilde{A}} : R \to [0,1]$ has the following characteristics.

$$\mu_{\widetilde{A}}(\mathrm{x}) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1 \quad , & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0 \quad , & otherwise \end{cases}$$

It is denoted by $\widetilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ where $a^{(1)}$ is Core ($\widetilde{A}$), $a^{(2)}$ is left width and $a^{(3)}$ is right width. The geometric representation of Triangular Fuzzy number is shown in figure. Since, the shape of the triangular fuzzy number $\widetilde{A}$ is usually in triangle it is called so.

Membership function of triangular fuzzy number

The Parametric form of a triangular fuzzy number is represented by

$\widetilde{A} = \left[a^{(1)} - a^{(2)}(1-r),\ a^{(1)} + a^{(3)}(1-r) \right]$

3. Main results

Ranking of triangular fuzzy number with distance method

Let all of fuzzy numbers be either positive or negative. Without less of generality,

Assume that all of the m are positive. The membership function of, $a \in R$ is $u_0(x) = 1$ if $x = a : and\ u_0(x) = 0, if\ x \ne a. Hence if\ a = 0$ we have the following

$$u_0(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0. \end{cases}$$

Since $u_0(x) \in E$, left fuzziness σ and right fuzziness β are 0, so for each $u_0(x) \in E$

$$d_2(u,u_0) = \left[\int_0^1 \underline{u}(r)^2 \, dr + \int_0^1 \overline{u}(r)^2 \, dr + \right]^{\frac{1}{2}}$$

Thus we have the following definition.

Definition 3.1. For $u \text{ and } v \in E$, define the ranking of $u \text{ and } v$ by saying

$$d(u,u_0) > d(v,u_0) \quad \text{if and only if } u \succ v$$
$$d(u,u_0) < d(v,u_0) \quad \text{if and only if } u \prec v$$
$$d(u,u_0) = d(v,u_0) \quad \text{if and only if } u \approx v.$$

Property 3.1. Suppose $u \text{ and } v \in E$ are arbitrary then:

1. If $u = v \text{ then } u \approx v$
2. If $v \subseteq u \text{ and } \underline{u}(r)^2 + \overline{u}(r)^2 > \underline{v}(r)^2 + \overline{v}(r)^2 \text{ for all } r \in [0, 1] \text{ then } v \prec u.$

Remark 3.1. The distance triangular fuzzy number $u = (x_0, \sigma, \beta) \text{ of } u_0$ is defined as following

$$d(u,u_0) = \left[2x_0^2 + \sigma^2/3 + \beta^2/3 + x_0(\beta - \sigma) \right]^{\frac{1}{2}}$$

Remark 3.2. The distance trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta) \text{ of } u_0$ is defined as following

$$d(u,u_0) = \left[x_0^2 + y_0^2 + \sigma^2/3 + \beta^2/3 - x_0\sigma + y_0\beta \right]^{\frac{1}{2}}$$

Remark 3.3. If $u \approx v$, it is not necessary that $u = v$. Since if $u \neq v$ and

$$\left(\underline{u}(r)^2 + \overline{u}(r)^2 \right)^{\frac{1}{2}} = \left(\underline{v}(r)^2 + \overline{v}(r)^2 \right)^{\frac{1}{2}} \text{ then } u \approx v.$$

4. Procedure

A popular fuzzy number u is the symmetric triangular fuzzy number $S[x_0, \sigma]$ centered at x_0 with basis 2σ

$$u(x)=\begin{cases}\frac{1}{\sigma}(x-x_0+\sigma), & x_0-\sigma\le x\le x_0\\ \frac{1}{\sigma}(x-x_0+\sigma), & x_0\le x\le x_0+\sigma\\ 0, & otherwise\end{cases}$$

which its parametric form is $\underline{u}(r)=x_0-\sigma+\sigma r \quad \bar{u}(r)=x_0+\sigma-\sigma r$

Remark 4.1. The distance triangular fuzzy number $S[x_0,\sigma]$ of u_0 is defined as

$$d(S[x_0,\sigma],u_0)=\left[2x_0^2+\frac{2}{3}\sigma^2\right]^{\frac{1}{2}}$$

If for ranking fuzzy numbers $S[x_0,\sigma]$, $S[x_0,\beta]$ and $\sigma\ne\beta$ we used Yao and Wu [8], method we would have $S[x_0,\sigma]=S[x_0,\beta]$. But with our method we have $S[x_0,\sigma]\ne S[x_0,\beta]$.

We shall now compare the methods used by other authors in [7,8,9,12,13] and our method with four sets of examples taken from Yao and Wu [13].

Set 1: $A=(0.5,0.1,0.5)$, $B=(0.7,0.3,0.3)$, $C=(0.9,0.5,0.1)$.

