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Solving Critical Path Problem using Triangular Intuitionistic Fuzzy Number

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Abstract. This paper presents a method for finding critical path with Intuitionistic fuzzy project network. Triangular Intuitionistic fuzzy numbers are used to represent activity times in the project network and we propose an algorithm for finding critical path. A solution is also given to demonstrate our proposed approach.

Keywords: Network, Triangular Intuitionistic fuzzy number, Intuitionistic fuzzy critical path analysis, Method of magnitude.

AMS Mathematics Subject Classification (2010): 52B05

1. Introduction

Critical Path Method (CPM) is one of the many network techniques which is widely used for planning, scheduling and controlling the projects. First of all it consists in the identification of the so-called critical paths, critical activities and critical events in the network , which is the project model , assuming the earliest possible completion time of the whole project .There are many literatures devoted to research about the fuzzy CPM theories and applications . Starting with the second part of the 1970s [sec (Chanas and Radosinski, 1976; Prade , 1979)] the other approach to the network project analysis, usually called the fuzzy PERT method or the fuzzy CPM ,has been developed,in which it is suggested to use fuzzy numbers (sets) to model the activity times.

The main purpose of CPM is thus to identify critical activities on the critical path. However, the vagueness of the time parameters in the problem has led to the development of fuzzy CPM. The unknown problem that could occur in practical situation can be very well managed using this fuzzy CPM. Chanas and Kamburowski [2] introduced FPERT, they used fuzzy numbers to represent activity durations in project networks. Mon et al [9] , assumed the duration of each activity as a positive fuzzy numbers and using the α – cut of each fuzzy duration they exploited a linear combination of the duration bounds to represent the operation time of each activity and to determine the critical critical activities and paths by use of the traditional (crisp) PERT technique . However,based on the α values different critical activities and paths are obtained. Liberatore and Connelly [8], proposed a new straight forward method for applying fuzzy logic to assess uncertainty in critical path analysis.Chanas and Zielinski [5], assumed that the operation time of each activity can be represented as a crisp value, interval or a fuzzy number and discussed the complexity of criticality.

This paper is organized as follows. In Section 2 some elementary concepts and definitions in intuitionistic fuzzy set theory. An algorithm is presented in section 3. An illustrative example to find the critical path is explained in section 4. The last section draws some concluding remarks.

2. Preliminaries

In this section, some basic definitions used throughout the paper are presented.

2.1. Fuzzy set[12]

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. In the pair (x, $\mu_A(x)$) the first element x belongs to the classical set A, the second element $\mu_A(x)$, belongs to the interval [0, 1], called Membership function. It can also be denoted by $\tilde{A} = \{\mu_A(x)/x : x \in A, \mu_A(x) \in [0,1]\}$

2.2. Fuzzy number

The notion of fuzzy numbers was introduced by Dubois.D and Prade.H (1980). A fuzzy subset A of the real line R with membership function $\mu_A : R \to [0,1]$ is called a fuzzy number if

i. A is normal, i.e., there exists an element $x_0 \in A$ such that $\mu_A(x_0) = 1$

ii. A is fuzzy convex,

$i.e., \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2) \forall (x_1, x_2) \in R \;\&\; \forall \lambda \in [0,1]$

iii. μ_A is upper semi continuous

iv. Supp A is bounded where Supp $A = \{x \in R : \mu_A(x) > 0\}$

2.3. Triangular fuzzy number

A triangular fuzzy number $\tilde{A}$ is a fuzzy number fully specified by 3-tuples (a_1, a_2, a_3) such that $a_1 \leq a_2 \leq a_3$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, if a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2}, if a_2 \leq x \leq a_3 \\ 0, otherwise \end{cases}$$

This is represented diagrammatically as

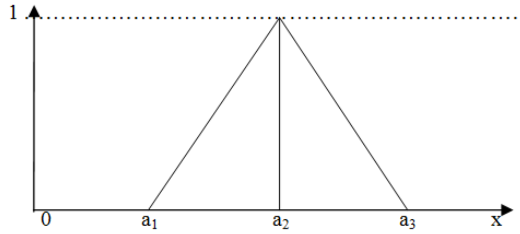

Figure 1: Triangular fuzzy number

2.4. Intuitionistic fuzzy set (IFS)

Let X be the universe of discourse, then an intuitionistic fuzzy set A in X is given by $A=\{x,\mu_A(x),\gamma_A(x)/x\in X\}$ where μ_A (x) : X $\rightarrow$[0 , 1] and γ_A (x) : X $\rightarrow$[0 , 1] determine the degree of membership and non membership of the element x $\in$X, respectively and for every $x\in X, 0\leq\mu_A(x)+\gamma_A(x)\leq 1$.

