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A New Approach on Multiple Attribute Decision Making with Interval-Valued Knowledge Measure

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Abstract. In this paper, firstly some entropy measures and knowledge measure for the interval-valued intuitionistic fuzzy sets are discussed. Also interval-valued intuitionistic fuzzy weighted geometric averaging operator (IVIFWA), interval-valued intuitionistic fuzzy ordered weighted geometric averaging operator (IVIFOWGA) and interval-valued intuitionistic hybrid geometric averaging operator (IVIFHGA) are discussed. Based on the concepts mentioned above, a new multi attribute decision making method for interval-valued intuitionistic fuzzy numbers is proposed. Finally some illustrations are performed to demonstrate the effectiveness of the proposed method.

Keywords: Fuzzy sets, knowledge measure, interval-valued intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy averaging operator, multiple attribute decision making.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

In [2] Attanassov and Gargov presented the concept of interval valued intuitionistic fuzzy sets (IVIFSs), which is an extension of the concept if intuitionistic fuzzy sets where the membership and non-membership degree are represented by interval-valued intuitionistic fuzzy values (IVIFVs) respectively. Many multiple attributes decision making methods have been presented on IVIFSs so far. In [5] Chen et al presented a multiple attribute decision making method based on the interval-valued intuitionistic fuzzy weighted average operator and the ranking method of intuitionistic fuzzy values. In [5] Chen et al presented a method for ranking IVIFSs for multiple attribute decision making. In [2013] Jin et al presented a multiple criteria fuzzy group decision making method based on Interval-valued intuitionistic fuzzy continuous weighted entropies. Ye (2013) presented a multiple attribute decision making method based on the accuracy function in an Interval-valued intuitionistic fuzzy environment. In 2014, Zhang et al presented a multiple criteria decision making method based on interval- valued intuitionistic fuzzy soft sets.In[2014], Wang and Liu presented the Interval-valued intuitionistic fuzzy Einstein weight averaging ($\text{IVIFWA}^{€}$) operator , the interval- valued intuitionistic fuzzy Einstein ordered weighted averaging ($\text{IVIFOWA}^{€}$) operator and interval- valued intuitionistic fuzzy Einstein hybrid weight averaging ($\text{IVIFHWA}^{€}$) operator based on Einstein operations for multiple attribute decision making. In [7] Xu and Chen presented the interval-valued intuitionistic fuzzy weight geometric (IVIFWG) operator, interval- valued intuitionistic fuzzy ordered weighted geometric (IVIFOWG) operator and the interval-valued intuitionistic fuzzy ordered hybrid geometric (IVIFHG) operator for multiple attribute decision making . Xu, Chen and some other author methods have the drawback on their corresponding operators. It was overcome by Shyi-mingchen [6] and Wei-Hsiang Tsai by introducing a multiple attribute decision making based on novel interval-valued intuitionistic fuzzy geometric operators. Also at present there are several measures for IVIFS. In [4] Hoang Nguyen extended the knowledge measure for IFSs developed to present a novel interval-valued knowledge measure for IVIFSs.

Based on the above discussed things we have developed a new multiple attribute decision making under interval- valued intuitionistic fuzzy environment. This paper is organized as follows: In section 2, we review some basic concepts of IVIFSs and the existing measures for ranking the alternatives. Section 3 provides interval- valued intuitionistic fuzzy weighted geometric averaging (IVIFWGA) operator, the interval-valued intuitionistic fuzzy ordered weighted geometric (IVIFOWGA) operator and the interval- valued intuitionistic fuzzy ordered hybrid geometric (IVIFHG) operator. Section 4 explains the entropy measure and knowledge measure for IVIFSs. Section 5 we propose a new multiple attribute decision making method based on interval- valued intuitionistic fuzzy weighted geometric averaging (IVIFWGA) operator, the interval- valued intuitionistic fuzzy ordered weighted geometric (IVIFOWGA) operator and the alternatives are ranked by knowledge measure. Finally an illustration is explained to prove the effectiveness of the proposed method.