Set 2: $A=(0.4,0.7,0.1,0.2)$ (Trapezoidal fuzzy number)

$B=(0.7,0.4,0.2)$, $C=(0.7,0.2,0.2)$

Set 3: $A=(0.5,0.2,0.2)$, $B=(0.5,0.8,0.2,0.1)$, $C=(0.5,0.2,0.4)$.

Set 4: $A=(0.4,0.7,0.4,0.1)$, $B=(0.5,0.3,0.4)$, $C=(0.6,0.5,0.2)$.

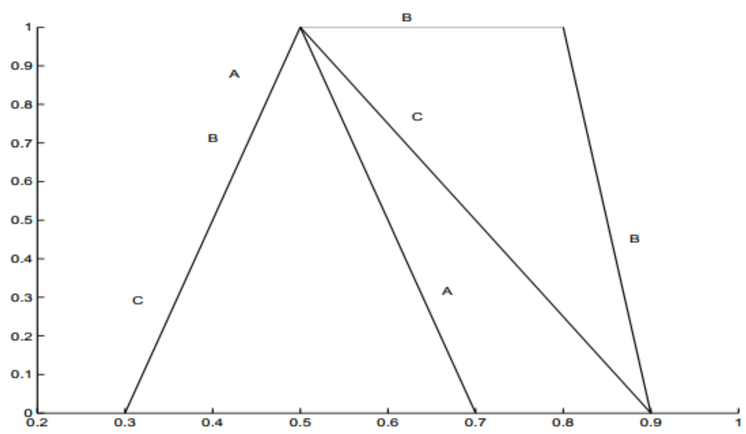

Figure 1:

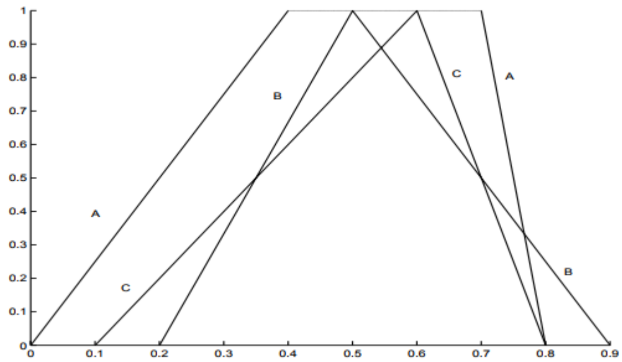

Figure 2:

Table 1:

set	Fuzzy number	Choobinech and Li	Yager	Chen	Baldwin and Guild	Wu
1	A B C	0.33 0.50 0.667 A < B < C	0.60 0.70 0.667 A < B < C	0.3375 0.50 0.667 A < B < C	0.30 0.33 0.44 A < B < C	0.60 0.70 0.80 A < B < C
2	A B C	0.458 0.583 0.667 A < B < C	0.575 0.65 0.7 A < B < C	0.4315 0.5625 0.625 A < B < C	0.27 0.27 0.37 A ≈ B < C	0.575 0.65 0.70 A < B < C
3	A B C	0.33 0.4167 0.5417 A < B < C	0.50 .055 0.625 A < B < C	0.375 0.425 0.55 A < B < C	0.27 0.37 0.45 A < B < C	0.50 0.625 0.55 A < C < B
4	A B C	0.50 0.5833 0.6111 A < B < C	0.45 0.525 0.55 A < B < C	0.52 0.57 0.625 A < B < C	0.40 0.42 0.42 A < B ≈ C	0.475 0.525 .0525 A < B ≈ C

By using our method, we have the following results:

Set 1: $d(A,u_0)$=0.8869, $d(B,u_0)$=1.0194, $d(C,u_0)$=1.1605, and by definition: A < B < C

Set 2: $d(A,u_0)$=0.8756, $d(B,u_0)$=0.9522, $d(C,u_0)$=1.0033, and by definition: A < B < C

Set 3: $d(A,u_0)=0.7257$, $d(B,u_0)=0.9416$, $d(C,u_0)=0.8165$, and by definition: A < C < B

Set 4: $d(A,u_0)=0.7853$, $d(B,u_0)=0.7958$, $d(C,u_0)=0.8386$, and by definition: A < B < C

5. Conclusion

We construct a new ranking system for fuzzy triangular number which is very realistic and also matching our intuition as the crisp ranking system on R. In table 1, we have the following results: In set 1, our method has the same result as in other five papers. In set 2, our method has the same result as in the other four papers. However in set 3, we and Yao and Wu have A < C < B, but all other the four papers have A < B < C. We can see from Fig 1, that define ordering A < C < B is better than define ordering A < B < C. In set 4, Fig 2, A < B < C, our method leads to the same result as that of Choobinech and Li, Yager and Chen. The result of Yao and Wu, Baldwin and Guild have A < B ≈ C.

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