

## On Fuzzy $e^*$ -Open Sets and Fuzzy $(\mathcal{D}, S)^*$ -Sets

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**Abstract.** The aim of this paper is to introduce some new classes of fuzzy sets and some new classes of fuzzy continuity namely fuzzy  $e^*$ -open sets, fuzzy  $(\mathcal{D}, S)$ -sets, fuzzy  $(\mathcal{D}S, \varepsilon)$ -sets, fuzzy  $(\mathcal{D}, S)^*$ -sets, fuzzy  $(\mathcal{D}S, \varepsilon)^*$ -sets, fuzzy  $e$ -continuity, fuzzy  $(\mathcal{D}, S)^*$ -continuity and fuzzy  $(\mathcal{D}S, \varepsilon)^*$ -continuity. Properties of these new concepts are investigated. Moreover, some new decompositions of fuzzy continuity are provided.

**Keywords:** Fuzzy Continuity, Fuzzy  $e^*$ -open sets, Fuzzy  $(\mathcal{D}, S)$ -sets, Fuzzy  $(\mathcal{D}S, \varepsilon)$ -sets, Fuzzy  $(\mathcal{D}, S)^*$ -sets, Fuzzy  $(\mathcal{D}S, \varepsilon)^*$ -sets, Fuzzy  $e$ -continuity, Fuzzy  $(\mathcal{D}, S)^*$ -continuity and Fuzzy  $(\mathcal{D}S, \varepsilon)^*$ -continuity.

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### 1. Introduction and preliminaries

After Chang's paper [4], mathematicians introduced different new decompositions of fuzzy continuous functions and some weaker forms of fuzzy continuous functions. The main purpose of this paper is to establish some new decompositions of fuzzy continuous functions. Firstly, we introduce a new classes of sets called fuzzy  $e^*$ -open sets. The class of fuzzy  $e^*$ -open sets generalize the classes of fuzzy  $e$ -open sets, fuzzy  $\delta$ -semiopen sets and fuzzy  $\delta$ -preopen sets. Properties and the relationships of fuzzy  $e^*$ -open sets are investigated. On the other hand, we introduce the notions of fuzzy  $(\mathcal{D}, S)$ -sets, fuzzy  $(\mathcal{D}S, \varepsilon)$ -sets, fuzzy  $(\mathcal{D}, S)^*$ -sets, fuzzy  $(\mathcal{D}S, \varepsilon)^*$ -sets, fuzzy  $e^*$ -continuity, fuzzy  $(\mathcal{D}, S)^*$ -continuity and fuzzy  $(\mathcal{D}S, \varepsilon)^*$ -continuity. Finally, we obtain some new decompositions of fuzzy continuous functions via these new concepts.

In this paper  $(X, \tau)$  and  $(Y, \sigma)$  represent fuzzy topological spaces. For a subset  $A$  of a space  $X$ ,  $\text{cl}(A)$  and  $\text{int}(A)$  denote the fuzzy closure of  $A$  and the fuzzy interior of  $A$ , respectively. A subset  $A$  of a space  $(X, \tau)$  is called fuzzy  $\alpha$ -open [8] (resp. Fuzzy  $\beta$ -open [1], fuzzy preopen [3], fuzzy regular open [2], fuzzy regular closed [2]) if:

$$A \leq \text{int}(\text{cl}(\text{int}(A))) \text{ (resp. } A \leq \text{cl}(\text{int}(\text{cl}(A))), \\ A \leq \text{int}(\text{cl}(A)), A = \text{int}(\text{cl}(A)), A = \text{cl}(\text{int}(A))).$$

