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On Fuzzy arω-Continuous Maps in Fuzzy Topological Spaces

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Abstract: In this paper, we introduced new class of continuous maps called fuzzy αrω-continuous maps in fuzzy topological spaces and studied some of their basic properties. Also we introduce fuzzy αrω-irresolute functions in fuzzy topological spaces and studied some of their properties.

Keywords: Fuzzy αrω-closed sets, Fuzzy αrω-open sets.

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1. Introduction

Chang [6] introduced fuzzy topological spaces. Several topologists extended the concepts in topological spaces to fuzzy topological spaces. The study of continuity and its weaker forms constitutes an established branch of investigation in general topological spaces. Recently some researchers extended to these studies to the broader framework of fuzzy topological spaces. Fuzzy semi continuity, fuzzy almost continuity has been introduced and studied by Azad [1], fuzzy strong semi continuity and fuzzy pre-continuity has been introduced and studied by Shahna [5]. We introduce Fαrω-continuous and Fαrω-irresolute mappings and study some of their fundamental properties.

Definition 1.1. A fuzzy set A of (X,τ) is called

1. Fuzzy semi open (in short, Fs-open) [1] if $A \le Cl(Int(A))$ and a fuzzy semi-closed (in short, Fs-closed) if $Int(Cl(A)) \le A$ or if and only if there exist exists a fuzzy open set V in X such that $V \le A \le cl(V)$.
2. Fuzzy pre-open (in short, Fp-open)[5] if $A \le Int(Cl(A))$ and a fuzzy pre-closed (in short, Fp-closed) if $Cl(Int(A)) \le A$
3. Fuzzy α-open (in short, Fα-open)[5] if $A \le Int(Cl(Int(A)))$ and a fuzzy α-closed (in short, Fα-closed) if $Cl(Int(Cl(A))) \le A$
4. Fuzzy semi-pre-open (in short, Fsp-open)[20] if $A \le Cl(Int(Cl(A)))$ and a fuzzy semi-pre-closed (in short, Fsp-closed) if $Int(Cl(Int(A))) \le A$
5. Fuzzy regular-open set of X if $int(cl(\mu)) = \mu$.
6. Fuzzy regular-closed set of X if $cl(int(\mu)) = \mu$.

Definition 1.2. A fuzzy set A of (X,τ) is called

1. fuzzy rw-closed[4] if $cl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ & μ is fuzzy regular semi-open in X.
2. fuzzy gp-closed [7] if $pcl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in X.
3. fuzzy gs-closed [11] if $scl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in X.
4. fuzzy αg-closed [3] if $\alpha cl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy α-open in X.
5. fuzzy ω-closed [12] if $cl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi-open in X.
6. fuzzy g-closed [2] if $cl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in X.
7. fuzzy gsp-closed [9] if $spcl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in X.
8. fuzzy sg-closed [10] if $scl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi-open in X.
9. fuzzy $\alpha r\omega$-closed [15] if $\alpha cl(\lambda)\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy rw-open in X.

Definition 1.3. [8] A fuzzy point $x_\lambda \in A$ is said to be **quasi-coincident** (in short q-coincident) with the fuzzy set A is denoted by $x_\lambda qA$ if and only if $\lambda + A(x) > 1$.

A fuzzy set A is quas-coincident with a fuzzy set B denoted by AqB if and only if there exists $x\in X$ such that $A(x) + B(x) > 1$.

If the fuzzy sets A and B are not quasi-coincident then we write $A\bar{q}B$.
A fuzzy set B is said to be a **q-neighbourhood** (in short, q-nbd) of a fuzzy set A if there is a fuzzy open sets U with $AqU \leq B$.

Definition 1.4. [14] A fuzzy set A in (X,τ) is called F$\alpha r\omega$-nhd of a fuzzy point x_λ if there exists a F$\alpha r\omega$-open set B such that $x_\lambda \in B \leq A$.

A fuzzy set A in (X,τ) is called fuzzy $\alpha r\omega$-q-nhd of a fuzzy point x_λ (resp. fuzzy set B), if there exists a F$\alpha r\omega$-open set U in (X,τ) such that $x_\lambda qU \leq A$ (resp. $BqU\leq A$).

2. F$\alpha r\omega$-continuous and F$\alpha r\omega$-irresolute functions

We introduce F$\alpha r\omega$-continuous and F$\alpha r\omega$-irresolute mappings and study some of their fundamental properties and also we introduce fuzzy $T_{\alpha r\omega}$-space as an application of fuzzy $\alpha r\omega$-closed set.

Definition 2.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is called fuzzy $\alpha r\omega$-continuous (in short, F$\alpha r\omega$-continuous) if $f^{-1}(V)$ is F$\alpha r\omega$-closed in (X,τ) for every fuzzy closed set V of (Y,σ).