2. Preliminaries

Definition 2.1. Interval-valued intuitionistic fuzzy sets

Let a set X be fixed, an AIFS A in X is defined as $A= \{(x, \mu_A(x), \nu_A(x)), x\in X\}$ where $\mu_A$and $\nu A$are mappings from X to the closed interval [0,1] such that $0 \leqslant \mu_A(x) \leqslant 1$, $0 \leqslant \nu_A(x) \leqslant 1$ and $0 \leqslant \mu_A(x)+\nu_A(x) \leqslant 1$, for all $x\in X$, and they denote the degrees of membership and non-membership of element $x\in X$ to set A, respectively.

The intervals μ_A (x) and ν_A(x) denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to A. Then for each x ∈ X, μ_A (x) and (x) are closed intervals and their lower and upper end points are denoted byμ_{AL}(x), μ_{AU}(x), $\nu_{AL}(x)$ and $\nu_{AU}(x)$, respectively, and thus we can replace Eq. with

A ={<x, [μ_{AL}(x), μ_{AU}(x)], [$\nu_{AL}(x)$,$\nu_{AU}(x)$]>}: x ∈X},

where $0 \leq \mu_{AU}(x)+ \nu_{AU}(x)) \leq 1$

Xu (2007a) called $\tilde{a}$= <[a, b], [c, d]>an interval-valued intuitionistic fuzzy number (IVIFN), where [a, b] ∈ [0, 1], [c, d] ∈ [0, 1] and b +d ≤ 1.

Definition 2.2. Score function

Let ã= <[a, b], [c, d]> be an IVIFN, then the score function is defined as

$$S(\tilde{a})=\frac{1}{2}(a-c+b-d), \quad (1)$$

whereas (ã) ∈ [−1, 1]. The larger the value of s(ã), the higher the IVIFN ã.

Definition 2.3. Accuracy function

Let ã= <[a, b], [c, d]> be an IVIFN, then the accuracy function is defined as

` $$h(\tilde{a})=\frac{1}{2}(a+c+b+d), \quad (2)$$

where h(ã) ∈ [0, 1]. The larger the value of h(ã), the higher the accuracy degree of the IVIFN ã.

Definition 2.4. Hesitancy degree

Let ã= <[a, b], [c, d]> be an IVIFN, then the hesitancy degree, the mid-point of intuitionistic fuzzy number is defined as

$$\pi(\tilde{a}) = [1\text{-}a\text{-}c,\ 1\text{-}b\text{-}d] \quad (3)$$

Definition 2.5. Comparison of two interval-valued intuitionistic fuzzy numbers

Let $\tilde{a}_1$=<[a_1, b_1], [c_1, d_1]> and $\tilde{a}_2$=<[a_2, b_2], [c_2, d_2]>be two interval valued intuitionistic fuzzy numbers. Let S($\tilde{a}_1$) and S($\tilde{a}_2$) denote the Score function of $\tilde{a}_1$ and $\tilde{a}_2$ respectively. Let H($\tilde{a}_1$) and H($\tilde{a}_2$) denote the accuracy functions of $\tilde{a}_1$ and $\tilde{a}_2$ respectively.

Then ,

(i) If S ($\tilde{a}_1$)>S ($\tilde{a}_2$), then $\tilde{a}_1$ is greater than$\tilde{a}_2$, denoted by $\tilde{a}_1$>$\tilde{a}_2$.

(ii) If S($\tilde{a}_1$)=S($\tilde{a}_2$), then
- If H ($\tilde{a}_1$) =H ($\tilde{a}_2$), then $\tilde{a}_1$ and $\tilde{a}_2$ represent the same information.
- If H ($\tilde{a}_1$) >H ($\tilde{a}_2$), then $\tilde{a}_1$is greater than $\tilde{a}_2$, denoted by $\tilde{a}_1$>$\tilde{a}_2$.

Knowledge measure for IVIFSs

Most of the measures for IFSs and IVIFSs cannot distinguish the cases, whose membership and non-membership degrees are identical. Nguyen (2015) proposed a new knowledge measure for IFSs as follows:

Definition 2.6. Knowledge measure for IFSs

Let IFS(X) denote the family of all the IFSs, the knowledge measure of an IFS A€ IFS(X) is define as its normalized Euclidean distance from reference level 0 of information F(x, 0,0);

$$K_F(A) = \frac{1}{n\sqrt{2}}\sum_{i=1}^{n}\sqrt{((\mu_A(x_i))^2 + (\nu_A(x_i))^2((\mu_A(x_i) + \nu_A(x_i))^2} \quad (4)$$

For instance, knowledge measure is equal to 1 for the crisp sets and 0 for the most fuzzy intuitionistic set F(x, 0,0).