2.5. intuitionistic fuzzy number (IFN)

Let $A=\{(x,\mu_A(x),\gamma_A(x))/x\in X\}$ be an IFS, then we call the pair $(\mu_A(x),\gamma_A(x))$ an intuitionistic fuzzy number. We denote an intuitionistic fuzzy number by $(\langle a,b,c\rangle,\langle l,m,n\rangle)$, where $\langle a,b,c\rangle\in F(I),\langle l,m,n\rangle\in F(I), I=[0,1], 0\leq c+n\leq 1$.

2.6. Triangular intuitionistic fuzzy number (TIFN) and its arithmetic

A TIFN 'A' is given by $A=\{(\mu_A,\gamma_A)/x\in R\}$, where μ_A and γ_A are triangular fuzzy numbers with $\gamma_A\leq{\mu_A}^C$. So a triangular intuitionistic fuzzy number A is given by

A = $(\langle a,b,c\rangle,\langle e,f,g\rangle)$ with $\langle e,f,g\rangle\leq\langle a,b,c\rangle^c$ i.e., either e $\geq$ band f $\geq$ cor f $\leq$ aand g $\leq$ b are membership and non-membership fuzzy numbers of A.

An intuitionistic fuzzy number $(\langle a,b,c\rangle,\langle e,f,g\rangle)$ with e $\geq$ band f$\geq$ c is shown in the following figure:

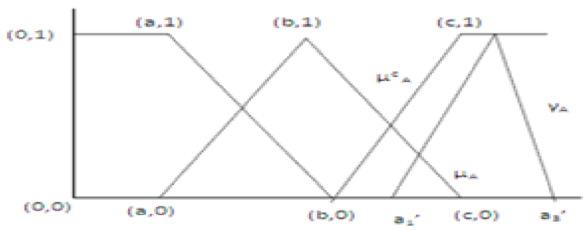

Figure 2: Triangular Intuitionstic fuzzy number

Addition

For two triangular Intuitionistic fuzzy numbers

A = $(\langle a_1,b_1,c_1\rangle:\mu_A,\langle e_1,f_1,g_1\rangle:\gamma_A)$ and B = $(\langle a_2,b_2,c_2\rangle:\mu_B,\langle e_2,f_2,g_2\rangle:\gamma_B)$

with $\mu_A \neq \mu_B$ and $\gamma_A \neq \gamma_B$, define A+B =
$$\left(\langle a_1 + a_2, b_1 + b_2, c_1 + c_2\rangle : Min(\mu_A, \mu_B), \langle e_1 + e_2, f_1 + f_2, g_1 + g_2\rangle : Max(\gamma_A, \gamma_B)\right)$$

Subtraction

A-B =
$$\left(\langle a_1 - a_2, b_1 - b_2, c_1 - c_2\rangle : Min(\mu_A, \mu_B), \langle e_1 - e_2, f_1 - f_2, g_1 - g_2\rangle : Max(\gamma_A, \gamma_B)\right)$$
where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

2.7. Forward pass calculations

Forward Pass Calculations are employed to calculate the Triangular Intuitionistic fuzzy earliest starting time TrIFESTj in the project network
$TrIFEST_j = Max_i\{TrIFEST_i \oplus TrIFt_{ij}\}$, i = number of preceding nodes.

Triangular Intuitionistic Fuzzy earliest finishing time TrIFEFTj in the project network $TrIFEFT = TrIFEST \oplus$ Fuzzy activity time

2.8. Backward pass calculations

Backward Pass Calculations are employed to calculate the Triangular Intuitionistic fuzzy latest finishing time $TrIFLFT_i$ in the project network
$TrIFLFT_i = Min_j\{TrIFLFT_j \Theta TrIF\tilde{t}_{ij}\}$, j = number of succeeding nodes Triangular Intuitionistic Latest starting time $TrIFLST = TrIFLFT \Theta$ Fuzzy activity time.