Definition 2.2. A mapping $f :(X,\tau)\rightarrow(Y,\sigma)$ is called fuzzy $\alpha r\omega$-irresolute(in short, F$\alpha r\omega$-irresolute) if $f^{-1}(V)$ is F$\alpha r\omega$-closed in (X,τ) for every F$\alpha r\omega$-closed set V of (Y,σ).

Definition 2.3. A fuzzy topological space (X,τ) is called a fuzzy $T_{\alpha r\omega}$ space if every F$\alpha r\omega$-closet set in it is fuzzy closed.

Theorem 2.4. 1) If $f :(X,\tau)\rightarrow(Y,\sigma)$ be fuzzy continuous then f is F$\alpha r\omega$-continuous.
2) If $f:(X,\tau)\rightarrow(Y,\sigma)$ be Fα-continuous then f is F$\alpha r\omega$-continuous.
Proof: 1) Let V be a fuzzy closed set in (Y,σ). Since f is fuzzy-continuous, $f^{-1}(V)$ is F-closed in (X,τ). Every F-closed is F$\alpha r\omega$-closed set, $f^{-1}(V)$ is F$\alpha r\omega$-closed in (X,τ). Hence f is F$\alpha r\omega$-continuous.

2) Let V be a fuzzy closed set in (Y,σ). Since f is Fα-continuous, $f^{-1}(V)$ is Fα-closed in (X,τ). Every Fα-closed is Fαrω-closed set, $f^{-1}(V)$ is Fαrω-closed in (X,τ). Hence f is Fαrω-continuous.

Example 2.5. Let P=Q={a,b,c,d} and the fuzzy sets A, B,C,D,E**:**P→[0, 1] be defined as

$$A(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{cases} \quad B(x) = \begin{cases} 1 \text{ if } x = b \\ 0 \text{ otherwise} \end{cases} \quad C(x) = \begin{cases} 1 \text{ if } x = a, b \\ 0 \text{ otherwise} \end{cases}$$

$$D(x) = \begin{cases} 1 \text{ if } x = a, b, c \\ 0 \quad \text{otherwise} \end{cases} \quad E(y) = \begin{cases} 1 \text{ if } y = c, d \\ 0 \text{ otherwise} \end{cases} \quad G(x) = \begin{cases} 1 \text{ if } x = a, d \\ 0 \text{ otherwise} \end{cases}$$

Consider T_1 and T_2 ={1,0,A,B,C,D} then (P,T_1) and (Q,T_2) are fuzzy topological space. Let map f: P→Q defined by f(a)=c, f(b)= a, f(c)=b, f(d)=d, then f is Fαrω-continuous but not fuzzy-continuous and not Fα-continuous, as fuzzy closed set E in Q, then $f^{-1}(E)$=G in P which is not Fα-closed , not F-closed set in P.

Theorem 2.6. If f : (X,τ)→(Y,σ) be Fαrω-continuous. Then f is Fαg-continuous.
Proof: Let V be a fuzzy closed set in (Y,σ). Since f is Fαrω-continuous, $f^{-1}(V)$ is Fαrω-closed in (X,τ). Every Fαrω-closed is Fαg-closed set, $f^{-1}(V)$ is Fαg-closed in (X,τ). Hence f is Fαg-continuous.

Example 2.7. Let X=Y={a,b,c} and the fuzzy sets A, B, G**:**X→ [0, 1]

$$A(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{cases} \quad B(x) = \begin{cases} 1 \text{ if } x = b, c \\ 0 \quad \text{otherwise} \end{cases} \quad G(x) = \begin{cases} 1 \text{ if } x = a, c \\ 0 \text{ otherwise} \end{cases}$$

Consider T_1={1,0,A,B}, T_2={0,1,A} Then (X,T_1) and (Y,T_2) are fuzzy topological space. Let map f: X→Y defined by f(a)=b, f(b)=a, f(c)=c then f is Fαg-continuous but not Fαrω-continuous as F-closed set B in Y, then $f^{-1}(B)$=G in X which is not Fαrω-closed set in X

Theorem 2.8. Let f:(X, τ)→(Y,σ) be Fαrω-continuous if and only if inverse image of each fuzzy open set of (Y,σ) is Fαrω-open in (X,τ).
Proof: Let f be Fαrω-continuous. If V is any fuzzy open set in (Y,σ) then $f^{-1}(1-V)$ $=1-f^{-1}(V)$ is Fαrω-closed. Hence $f^{-1}(V)$ is Fαrω-open in (X, τ).
Conversely Let V be a fuzzy closed set in (Y,σ). By hypothesis, $f^{-1}(1 - V)$ is Fαrω-open in (X,τ). This gives $f^{-1}(V)$ is Fαrω-closed.
Hence f is Fαrω-continuous.