Definition 2.7. Knowledge measure for IVIFSs

The knowledge measure of an interval- valued intuitionistic fuzzy set is defined as

K_F^{I} = [K_F(<x, $\mu^-_A(x_i)$, $\nu^-_A(x_i)$] , K_F(<x, $\mu^+_A(x_i)$, $\nu^+_A(x_i)$]Where

K_F(<x, $\mu^-_A(x_i)$, $\nu^-_A(x_i)$=

$$\frac{1}{\sqrt{2}}\sum_{i=1}^{n}\sqrt{((\mu^-{}_A(x_i))^2 + (\nu^-{}_A(x_i))^2((\mu^-{}_A(x_i) + \nu^-{}_A(x_i))^2}$$

And

K_F(<x, $\mu^+_A(x_i)$, $\nu^+_A(x_i)$=

$$\frac{1}{\sqrt{2}}\sum_{i=1}^{n}\sqrt{((\mu^+{}_A(x_i))^2 + (\nu^+{}_A(x_i))^2((\mu^+{}_A(x_i) + \nu^+{}_A(x_i))^2} \quad (5)$$

We denote $A^- = <x, \mu^-_A(x_i), \nu^-_A(x_i)>$ and $A^+ = <x, \mu^+_A(x_i), \nu^+_A(x_i)>$ two IFSs in the interval [0,1] is define as : $K_F^I(A^I) = [K_F(A^-), K_F(A^+)]$.

3. Interval-valued intuitionistic fuzzy geometric aggregation operators

In 2016, Chen and Tsai proposed a new interval-valued intuitionistic fuzzy geometric averaging operators of IVIFSs.

Definition 3.1. IVIFWGA operator

Let $\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n$be IVIFVs where $\tilde{a}_i$= ($[a_i, b_i], [c_i, d_i]$), $0 \le a_i \le b_i \le 1$, $0 \le c_i \le d_i \le 1$, $0 \le b_i + d_i \le 1$ and $1 \le i \le n$. Let w_i be the weight of α_i, where $w_i \in [0,1]$, $1 \le i \le n$ and $\sum_{i=1}^{n} w_i = 1$. Then the IVIFWGA operator of the IVIFVs is defined as follows:

IVIFWGA $(\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n)$ =

$$([\prod_{i=1}^{n}(1-c_i)^{w_i} - \prod_{i=1}^{n}(1-a_i-c_i)^{w_i}, \prod_{i=1}^{n}(1-d_i)^{w_i} - \prod_{i=1}^{n}(1-b_i-d_i)^{w_i}], [1-\prod_{i=1}^{n}(1-c_i)^{w_i}, 1-\prod_{i=1}^{n}(1-d_i)^{w_i}]) \quad (6)$$

where w= $(w_1, w_2, \ldots\ldots w_n)^T$ be the weight vector of $\tilde{a}_i$ (i=1,2,…n) and w_i>0, $\sum_{i=1}^{n} w_i = 1$.

Definition 3.2. IVIFOWGA operator

Let $\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n$ be IVIFVs where $\tilde{a}_i$= ($[a_i, b_i], [c_i, d_i]$), $0 \le a_i \le b_i \le 1$, $0 \le c_i \le d_i \le 1$, $0 \le b_i + d_i \le 1$ and $1 \le i \le n$. Let w_i be the weight of α_i, where $w_i \in [0,1]$, $1 \le i \le n$ and $\sum_{i=1}^{n} w_i = 1$. Then the IVIFWGA operator of the IVIFSs is defined as follows:

IVIFOWGA $(\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n)$ =

$$([\prod_{i=1}^{n}(1-c_{\sigma(i)})^{w_i} - \prod_{i=1}^{n}(1-a_{\sigma(i)}-c_{\sigma(i)})^{w_i}, \prod_{i=1}^{n}(1-d_{\sigma(i)})^{w_i} - \prod_{i=1}^{n}(1-b_{\sigma(i)}-d_{\sigma(i)})^{w_i}], [1-\prod_{i=1}^{n}(1-c_{\sigma(i)})^{w_i}, 1-\prod_{i=1}^{n}(1-d_{\sigma(i)})^{w_i}] \quad (7)$$

where w= $(w_1, w_2, \ldots\ldots w_n)^T$ be the weight vector of $\tilde{a}_i$ (i=1,2,…n) and w_i>0, $\sum_{i=1}^{n} w_i = 1$, and