A subset  $A$  of a space  $(X, \tau)$  is called fuzzy  $\delta$ -open [5] if for each  $x \in A$  there exists a fuzzy regular open set  $V$  such that  $x \in V \leq A$ . A set  $A$  is said to be fuzzy  $\delta$ -closed if its complement is fuzzy  $\delta$ -open. A point  $x \in X$  is called a fuzzy  $\delta$ -cluster points of  $A$  [5]

if  $A \wedge \text{int}(\text{cl}(V)) \neq \emptyset$  for each fuzzy open set  $V$  containing  $x$ . Set of all fuzzy  $\delta$ -cluster points of  $A$  is called the fuzzy  $\delta$ -closure of  $A$  and is denoted by  $\delta\text{-cl}(A)$ . The fuzzy  $\delta$ -interior of  $A$  is the union of all fuzzy regular open sets contained in  $A$  and is denoted by  $\delta\text{-int}(A)$ . A subset  $A$  of a space  $(X, \tau)$  is called fuzzy  $\delta$ -preopen [5] (resp. fuzzy  $\delta$ -semiopen [5], fuzzy  $e$ -open [7], fuzzy  $e$ -closed [7]) if:

$$A \leq \text{int}(\delta\text{-cl}(A)) \text{ (resp. } A \leq \text{cl}(\delta\text{-int}(A)), \\ A \leq \text{cl}(\delta\text{-int}(A)) \vee \text{int}(\delta\text{-cl}(A)), \text{cl}(\delta\text{-int}(A)) \wedge \text{int}(\delta\text{-cl}(A)) \leq A).$$

The complement of a fuzzy  $\delta$ -semiopen (resp. fuzzy  $\delta$ -preopen) set is called fuzzy  $\delta$ -semiclosed (resp. fuzzy  $\delta$ -preclosed). The intersection of all fuzzy  $e$ -closed (resp. fuzzy  $\delta$ -semiclosed, fuzzy  $\delta$ -preclosed) sets, each containing a set  $A$  in a fuzzy topological space  $X$  is called the fuzzy  $e$ -closure [7] (resp. fuzzy  $\delta$ -semiclosure [8], fuzzy  $\delta$ -preclosure [9]) of  $A$  and it is denoted by  $e\text{-cl}(A)$  (resp.  $\delta\text{-scl}(A)$ ,  $\delta\text{-pcl}(A)$ ). The union of all fuzzy  $e$ -open (resp. Fuzzy  $\delta$ -semiopen, fuzzy  $\delta$ -preopen) sets, each contained in a fuzzy set  $A$  in a fuzzy topological space  $X$  is called the fuzzy  $e$ -interior [7] (resp. fuzzy  $\delta$ -semiinterior [5], fuzzy  $\delta$ -preinterior [5]) of  $A$  and it is denoted by  $e\text{-int}(A)$  (resp.  $\delta\text{-sint}(A)$ ,  $\delta\text{-pint}(A)$ ).

**Lemma 1.1.** [8] The following hold for a subset  $A$  of a space  $X$ :

- (1)  $\delta\text{-sint}(A) = A \wedge \text{cl}(\delta\text{-int}(A))$  and  $\delta\text{-scl}(A) = A \vee \text{int}(\delta\text{-cl}(A))$ ;
- (2)  $\delta\text{-pcl}(A) = A \vee \text{cl}(\delta\text{-int}(A))$ ;
- (3)  $\delta\text{-scl}(\delta\text{-sint}(A)) = \delta\text{-sint}(A) \vee \text{int}(\text{cl}(\delta\text{-int}(A)))$  and  $\delta\text{-sint}(\delta\text{-scl}(A)) = \delta\text{-scl}(A) \wedge \text{cl}(\text{int}(\delta\text{-cl}(A)))$ ;
- (4)  $\delta\text{-cl}(\delta\text{-sint}(A)) = \text{cl}(\delta\text{-int}(A))$ ;
- (5)  $\delta\text{-scl}(\delta\text{-int}(A)) = \text{int}(\text{cl}(\delta\text{-int}(A)))$ .

**Theorem 1.1.** [3] Let  $A$  be a subset of a space  $X$ . Then:

- (1)  $e\text{-cl}(A) = \delta\text{-pcl}(A) \wedge \delta\text{-scl}(A)$ ;
- (2)  $\delta\text{-int}(e\text{-cl}(A)) = \text{int}(\text{cl}(\delta\text{-int}(A)))$ .