Theorem 2.9. If f:(X,τ)→(Y,σ) is Fαrω-continuous then for each fuzzy point x_λ of X and A□σ such that $f(x_\lambda) \in A$, there exists a Fαrω-open set B of X such that $x_\lambda \in B$ and $f(B) \leq A$.
Proof: Let x_λ be a fuzzy point of X and $A \in \sigma$ such that $f(x_\lambda) \in A$. Take $B=f^{-1}(V)$. Since 1− A is fuzzy closed in (Y, σ) and f is Fαrω-continuous, we have $f^{-1}(1-A)= 1-f^{-1}(A)$ is Fαrω-closed in (X,τ). This gives $B=f^{-1}(A)$ is Fαrω-open in (X,τ) and $x_\lambda \in B$ and $f(B)= f(f^{-1}(A)) \leq A$.

Theorem 2.10. If f:(X,τ)→(Y,σ) is Fαrω-continuous then for each fuzzy point x_λ of X and $A \in \sigma$ such that $f(x_\lambda)qA$, there exists a Fαrω-open set B of X such that $x_\lambda qB$ and $f(B) \leq A$.

Proof: Let x_λ be a fuzzy point of X and $A\in\sigma$ such that $f(x_\lambda)qA$. Take $B=f^{-1}(A)$. By above thereon 2.9, B is Fαrω-open in (X,τ) and $x_\lambda qB$ and $f(B)= f(f^{-1}(A))\leq A$.

Theorem 2.11. If $f:(X,\tau)\to(Y,\sigma)$ is Fαrω-irresolute then f is Fαrω-continuous.
Proof: As every fuzzy closed set is Fαrω-closed and f is Fαrω-irresolute map.

Remark 2.12. The following example shows that reverse implication is not true.

Example 2.13. Let X={a,b,c,d},Y={a,b,c} and
fuzzy sets A,B,C,D:X→[0,1] E,F:Y→[0,1] be defined as

$$A(x) = \begin{matrix} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{matrix} \quad B(x) = \begin{matrix} 1 \text{ if } x = b \\ 0 \text{ otherwise} \end{matrix} \quad C(x) = \begin{matrix} 1 \text{ if } x = a, b \\ 0 \text{ otherwise} \end{matrix}$$

$$D(x) = \begin{matrix} 1 \text{ if } x = a, b, c \\ 0 \quad \text{otherwise} \end{matrix} \quad E(y) = \begin{matrix} 1 \text{ if } y = a \\ 0 \text{ otherwise} \end{matrix} \quad G(y) = \begin{matrix} 1 \text{ if } y = b \\ 0 \text{ otherwise} \end{matrix}$$

τ = {0,1,A,B,C,D} σ ={0,1,E} Then (X,τ) and (Y,σ) are fuzzy topological space. Let map f:X→Y defined by f(a)=b, f(b)=a, f(c)=a, f(d)=c then f is Fαrω-continuous but f is not Fαrω-irresolute, as Fαrω-closed set G in Y, then $f^{-1}(G)=A$ in X , which is not Fαrω-closed set in X.

Theorem 2.14. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be any two functions. Then

i) $g\circ f:(X,\tau)\to(Z,\eta)$ is Fαrω-continuous if g is fuzzy-continuous and f is Fαrω-irresolute.
ii) $g\circ f:(X,\tau)\to(Z,\eta)$ is Fαrω-irresolute if g is Fαrω-irresolute and f is Fαrω-irresolute.
iii) $g\circ f:(X,\tau)\to(Z,\eta)$ is Fαrω-continuous if g is Fαrω-continuous and f is Fαrω-irresolute.

Proof:

(i) Let U be a Fopen set in (Z,η). Since g is Fuzzy-continuous, $g^{-1}(U)$ is Fuzzy-open set in (Y,σ). Since every Fuzzy-open is Fαrω-open then $g^{-1}(U)$ is Fαrω-open in Y, siince f is Fαrω-irresolute $f^{-1}(g^{-1}(U))$ is an Fαrω-open set in (X,τ). Thus $(g\circ f)^{-1}(U)= f^{-1}(g^{-1}(U))$ is an Fαrω-open set in (X, τ) and hence gof is Fαrω-continuous.
(ii) Let U be a Fαrω–open set in (Z,η). Since g is Fαrω-irresolute, $g^{-1}(U)$ is Fαrω-open set in (Y, σ). Since f is Fαrω–irresolute, $f^{-1}(g^{-1}(U))$ is an Fαrω-open set in (X,τ). Thus $(g\circ f)^{-1}(U)=f^{-1}(g^{-1}(U))$ is an Fαrω-open set in (X,τ) and hence gof is Fαrω-irresolute.
(iii) Let U be a fuzzy-open set in (Z,η). Since g is fuzzy-continuous, $g^{-1}(U)$ is fuzzy-open set in (Y,σ). As every Fopen set is Fαrω–open, $g^{-1}(U)$ is Fαrω–open set in (Y,σ). Since f is Fαrω–irresolute $f^{-1}(g^{-1}(U))$ is an Fαrω–open set in (X,τ). Thus$(g\circ f)^{-1}(U)= f^{-1}(g^{-1}(U))$ is an Fαrω-open set in (X,τ) and hence $g\circ f$ is Fαrω-continuous.

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