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A Computational Method for Rice Production Forecasting Based on High-Order Fuzzy Time Series

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Abstract. This paper presents a new method of forecasting based on high-order fuzzy logical relationships in the fuzzy time series. The objective of the present study is to develop a computational method for various high orders forecasting to remove the computational drawback of the existing high-order fuzzy time series forecasting methods. The developed method has been presented in form of computational algorithm. This algorithm has been implemented in forecasting of the rice production to examine suitability of these proposed high-order forecasting models on the basis of its average forecasting errors. The forecasting accuracy of the proposed computational method is better than that of existing methods and the forecasted production is much closer to the actual production.

Keywords: Fuzzy time series; time invariant; time variant; linguistic variables; fuzzy logical relationships

AMS Mathematics Subject Classification (2010): 03E72, 37M10

1. Introduction

Time series forecasting is an important and interesting problem in variety of applications and has been widely studied in the area of statistics with a bottleneck of dealing only with numerical data. The concept of fuzzy time series, capable of dealing with vague and imprecise data presented in terms of linguistic variables was developed by Song and Chissom [25] by using the theory of sets and linguistic variable given by Zadeh [38,39]. Song and Chissom [26,27] further extended his theory of fuzzy time series to be capable of dealing with numerical data by introducing the concept of fuzzification and defuzzification and applied it to forecast the student enrollments of university of Alabama. Chen [2] resolved the problem of large computational requirements of Song and Chissom method of computing fuzzy relations using max-min composition by replacing it with the simplified arithmetic operations and applied the method on the student enrollments of University of Alabama.

During the last few decades, several methods for time series forecasting have been presented by Tsai and Wu [29], Chen and Hwang [4], Huarng [9], Chen and Hsu [5], Lee and Chou [15] , Li and Chen [18], Sullivan and Woodall [28] made a comparative study of fuzzy time series forecasting and Markov modeling. Kim and Lee [14] proposed a fuzzy time series prediction method based on consecutive values. Tsaur et al. [32] used the concept of entropy to measure the degree of fuzziness to obtain a time invariant relation matrix for fuzzy time series forecasting. Yu [36,37] presented refined fuzzy time series model and a weighted fuzzy time series model to improve the TAIEX forecasting. Huang and Yu [10] applied a ratio based length of intervals to improve the enrollments forecast. Cheng et al. [6] studied the fuzzy time series forecasting of enrollments using: minimize entropy principle approach (MEPA) and Trapezoid fuzzification approach (TFA).Regarding the improvement in forecasting the aspect explored in the majority of the above mentioned methods are the refinement in the length of intervals.

Another direction for the improvement of fuzzy time series forecasting emerged as the time variant models by application of the high-order methods in the fuzzy time series forecasting. The major problem associated with Song and Chissom [27] is the need of large computational requirements. However, Hwang et al. [11] tried to minimize the computational complexity by using heuristic rules. To forecast enrollments of year t, the number of past years of the enrollments data used was called the window basis and obtained the enrollments forecast for various window basis and succeeded in improvement in the forecasting errors. Chen [3] considered the fuzzy logical relations of various high- orders and presented some rules for forecasting based on high-order fuzzy time series. In his study he found certain ambiguity in forecasting and the method depends strongly on the deviation of highest order in the time series. Tsai et al. [31] studied the effect of membership function in high order fuzzy time series forecasting and Tsai and Wu [30] applied the high-order-fuzzy time series model in a local region forecasting. Own and Yu [19] extended the Chen's model as a heuristic high-order fuzzy time series model to overcome the deficiency of the Chen's model. Lee et al. [16] studied the temperature and TAIFEX forecasting based on two-factor high-order fuzzy time series. Singh [22] presented a simple method of forecasting based on fuzzy time series of order three and tested its suitability in students enrollments forecasting of university of Alabama in comparison with existing methods and implemented it in wheat production forecasting. Singh [23,24] have further presented a robust method and time variant method for fuzzy time series forecasting.