2.9. Triangular intuitionistic fuzzy total float (TrIFTF)

$TrIFTF_{ij}$= $TrIFLFT_j \Theta$ $TrIFEST_i \Theta$ $TrIFt_{ij}$

2.10. Ranking function

A ranking function $\mathcal{R}$: F(R) $\rightarrow$ R ,where F(R) (a set of all fuzzy numbers defined on set of
real numbers) , maps each fuzzy number into a real of F(R) .

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers in F(R) , then

(i) $\tilde{a} \geq_{\mathcal{R}} \tilde{b}$ if and only if $\mathcal{R}(\tilde{a}) \geq \mathcal{R}(\tilde{b})$

(ii) $\tilde{a} >_{\mathcal{R}} \tilde{b}$ if and only if $\mathcal{R}(\tilde{a}) > \mathcal{R}(\tilde{b})$

(iii) $\tilde{a} =_{\mathcal{R}} \tilde{b}$ if and only if $\mathcal{R}(\tilde{a}) = \mathcal{R}(\tilde{b})$

2.11. Method of magnitude

Let $\tilde{A} = (\langle l, m, n\rangle, \langle p, q, r\rangle)$ be a TrIFN , then

$$\text{Magnitude}(\tilde{A}) = \frac{5}{12}[(m+q)+n+r] + \frac{1}{2}[l+p]$$

3. Intuitionistic fuzzy critical path method

3.1. Notation

N : {1,2,3,…n} , the set of all the nodes in a project network.

$TrIFEST_j$: Triangular Intuitionistic fuzzy earliest starting time of j

$TrIFLFT_j$: Triangular Intuitionistic fuzzy latest finishing time of j
$TrIFTF_{ij}$: Triangular Intuitionistic fuzzy total fioat time of activity i -j
$TrIFt_{ij}$: Triangular Intuitionistic fuzzy activity time of nodes i and j
TrIECP : Triangular Intuitionistic fuzzy completion time of path
TrIFP(j) = The set of nodes connected to all predecessor activites of node j
TrIFS(j) = The set of all nodes connected to all successor activites of node j.

3.2. Procedure for finding the critical path using TrIFN

Step 1: Calculate TrIFEST = Max $\{TrIFEST_i \oplus TrIFt_{ij} / i \in TrIFP(j), j \neq 1, j \in N_{ij}\}$ and$TrIFEST_1 = TrIFLFT_1 = 0$

Step 2: Calculate $TrIFLFT_i$ = Min $\{TrIFLFT_j \Theta\, TrIFt_{ij} / j \in TrIFS(j), j \neq n, j \in N_{ij}\}$ and$TrIFLFT_n = TrIFEST_n$

Step 3: Calculate $TrIFTF_{ij} = (TrIFLFT_j \Theta TrIFEST_i \,\Theta\, TrIFt_{ij}\ ; i, j \in N_{ij}\}$

Step 4: Find all the possible paths and calculate TrIFCP in a project network

Step 5: Find the ranking value of $TrIFCP(P_i)$ i = 1,2,3,4,5 and compute the critical path.

4. Numerical examples

Figure 1 shows the network representation of a fuzzy project network.

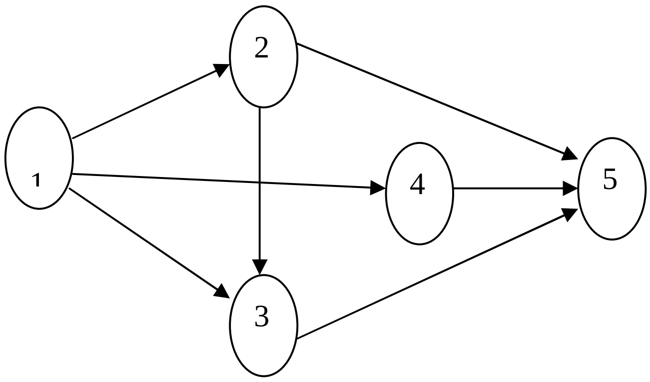

Figure 1:

Table 1:

Activity $TrIFP_{ij}$	Intuitionistic Fuzzy Activity Time
P_{12} : 1-2	$(\langle 2,3,4 \rangle, \langle 5,6,7 \rangle)$
P_{13} : 1-3	$(\langle 1,3,4 \rangle, \langle 4,5,6 \rangle)$
P_{14} : 1-4	$(\langle 1,3,5 \rangle, \langle ,4,6,7 \rangle)$