$\tilde{a}_{\sigma(i)} = ([a_{\sigma(i)}, b_{\sigma(i)}], [c_{\sigma(i)}, d_{\sigma(i)}])$ and $\tilde{a}_{\sigma(1)}, \tilde{a}_{\sigma(2)}, \ldots\ldots\ldots\ldots \tilde{a}_{\sigma(n)}$ is the permutation of $\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n$ such that $\tilde{a}_{\sigma(i)}$is the largest IVIFV among the IVIFVs.

We now analyze the drawbacks of the operators developed by Xu and Chen with the one proposed by Chen and Tsai.

Definition 3.3. Interval-valued intuitionistic fuzzy weighted geometric aggregation operator (IVIFWG)

Let $\tilde{a}_i$=<$[a_i, b_i], [c_i, d_i]$>, i=1,2,….n be a collection of interval-valued intuitionistic fuzzy values, then the IVIFWG operator of the IVIFSs is defined as follows:

$$\text{IVIFWG}(\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n) = ([\prod_{i=1}^{n} a_i^{w_i}, \prod_{i=1}^{n} b_i^{w_i}], [1-\prod_{i=1}^{n}(1-c_i)^{w_i}, 1-\prod_{i=1}^{n}(1-d_i)^{w_i}]) \quad (8)$$

where w= $(w_1, w_2, \ldots\ldots w_n)^T$ be the weight vector of $\tilde{a}_i$ (i=1,2,…n) and w_i>0, $\sum_{i=1}^{n} w_i = 1$

Definition 3.4. Interval-valued intuitionistic fuzzy ordered weighted arithmetic aggregation operator (IVIFWAG)

Let $\tilde{a}_i$=<$[a_i, b_i], [c_i, d_i]$>, i=1,2,….n be a collection of interval-valued Intuitionistic fuzzy values, then the IVIFOWG operator of the IVIFSs is defined as follows:

IVIFWAG$(\tilde{a}_1, \tilde{a}_2 \ldots\ldots \tilde{a}_n)$ = $([1-\prod_{i=1}^{n}(1-a_i)^{w_i}, 1-\prod_{i=1}^{n}(1-b_i)^{w_i}],$
$[\prod_{i=1}^{n} c_i^{w_i}, \prod_{i=1}^{n} d_i^{w_i}])$ (9)
where w= $(w_1, w_2, \ldots\ldots w_n)^T$ be the weight vector of $\tilde{a}_i$ (i=1,2,…n) and $w_i>0$, $\sum_{i=1}^{n} w_i = 1$.

The drawback is that when there is only one membership degree of an IVIFV equal to [0,0], the membership degree $[\prod_{i=1}^{n} a_i^{w_i}, \prod_{i=1}^{n} b_i^{w_i}]$ of the aggregated IVIFSs of n IVIFVs becomes [0,0] even though all the other membership degree of other n-1 IVIFVs are not equal to zero.

Similarly when there is only one non membership degree of an IVIFV equal to [0,0], the non-membership degree $[\prod_{i=1}^{n} c_i^{w_i}, \prod_{i=1}^{n} d_i^{w_i}]$ of the aggregated IVIFSs of n IVIFVs becomes [0,0] even though all the other non-membership degree of other n-1 IVIFVs are not equal to zero.

4. The proposed method for multiple attribute group decision making based on new interval-valued intuitionistic fuzzy geometric averaging operators with knowledge measure.