## 2. Fuzzy $e^*$ -open sets, Fuzzy $(\mathcal{D}, \mathcal{S})$ -sets and Fuzzy $(\mathcal{DS}, \varepsilon)$ -sets

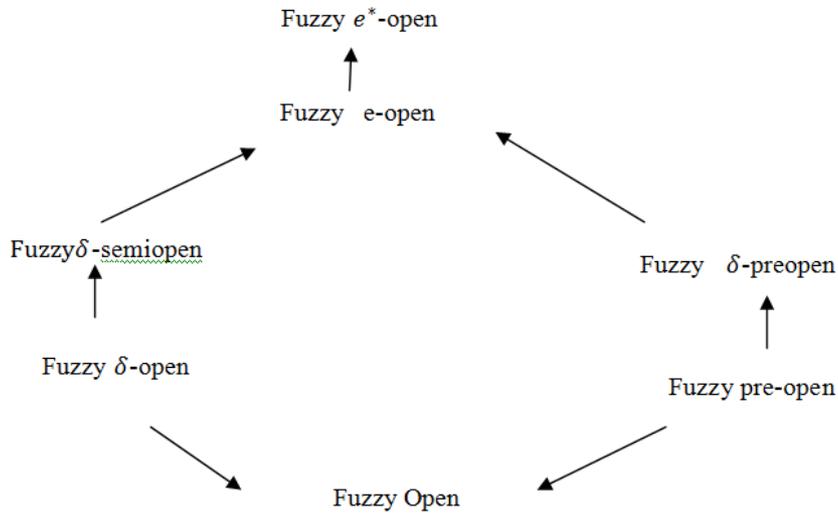
**Definition 2.1.** A subset  $A$  of a space  $X$  is called Fuzzy  $e^*$ -open if  $A \leq \text{cl}(\text{int}(\delta\text{-cl}(A)))$ .

**Theorem 2.1.** The following are equivalent for a fuzzy subset  $A$  of a space  $X$ :

- (1)  $A$  is fuzzy  $e^*$ -open,
- (2) there exists a fuzzy  $\delta$ -preopen set  $U$  such that  $U \leq \delta\text{-cl}(A) \leq \delta\text{-cl}(U)$ ,
- (3)  $\delta\text{-cl}(A)$  is fuzzy regular closed.

**Remark 2.1.** (1) Let  $A$  be a fuzzy subset of a space  $X$ . Then the following diagram holds:

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- (2) None of these implications is reversible as shown in the following example and in [3].  
 (3) Every fuzzy  $\beta$ -open set is fuzzy  $e^*$ -open but the converse is not true in general as shown in the following example.

**Example 2.1.** Let  $\lambda_1, \lambda_2, \lambda_3$  &  $\lambda_4$  be fuzzy sets on  $X=\{a,b,c\}$  define as

$$\lambda_1(a)=0.3, \lambda_1(b)=0.4, \lambda_1(c)=0.5,$$

$$\lambda_2(a)=0.6, \lambda_2(b)=0.5, \lambda_2(c)=0.5,$$

$$\lambda_3(a)=0.6, \lambda_3(b)=0.5, \lambda_3(c)=0.4,$$

$$\lambda_4(a)=0.3, \lambda_4(b)=0.4, \lambda_4(c)=0.4,$$

$$\lambda_5(a)=0.7, \lambda_5(b)=0.6, \lambda_5(c)=0.4,$$

Consider fuzzy topologies  $\tau = \{0_x, 1_x, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$  &  $\sigma = \{0_x, 1_x, \lambda_5\}$ . The identity function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $(\mathcal{D}, S)^*$ -but it is not fuzzy  $(\mathcal{D}, S)$ -set.

**Definition 2.2.** A fuzzy subset  $A$  of a space  $X$  is called fuzzy  $e^*$ -closed if  $\text{int}(\text{cl}(\delta - \text{int}(A))) \leq A$ .

- Theorem 2.2.** (1) The union of any family of fuzzy  $e^*$ -open sets is an fuzzy  $e^*$ -open set;  
 (2) The intersection of any family of fuzzy  $e^*$ -closed sets is an fuzzy  $e^*$ -closed set.