P_{23} : 2-3	$(\langle 3,4,6\rangle,\langle 5,7,8\rangle)$
P_{25} : 2-5	$(\langle 2,4,5\rangle,\langle 6,7,8\rangle)$
P_{35} : 3-5	$(\langle 3,6,8\rangle,\langle 7,9,11\rangle)$
P_{45} : 4-5	$(\langle 1,4,5\rangle,\langle 5,6,7\rangle)$

The computational procedures are as follows:

Step 1: To calculate the Triangular Intuitionistic Fuzzy Earliest Starting Time:

Set $TrIFEST_1$ = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$ and

Calculate $TrIFEST_j$, j = 2,3,4,5 ,by using definition (2.7)

$TrIFEST_2$= $(\langle 2,3,4\rangle,\langle 5,6,7\rangle)$

$TrIFEST_3$ = $(\langle 5,7,10\rangle,\langle 10,13,15\rangle)$

$TrIFEST_4$ = $(\langle 1,3,5\rangle,\langle 4,6,7\rangle)$

and $TrIFEST_5$ = $(\langle 8,13,18\rangle,\langle 17,22,26\rangle)$

Step 2: To calculate the Triangular IntuitionisticFuzzy Latest Finishing Time:
Setby using definition (2.8)

$TrIFLFT_5$ = $(\langle 8,13,18\rangle,\langle 17,22,26\rangle)$; $TrIFLFT_4$ = $(\langle 7,9,13\rangle,\langle 12,16,19\rangle)$

$TrIFLFT_3$ = $(\langle 5,7,10\rangle,\langle 10,13,15\rangle)$; $TrIFLFT_2$ = $(\langle 2,3,4\rangle,\langle 5,6,7\rangle)$

$TrIFLFT_1$ = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$

Step 3: To calculate the Triangular Intuitionistic Fuzzy total float:

Calculate $TrIFTF_{ij}$ with respect to each activity by using

$TrIFTF_{ij}$= $TrIFLF_j \Theta$ $TrIFEST_i \Theta$ $TrIFt_{ij}$

$TrIFTF_{12}$ = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$; $TrIFTF_{13}$ = $(\langle 4,4,6\rangle,\langle 6,9,9\rangle)$

$TrIFTF_{14}$ = $(\langle 6,6,8\rangle,\langle 8,10,12\rangle)$; $TrIFTF_{23}$ = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$

$TrIFTF_{25}$ = $(\langle 4,6,9\rangle,\langle 6,9,11\rangle)$; $TrIFTF_{35}$ = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$

$TrIFTF_{45}$ = $(\langle 6,6,8\rangle,\langle 8,10,12\rangle)$

Step 4 :Find all the possible and calculate TrIFCP is a project network.

P_i = {(1,2,5),(1,2,3,5),(1,3,5),(1,4,5)}

$TrIFCP(P_1)$ = $TrIFTF_{12} \oplus$ $TrIFTF_{25}$ = $(\langle 4,6,9\rangle,\langle 6,9,11\rangle)$

$TrIFCP(P_2)$ = $TrIFTF_{12} \oplus$ $TrIFTF_{23} \oplus$ $TrIFTF_{35}$ = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$

$\text{TrIFCP}(P_3) = \text{TrIFTF}_{13} \oplus \text{TrIFTF}_{35} = (\langle 4,4,6\rangle, \langle 6,9,9\rangle)$

$\text{TrIFCP}(P_4) = \text{TrIFTF}_{14} \oplus \text{TrIFTF}_{45} = (\langle 12,12,16\rangle, \langle 16,20,24\rangle)$

Step 5: To obtain the critical path using Ranking procedure (2.11)
$R(P_1) = 15.4$; $R(P_2) = 0$; $R(P_3) = 12.5$; $R(P_5) = 32.3$
Since $R(P_2) < R(P_3) < R(P_1) < R(P_5)$
Intuitionistic fuzzy critical path $P_2$is 1-2-3-5 .

5. Conclusion

In this paper, a simple approach is provided to find the triangular intuitionistic fuzzy total duration times and the critical path when the activity times are triangular intuitionistic fuzzy numbers. Ranking techniques can be implemented in the above form of project for getting better approximation. Intuitionistic fuzzy models are more effective in determining the critical path in a real project network

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