Let D=$\widetilde{(a_{kl})}_{mxn}$ be a decision matrix, shown as follows:

$$D_j\widetilde{(a_{ij})}_{mxn} = \begin{matrix} & A_1 & A_2 & . & . & A_1 \\ x1 & \tilde{a}_{11} & \tilde{a}_{12} & . & . & \tilde{a}_{1n} \\ x2 & \tilde{a}_{21} & \tilde{a}_{22} & . & . & \tilde{a}_{11} \\ x3 & \vdots & . & . & . & . \\ x4 & & . & . & . & . \\ x5 & \tilde{a}_{m1} & \tilde{a}_{m2} & . & . & \tilde{a}_{mn} \end{matrix}$$

Given by the decision makers D_j where $\tilde{a}_{kl}$ is an evaluating IVIFV of attribute A_i with respect to alternative x_k represented by an IVIFV $\tilde{a}_{ij}$= ([a_{ij}, b_{ij}], [c_{ij},dj_i]), $0\le a_{ij}\le b_{ij}\le 1$, $0\le c_{ij}\le d_{ij}\le 1$, $0\le b_{ij}+d_{ij}\le 1$ and $1\le i\le n$. Let w_i be the weight of α_i, where w_i€[0,1], $1\le i\le n$ and $\sum_{i=1}^{n} w_i = 1$.

The proposed multiple attribute decision making method is as follows:

Step 1: Based on the IVIFWGA operator shown in eq (6) aggregate the evaluating IVIFVs for each decision matrix in to a single decision matrix with the weight vectors assigned to each decision maker.

Step 2: Based on the IVIFWGA operator shown in eq (6) aggregate the elements appearing at the kth row of the aggregated decision matrix in to an IVIFV $\tilde{a}_k$, where $\tilde{a}_k$ = ([a_{ij}, b_{ij}], [c_{ij},dj_i])]), $0\le a_{ij}\le b_{ij}\le 1$, $0\le c_{ij}\le d_{ij}\le 1$, $0\le b_{ij}+d_{ij}\le 1$ and $1\le i\le n$. Let w_i be the weight of α_i, where w_i€[0,1], $1\le i\le n$ and $\sum_{i=1}^{n} w_i = 1$.Here w_j is the weight of the attribute.

Step 3: Calculate the interval-valued knowledge measureK_F^1 $(\tilde{a}_k)$ of alternatives from the overall collective evaluation using eq (5) for single- element IVIFSs:

Denote K_F^1 $(\tilde{a}_k)$ with $[\mu^-_{ak}, \mu^+_{ak}] \geq [\nu^-_{ak}, \nu^+_{ak}]$ by positive interval-valued knowledge measure K_F^1 $(\tilde{a}_k)$ and $[\mu^-_{ak}, \mu^+_{ak}] \leq [\nu^-_{ak}, \nu^+_{ak}]$ by negative interval-valued knowledge measure K_F^1 $(\tilde{a}_k)$.

Our goal is to select the best alternative with the most positive interval valued measure.

Step 4: Rank the alternatives in the descending order of their positive interval-valued knowledge measures and in the ascending order of their negative interval-valued knowledge measures.

The best alternative is the one with the biggest positive interval-valued knowledge measures and the worse alternative is the one with the biggest negative interval-valued knowledge measures.

Example. A firm needs to identify a best supplier from a set of four suppliers namely S_1, S_2, S_3 and S_4. Three criteria must be evaluated. They are Quality (C_1), Reliability (C_2), and Price (C_3). The interval valued intuitionistic decision matrix provided by the decision makers are given below:

Table 4.1: Interval- valued intuitionistic fuzzy decision matrix $R^{(1)}$ of the expert D_1

	A_1	A_2	A_3
X_1	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.1,0.2])	([0.6,0.8],[0.0,0.2])
X_2	([0.6,0.7],[0.2,0.3])	([0.5,0.7],[0.2,0.3])	([0.5,0.6],[0.2,0.3])
X_3	([0.4,0.5],[0.2,0.4])	([0.5,0.6],[0.2,0.3])	([0.4,0.6],[0.1,0.2])
X_4	([0.7,0.8],[0.1,0.2])	([0.6,0.8],[0.0,0.1])	([0.6,0.7],[0.1, 0.2])

Table 4.2: Interval- valued intuitionistic fuzzy decision matrix $R^{(2)}$ of the expert D_2

	A_1	A_2	A_3
X_1	([0.7,0.8],[0.1,0.2])	([0.8,0.9],[0.0,0.1])	([0.7,0.9],[0.0,0.1])
X_2	([0.5,0.7],[0.1,0.2])	([0.6,0.7],[0.1,0.3])	([0.4,0.5],[0.1,0.3])
X_3	([0.3,0.5],[0.1,0.3])	([0.4,0.5],[0.1,0.3])	([0.3,0.6],[0.3,0.4])
X_4	([0.6,0.7],[0.1,0.2])	([0.7,0.8],[0.1,0.2])	([0.5,0.7],[0.1, 0.3])