**Definition 2.3.** Let  $A$  be a fuzzy subset of a space  $X$ .

- (1) The intersection of all fuzzy  $e^*$ -closed sets containing  $A$  is called the fuzzy  $e^*$ -closure of  $A$  and is denoted by  $e^*\text{-cl}(A)$ ;  
 (2) The fuzzy  $e^*$ -interior of  $A$ , denoted by  $\text{fuzzy } e^*\text{-int}(A)$ , is defined by the union of all fuzzy  $e^*$ -open sets contained in  $A$ .

**Lemma 2.1.** The following hold for a subset A of a space X:

- (1)  $e^*$ -cl(A) is fuzzy  $e^*$ -closed,
- (2)  $X \setminus e^*$ -cl(A) =  $e^*$ -int( $X \setminus A$ ).

**Theorem 2.3.** The following hold for a subset A of a space X:

- (1) A is fuzzy  $e^*$ -open if and only if  $A = A \wedge \text{cl}(\text{int}(\delta\text{-cl}(A)))$ ;
- (2) A is fuzzy  $e^*$ -closed if and only if  $A = A \vee \text{int}(\text{cl}(\delta\text{-int}(A)))$ ;
- (3)  $e^*$ -cl(A) =  $A \vee \text{int}(\text{cl}(\delta\text{-int}(A)))$ ;
- (4)  $e^*$ -int(A) =  $A \wedge \text{cl}(\text{int}(\delta\text{-cl}(A)))$ .

**Proof.** (1) :

Let A be  $e^*$ -open. Then  $A \leq \text{cl}(\text{int}(\delta\text{-cl}(A)))$ . We obtain

$A \leq A \wedge \text{cl}(\text{int}(\delta\text{-cl}(A)))$ . Conversely, let  $A = A \wedge \text{cl}(\text{int}(\delta\text{-cl}(A)))$ . We have  $A = A \wedge \text{cl}(\text{int}(\delta\text{-cl}(A))) \leq \text{cl}(\text{int}(\delta\text{-cl}(A)))$  and hence, A is  $e^*$ -open.

(3) : Since  $e^*$ -cl(A) is  $e^*$ -closed,

$$\text{int}(\text{cl}(\delta\text{-int}(A))) \leq \text{int}(\text{cl}(\delta\text{-int}(e^*\text{-cl}(A)))) \leq e^*\text{-cl}(A).$$

Hence,  $A \vee \text{int}(\text{cl}(\delta\text{-int}(A))) \leq e^*\text{-cl}(A)$ .

Conversely, since:

$$\begin{aligned} \text{int}(\text{cl}(\delta\text{-int}(A \vee \text{int}(\text{cl}(\delta\text{-int}(A))))) &= \text{int}(\text{cl}(\delta\text{-int}(A \vee \delta\text{-int}(\delta\text{-cl}(\delta\text{-int}(A))))) \\ &= \text{int}(\text{cl}(\delta\text{-int}(A) \vee \delta\text{-int}(\delta\text{-int}(\delta\text{-cl}(\delta\text{-int}(A))))) \\ &= \text{int}(\text{cl}(\delta\text{-int}(A) \vee \delta\text{-int}(\delta\text{-cl}(\delta\text{-int}(A)))) \\ &= \text{int}(\text{cl}(\delta\text{-int}(\delta\text{-cl}(\delta\text{-int}(A)))) \\ &= \text{int}(\text{cl}(\delta\text{-int}(A))) \leq A \vee \text{int}(\text{cl}(\delta\text{-int}(A))), \end{aligned}$$

then  $A \vee \text{int}(\text{cl}(\delta\text{-int}(A)))$  is fuzzy  $e^*$ -closed containing A and hence:

$$e^*\text{-cl}(A) \leq A \vee \text{int}(\text{cl}(\delta\text{-int}(A))).$$

Thus, we obtain  $e^*\text{-cl}(A) = A \vee \text{int}(\text{cl}(\delta\text{-int}(A)))$ .

(2) follows from (1) and (4) follows from (3).