Another direction in fuzzy time series forecasting was given by Cheng [1] by presenting a fuzzy time series based adaptive expectation model for TAIEX forecasting followed by an average based fuzzy time series model for forecasting shanghai compound index was given by Xihao and Yimin [35]. Further, Lee et al. [17] presented a weighted fuzzy time series model for enrollment forecasting. Later Wong et al. [34] gave an adaptive time variant model for fuzzy time series forecasting. Qiu et al. [20] proposed a generalized method for forecasting based on fuzzy time series. Ismail and Efendi [12] proposed a modified weighted fuzzy time series model for enrollment forecasting. Gangwar and Kumar [8] proposed a partition based computational method for high order fuzzy time series forecasting followed by Joshi and Kumar [13] who used a computational method for fuzzy time series forecasting based on difference parameter. Singh and Borah [21] considered an efficient forecasting model based on fuzzy time series and a fuzzy time series approach based on weights determined by the number of recurrences of fuzzy relations was proposed by Uslu et al. [33]. Recently Cheng et al. [7]

presented a fuzzy time series forecasting model based on fuzzy logical relations and similarity measures.

The objective of the present work is to develop a computational method of forecasting based on high-order (order 4 and higher) fuzzy logical relations with a motivation to examine the nature and general suitability of this high-order forecasting based on fuzzy time series in agricultural production system to support the crop simulation models for tactical and forecasting applications barring its limitations of availability of weather, soil and crop management information. In tactical applications, the crop models are actually run prior to growing season to help the farmers, producers or decision makers. In forecasting applications of the crop models, the main interest is in the final expected yield and the gain for planning a crop in the season. Apart for crop producers it is also important for local area companies to have optimal plan for their required input of raw material. The application study of the developed model has been made on the agricultural production system which involves the uncertainty in the crop yield even though all the standard cropping practices are adopted and we have considered the time series data of rice(paddy) crop production of Pantnagar farm, G.B.Pant University of Agriculture and Technology, Pantnagar (INDIA). Here, the rice production has been recorded in terms of quintal per hectare. The study comprise of model development, its testing on rice production forecast to examine its suitability in forecasting over the other available models and then its implementation in agricultural crop (rice) production forecasting.

2. Basics of fuzzy time series

In view of making our exposition self contained, some basic definitions of fuzzy time series models presented in literature are summarized and is reproduced as [25-27].

Definition 2.1. A fuzzy set is a class of objects with a continuum of grade of membership. Let U be the Universe of discourse with U={ u_1, u_2, u_3, …u_n },where u_i are possible linguistic values of U, then a fuzzy set of linguistic variables A_i of U is defined by

$$A_i = \mu_{A_i}(u_1)/u_1 + \mu_{A_i}(u_2)/u_2 + \mu_{A_i}(u_3)/u_3 + ... + \mu_{A_i}(u_n)/u_n \qquad (1)$$

here, μ_{A_i} is the membership function of the fuzzy set A_i , such that μ_{A_i} :U→[0,1].

If u_j is the member of A_i , then μ_{A_i} (u_j) is the degree of belonging of u_j to A_i.

Definition 2.2. Let Y(t) (t = …,0,1,2,3,…), is a subset of R , be the universe of discourse on which fuzzy sets f_i(t) , (i= 1, 2, 3, …) are defined and F(t) is the collection of f_i, then F(t) is defined as fuzzy time series on Y(t).

Definition 2.3. Suppose F(t) is caused only by F(t-1) and is denoted by F(t-1) →F(t); then there is a fuzzy relationship between F(t) and F(t-1) and can be expressed as the fuzzy relational equation:

$$F(t) = F(t-1) \circ R(t, t-1) \qquad (2)$$

here, '∘' is Max –Min composition operator. The relation R is called first order model of F(t).

Further if Fuzzy relation R(t, t-1) of F(t) is independent of time t ,that is to say for different times t_1 and t_2, $R(t_1, t_1 - 1) = R(t_2, t_2 - 1)$, then F(t) is called a time invariant fuzzy time series.

Definition 2.4. If F(t) is caused by more fuzzy sets, F(t-n), F(t-n+1) , …, F(t-1), then the fuzzy relationship is represented by

A_{i1} , A_{i2} , ….,A_{in} $\rightarrow A_j$

here, F(t-n) = A_{i1} , F(t-n+1) = A_{i2} , … , F(t-1) = A_{in} . This relationship is called n^{th} order fuzzy time series model.

Definition 2.5. Suppose F(t) is caused by a F(t-1), F(t-2),…, and F(t-m) (m>0) simultaneously and the relations are time variant. The F(t) is said to be time variant fuzzy time series and the relation can be expressed as the fuzzy relational equation:

$$F(t) = F(t-1) \circ R^w(t, t-1) \qquad (3)$$

here, w > 1 is a time (number of years) parameter by which the forecast F(t) is being affected. Various complicated computational methods are available to for the computations of the Relation R^w (t, t-1).