Table 4.3: Interval- valued intuitionistic fuzzy decision matrix $R^{(3)}$ of the expert D_3

	A_1	A_2	A_3
X_1	([0.6,0.7],[0.1,0.3])	([0.7,0.9],[0.0,0.1])	([0.8,0.9],[0.0,0.1])
X_2	([0.4,0.6],[0.1,0.2])	([0.5,0.7],[0.1,0.2])	([0.6,0.7],[0.1,0.3])
X_3	([0.2,0.4],[0.2,0.3])	([0.3,0.6],[0.2,0.3])	([0.4,0.6],[0.2,0.4])
X_4	([0.7,0.8],[0.0,0.1])	([0.8,0.9],[0.0,0.1])	([0.4,0.7],[0.2, 0.3])

Let $\eta_k = (1/6,2/6,3/6)^T$ be the weight of each decision maker D_i (i=1,2,3) and $w=(0.3,0.4,0.3)^T$ be the weight of the attributes a_j(j=1,2,3).

Step 1: Derive the collective evaluation values using IVIFWA operator in eq (6) with the corresponding weight vector$\eta_k = (1/6,2/6,3/6)^T$ be the weight of each decision maker D_i (i=1,2,3).

Table 4.4: Collective evaluation values of alternatives on attributes by all experts

	A_1	A_2	A_3
X_1	([0.682,0.815], [0.035,0.184])	([0.738,0.883], [0.017,0.118])	([0.743,0.883], [0.000,0.117])
X_2	([0.484,0.782], [0.167,0.218])	([0.536,0.748], [0.117,0.252])	([0.518,0.665], [0.151,0.435])
X_3	([0.271,0.455], [0.167,0.300])	([0.373,0.575], [0.157,0.300])	([0.365,0.629], [0.219,0.371])
X_4	([0.239,0.849], [0.480,0.151])	([0.741,0.865], [0.035,0.135])	([0.466,0.716], [0.151, 0.234])

Step 2: Derive the overall collective evaluation values $\tilde{a}_{ij}$ of alternatives x_i by utilizing IVIFWA operator to aggregate the individual evaluation $\tilde{a}_k$ with the attribute weight vector $w=(0.3,0.4,0.3)^T$.

$\tilde{a}_1$=([0.723,0.862],[0.017,0.138])
$\tilde{a}_2$=([0.515,0.697],[0.142,0.303])
$\tilde{a}_3$=([0.342,0.678],[0.179,0.322])
$\tilde{a}_4$=([0.490,0.830],[0.223,0.170])

Step 3: Calculate the interval-valued knowledge measures of alternatives from their overall collective evaluation values $\tilde{a}_k$.

$K_F^l\widetilde{(a_1)}$= [0.732,0.939]
$K_F^l\widetilde{(a_2)}$= [0.461, 0.889]
$K_F^l\widetilde{(a_3)}$= [0.459, 0.884]
$K_F^l\widetilde{(a_4)}$= [0.632, 0.927]

Step 4: The positive interval-valued knowledge measures are

$K_F^l\widetilde{(a_1)}$= [0.732,0.939]
$K_F^l\widetilde{(a_2)}$= [0.461, 0.889],$K_F^l\widetilde{(a_3)}$= [0.459, 0.884],$K_F^l\widetilde{(a_4)}$= [0.632, 0.927]. There is no negative interval-valued knowledge measure.

Rank these positive interval-valued knowledge measures as follows: $K_F^l\widetilde{(a_1)}$>$K_F^l\widetilde{(a_4)}$>$K_F^l\widetilde{(a_2)}$>$K_F^l\widetilde{(a_3)}$. Thus ranking of the alternatives in accordance with the knowledge measures as : $x_1>x_4>x_2>x_3$ and the best alternative is x_1.

5. Conclusion

Thus we propose a new multiple attribute decision making method under intuitionistic fuzzy environment with the help of knowledge measure. This method overcomes all the drawbacks in the existing methods. Further this can be extended to type 2 interval fuzzy sets also.

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