**Theorem 2.4.** Let A be a subset of a space X. Then the following hold:

- (1)  $e^*\text{-cl}(\delta\text{-int}(A)) = \text{int}(\text{cl}(\delta\text{-int}(A)))$ ;
- (2)  $\delta\text{-int}(e^*\text{-cl}(A)) = \text{int}(\text{cl}(\delta\text{-int}(A)))$ ;
- (3)  $e^*\text{-int}(\delta\text{-cl}(A)) = \delta\text{-cl}(e^*\text{-int}(A)) = \text{cl}(\text{int}(\delta\text{-cl}(A)))$ ;
- (4)  $e^*\text{-int}(e\text{-cl}(A)) = \delta\text{-sint}(\delta\text{-scl}(A)) \wedge \delta\text{-pcl}(A)$ ;
- (5)  $e^*\text{-cl}(e\text{-int}(A)) = \delta\text{-scl}(\delta\text{-sint}(A)) \vee \delta\text{-pint}(A)$ ;
- (6)  $e\text{-cl}(e^*\text{-int}(A)) = \delta\text{-sint}(\delta\text{-scl}(A)) \wedge \delta\text{-pcl}(A)$ ;
- (7)  $e\text{-int}(e^*\text{-cl}(A)) = \delta\text{-scl}(\delta\text{-sint}(A)) \vee \delta\text{-pint}(A)$ .

**Proof:** Proof is similar to the proof of Theorem 2.15 in [3].

**Theorem 2.5.** Let A be a subset of a space X. Then  $\delta\text{-scl}(\delta\text{-sint}(A)) \leq \delta\text{-sint}(\delta\text{-scl}(A))$ .

**Proof:** Proof is similar to the proof of Theorem 3.2 in [3].

**Definition 2.4.** A subset A of a space X is said to be a (D, S)-set if  $\delta\text{-sint}(\delta\text{-scl}(A)) = \text{int}(A)$ .

**Theorem 2.6.** Let A be a fuzzy subset of a space X. Then if A is a fuzzy (D, S)-set, it is fuzzy  $\delta$ -semiclosed.

**Proof:** Since A is a fuzzy (D, S)-set, we have:

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$$\begin{aligned} A &\geq \text{int}(A) = \delta\text{-sint}(\delta\text{-scl}(A)) = \delta\text{-scl}(A) \wedge \text{cl}(\text{int}(\delta\text{-cl}(A))) \\ &\geq \text{int}(\delta\text{-cl}(A)) \wedge \text{cl}(\text{int}(\delta\text{-cl}(A))) \geq \text{int}(\delta\text{-cl}(A)). \end{aligned}$$

This implies that A is fuzzy  $\delta$ -semiclosed.

The following example shows that the implication in Theorem 2.6 is notreversible.

**Example 2.2.** Let  $\lambda_1, \lambda_2, \lambda_3$  &  $\lambda_4$  be fuzzy sets on  $X=\{a,b,c\}$  define as

$$\begin{aligned} \lambda_1(a)=0.5, \lambda_1(b)=0, \lambda_1(c)=0, \\ \lambda_2(a)=0, \lambda_2(b)=0.5, \lambda_2(c)=0, \\ \lambda_3(a)=0.5, \lambda_3(b)=0.5, \lambda_3(c)=0, \\ \lambda_4(a)=0.5, \lambda_4(b)=0.5, \lambda_4(c)=0.5 \end{aligned}$$

consider fuzzy topologies  $\tau = \{0_x, 1_x, \lambda_1, \lambda_2, \lambda_3\}$  &  $\sigma = \{0_x, 1_x, \lambda_4\}$ . The identity function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $\delta$ -semiclosed but it is not fuzzy  $(\mathcal{D}, S)$ -set.

**Definition 2.5.** A fuzzy subset K of a space X is said to be a fuzzy  $(\mathcal{D}, S)^*$ -set if there exist an fuzzy open set A and a fuzzy  $(\mathcal{D}, S)$ -set B such that  $K = A \wedge B$ .

**Remark 2.2.** Every  $(\mathcal{D}, S)$ - set and every open set is a  $(\mathcal{D}, S)^*$ -set but notconversely.

**Example 2.3.** Let  $\lambda_1, \lambda_2, \lambda_3$  &  $\lambda_4$  be fuzzy sets on  $X=\{a,b,c\}$  define as

$$\begin{aligned} \lambda_1(a)=0.5, \lambda_1(b)=0, \lambda_1(c)=0, \\ \lambda_2(a)=0, \lambda_2(b)=0.5, \lambda_2(c)=0, \\ \lambda_3(a)=0.5, \lambda_3(b)=0.5, \lambda_3(c)=0, \text{ consider fuzzy topologies } \tau = \{0_x, 1_x, \lambda_1, \lambda_2, \lambda_3\} \text{ \& } \\ \sigma = \{0_x, 1_x, \lambda_4\}. \text{ The identity function } f:(X, \tau) \rightarrow (Y, \sigma) \text{ is fuzzy } (\mathcal{D}, S)^*\text{-but it is not } \\ \text{fuzzy } (\mathcal{D}, S)\text{-set.} \end{aligned}$$

**Theorem 2.7.** Let X be a topological space and  $A \leq X$ . Then A is  $e^*$ -open if and only if  $A \leq \delta\text{-sint}(\delta\text{-scl}(A))$ .

**Proof:** Since A is  $e^*$ -open, then  $A \leq \text{cl}(\text{int}(\delta\text{-cl}(A)))$ . By Lemma 1.1:

$$\begin{aligned} A &\leq \text{cl}(\text{int}(\delta\text{-cl}(A))) \wedge A \\ &\leq \delta\text{-scl}(A) \wedge \text{cl}(\text{int}(\delta\text{-cl}(A))) \\ &= \delta\text{-sint}(\delta\text{-scl}(A)). \end{aligned}$$

Conversely, since  $A \leq \delta\text{-sint}(\delta\text{-scl}(A))$ , by Lemma 1.1 we obtain:

$$\begin{aligned} A &\leq \delta\text{-sint}(\delta\text{-scl}(A)) \\ &= \delta\text{-scl}(A) \wedge \text{cl}(\text{int}(\delta\text{-cl}(A))) \\ &\leq \text{cl}(\text{int}(\delta\text{-cl}(A))). \end{aligned}$$

Thus, A is fuzzy  $e^*$ -open.

**Theorem 2.8.** Let K be a fuzzy subset of a topological space X. Then the following are equivalent:

- (1) K is fuzzy open;

- (2) K is fuzzy  $\alpha$ -open and a fuzzy  $(\mathcal{D}, S)^*$ -set;
- (3) K is fuzzy preopen and a fuzzy  $(\mathcal{D}, S)^*$ -set;
- (4) K is fuzzy  $\delta$ -preopen and a fuzzy  $(\mathcal{D}, S)^*$ -set;
- (5) K is fuzzy e-open and a fuzzy  $(\mathcal{D}, S)^*$ -set;
- (6) K is fuzzy  $e^*$ -open and a fuzzy  $(\mathcal{D}, S)^*$ -set

**Proof:** (6)  $\Rightarrow$  (1) : Since K is fuzzy  $e^*$ -open and a fuzzy  $(\mathcal{D}, S)^*$ -set, then there exist an fuzzy open set A and a fuzzy  $(\mathcal{D}, S)$ -set B such that  $K = A \wedge B$ . On the other hand:

$$\begin{aligned} K &\leq \delta\text{-sint}(\delta\text{-scl}(A \wedge B)) \\ &\leq \delta\text{-sint}(\delta\text{-scl}(A)) \wedge \delta\text{-sint}(\delta\text{-scl}(B)) \\ &\leq \delta\text{-sint}(\delta\text{-cl}(A)) \wedge \text{int}(B) \\ &= \text{cl}(\text{int}(\delta\text{-cl}(A))) \wedge \text{int}(B) \\ &\leq \text{cl}(\text{cl}(\delta\text{-cl}(A))) \wedge \text{int}(B) \\ &= \delta\text{-cl}(A) \wedge \text{int}(B) \end{aligned}$$

by Lemma 1.1. We have  $K \leq \delta\text{-cl}(A) \wedge \text{int}(B) \wedge A = \text{int}(B) \wedge A$  and then  $K = A \wedge \text{int}(B)$ . Thus, K is fuzzy open.

The other implications are obvious.

**Definition 2.6.** A fuzzy subset K of a space X is said to be a fuzzy  $(\mathcal{DS}, \varepsilon)$ -set if  $\delta\text{-scl}(\delta\text{-sint}(K)) = \text{int}(K)$ .

**Theorem 2.9.** Let K be a fuzzy subset of a space X. Then if K is a fuzzy  $(\mathcal{DS}, \varepsilon)$ -set, then it is fuzzy  $e^*$ -closed.

**Proof:** Let K be a fuzzy  $(\mathcal{DS}, \varepsilon)$ -set. By Lemma 1.1:

$$\begin{aligned} K &\geq \text{int}(K) \\ &= \delta\text{-scl}(\delta\text{-sint}(K)) \\ &= \delta\text{-sint}(K) \vee \text{int}(\text{cl}(\delta\text{-int}(K))) \\ &\geq \text{int}(\text{cl}(\delta\text{-int}(K))). \end{aligned}$$

This implies that K is fuzzy  $e^*$ -closed.

The following example shows that this implication is not reversible.

**Example 2.4.** Let  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  &  $\lambda_5$  be fuzzy sets on  $X = \{a, b, c\}$  define as

$$\begin{aligned} \lambda_1(a) &= 0.3, \lambda_1(b) = 0.5, \lambda_1(c) = 0.5, \\ \lambda_2(a) &= 0.4, \lambda_2(b) = 0.2, \lambda_2(c) = 0.6, \\ \lambda_3(a) &= 0.4, \lambda_3(b) = 0.5, \lambda_3(c) = 0.6, \\ \lambda_4(a) &= 0.3, \lambda_4(b) = 0.2, \lambda_4(c) = 0.5, \\ \lambda_5(a) &= 0.3, \lambda_5(b) = 0.4, \lambda_5(c) = 0.4, \end{aligned}$$

Consider fuzzy topologies  $\tau = \{0_x, 1_x, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$  &  $\sigma = \{0_x, 1_x, \lambda_5\}$ . The identity function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $e^*$ -closed but it is not fuzzy  $(\mathcal{DS}, \varepsilon)$ -set.

**Definition 2.7.** A subset K of a space X is said to be a fuzzy  $(\mathcal{DS}, \varepsilon)^*$ -set if there exist an fuzzy open set A and a fuzzy  $(\mathcal{DS}, \varepsilon)$ -set B such that  $K = A \wedge B$ .

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**Remark 2.3.** Every fuzzy open and every fuzzy  $(\mathcal{D}, \varepsilon)$ -set is a fuzzy  $(\mathcal{D}, \varepsilon)^*$ -set but not conversely.

**Example 2.5.** Let  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$  be fuzzy sets on  $X=\{a,b,c\}$  define as

$$\lambda_1(a)=0.3, \lambda_1(b)=0.4, \lambda_1(c)=0.5,$$

$$\lambda_2(a)=0.6, \lambda_2(b)=0.5, \lambda_2(c)=0.5,$$

$$\lambda_3(a)=0.6, \lambda_3(b)=0.5, \lambda_3(c)=0.4,$$

$$\lambda_4(a)=0.3, \lambda_4(b)=0.4, \lambda_4(c)=0.4,$$

$$\lambda_5(a)=0.7, \lambda_5(b)=0.6, \lambda_5(c)=0.4,$$

consider fuzzy topologies  $\tau = \{0_x, 1_x, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$  &  $\sigma = \{0_x, 1_x, \lambda_5\}$ . The identity function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $(\mathcal{D}, \varepsilon)^*$ -but it is not fuzzy  $(\mathcal{D}, \varepsilon)$ -set.

**Theorem 2.10.** Let  $X$  be a fuzzy topological space and  $K \leq X$ . Then  $K$  is fuzzy  $\delta$ -semiopen if and only if  $K \leq \delta\text{-scl}(\delta\text{-sint}(K))$ .

**Proof:** Proof is similar to the proof of Theorem 2.7.

**Theorem 2.11.** Let  $K$  be a fuzzy subset of a space  $X$ . Then if  $K$  is fuzzy  $\delta$ -semiopen and a fuzzy  $(\mathcal{D}, \varepsilon)^*$ -set,  $K$  is fuzzy open.

**Proof:** Since  $K$  is fuzzy  $\delta$ -semiopen and a fuzzy  $(\mathcal{D}, \varepsilon)^*$ -set, there exist an fuzzy open set  $A$  and a fuzzy  $(\mathcal{D}, \varepsilon)$ -set  $B$  such that  $K = A \wedge B$  and:

$$\begin{aligned} K &\leq \delta\text{-scl}(\delta\text{-sint}(K)) \\ &= \delta\text{-scl}(\delta\text{-sint}(A \wedge B)) \\ &\leq \delta\text{-scl}(\delta\text{-sint}(A)) \wedge \delta\text{-scl}(\delta\text{-sint}(B)) \\ &= \delta\text{-scl}(\delta\text{-sint}(A)) \wedge \text{int}(B) \\ &\leq \delta\text{-cl}(\delta\text{-sint}(A)) \wedge \text{int}(B) \\ &= \text{cl}(\delta\text{-int}(A)) \wedge \text{int}(B) \\ &\leq \text{cl}(A) \wedge \text{int}(B). \end{aligned}$$

Since  $K \leq \text{cl}(A) \wedge \text{int}(B) \wedge A = A \wedge \text{int}(B)$ ,  $K = A \wedge \text{int}(B)$ . Thus,  $K$  is fuzzy open.

**Definition 2.8.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be fuzzy  $e^*$ -continuous (resp. fuzzy  $(\mathcal{D}, S)^*$ -continuous, fuzzy  $(\mathcal{D}, \varepsilon)^*$ -continuous) if  $f^{-1}(A)$  is fuzzy  $e^*$ -open (resp. A fuzzy  $(\mathcal{D}, S)^*$ -set, fuzzy  $(\mathcal{D}, \varepsilon)^*$ -set) in  $X$  for every  $A \in \sigma$ .

**Definition 2.9.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called fuzzy  $e$ -continuous [7] (resp. fuzzy  $\delta$ -almost continuous [5], fuzzy  $\delta$ -semicontinuous [5], fuzzy  $\alpha$ -continuous [8], fuzzy precontinuous [7]) if  $f^{-1}(A)$  is fuzzy  $e$ -open (resp. fuzzy  $\delta$ -preopen, fuzzy  $\delta$ -semiopen, fuzzy  $\alpha$ -open, fuzzy preopen) for each  $A \in \sigma$ .

The following remark is immediate from Theorem 2.8.

**Remark 2.4.** For a function  $f : X \rightarrow Y$ , the following are equivalent:

(1)  $f$  is fuzzy continuous;

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- (2)  $f$  is fuzzy  $\alpha$ -continuous and fuzzy  $(\mathcal{D}, S)^*$ -continuous;
- (3)  $f$  is fuzzy precontinuous and fuzzy  $(\mathcal{D}, S)^*$ -continuous;
- (4)  $f$  is fuzzy  $\delta$ -almost continuous and fuzzy  $(\mathcal{D}, S)^*$ -continuous;
- (5)  $f$  is fuzzy  $e$ -continuous and fuzzy  $(\mathcal{D}, S)^*$ -continuous;
- (6)  $f$  is fuzzy  $e^*$ -continuous and fuzzy  $(\mathcal{D}, S)^*$ -continuous.

## 6. Conclusion

The work in this paper step forward to strengthen the theoretical foundation of fuzzy  $e^*$ -open sets in fuzzy topological spaces, fuzzy  $(\mathcal{D}, S)$ -sets, fuzzy  $(\mathcal{DS}, \varepsilon)$ -sets, fuzzy  $(\mathcal{D}, S)^*$ -sets, fuzzy  $(\mathcal{DS}, \varepsilon)^*$ -sets, fuzzy  $e$ -continuity, fuzzy  $(\mathcal{D}, S)^*$ -continuity and fuzzy  $(\mathcal{DS}, \varepsilon)^*$ -continuity.

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