Proposed model

The proposed model is of order m, as F(t) is caused by F(t-1), F(t-2), …, F(t-m) and F(t) is computed as

$$F(t) = F(t-1) * R(t-1, t-2, \ldots t-m)$$

here, the fuzzy relation R is considered a numeric value rather than a fuzzy relational matrix and is being computed as difference between differences, defined as d^m, which works like backward difference operator but providing the absolute value of the differences, in the consecutives values of year n-1 with n-2 and of values of year n-2 with n-3 and so on. The computational procedure for forecasting the value of year n are presenting in form of computational algorithms in the next section. The model development and the computational algorithm for various high-order method of forecasting are given in the next section.

3. Computational algorithm of proposed high-order forecasting

In this section, we present the stepwise procedure of the proposed method for fuzzy time series forecasting based on historical time series data.

1. Define the Universe of discourse, U based on the range of available historical time series data, by rule

   $U = [D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are two proper positive numbers.

2. Partition the Universe of discourse into equal length of intervals: u_1 , u_2, …,u_m. The number of intervals will be in accordance with the number of linguistic variables (fuzzy sets) A_1 , A_2, ….,A_m to be considered.

3. Construct the fuzzy sets A_i in accordance with the intervals in Step2 and apply the triangular membership rule to each intervals in each fuzzy set so constructed.

4. Fuzzify the historical data and establish the fuzzy logical relationships by the rule:

   If A_i is the fuzzy production of year n and A_j is the fuzzify production of year n+1 , then the fuzzy logical relation is denoted as $A_i \rightarrow A_j$. Here A_i is called current state and A_j is next state.

5. Establishing the fuzzy logical relations of various orders as given below

i) If for year n-3, n-2, n-1 and n the fuzzified production are A_{i2} , A_{i1}, A_i and A_j respectively , then the third order fuzzy logical relation is represented as

$$A_{i2}, A_{i1}, A_i \rightarrow A_j$$

ii) Similarly if for year n-4, n-3, n-2, n-1 and n the fuzzified production are A_{i3}, A_{i2} , A_{i1}, A_i and A_j respectively , then the fourth order fuzzy logical relation is represented as

$$A_{i3}, A_{i2}, A_{i1}, A_i \rightarrow A_j$$

In a similar way we can find the various fifth, sixth, seventh, eighth and other higher order fuzzy logical relations.

6. Computation of fuzzy difference parameter d^m ; m= 2, 3, 4, … of different orders

i) Considering a difference operator $d^2 = | \nabla |$ and is being defined as

$d^2 E_i = | E_i - E_{i-1} |$
$d^3 E_i = | d^2 E_i - d^2 E_{i-1} |$
$d^4 E_i = | d^3 E_i - d^3 E_{i-1} |$
$d^5 E_i = | d^4 E_i - d^4 E_{i-1} |$
$d^6 E_i = | d^5 E_i - d^5 E_{i-1} |$
and so on.

ii) Having fuzzy logical relation $A_{i3}, A_{i2}, A_{i1}, A_i \rightarrow A_j$ for year n and if E_{n-4}, E_{n-3} , E_{n-2} , E_{n-1} are the production of the year n-4, n-3 , n-2, n-1 then the fuzzy difference parameter(d^4E_i) of order 4 is to be used for forecasting and the model is said to be model of order 4 and is computed as given in step 6(i)

7. Computations for forecasting

Some notations used are defined as

[*A_j] is corresponding interval u_j for which membership in A_j is supremum (i.e. 1).

L[*A_j] is the lower bound of interval u_j

U[*A_j] is the upper bound of interval u_j

l[*A_j] is the length of the interval u_j whose membership in A_j is supremum (i.e. 1)

M[*A_j] is the mid value of the interval u_j having Supremum value in A_j

For a Fuzzy logical relation $A_i \rightarrow A_j$:

A_i is the fuzzified production of year n-1
A_j is the fuzzified of production year n
E_j is the actual production of year n
E_i is the actual production of year n-1
E_{i-1} is the actual production of year n-2
E_{i-2} is the actual production of year n-3
E_{i-3} is the actual production of year n-4
F_j is the crisp forecasted production of the year n

Here, we take the model of order four utilizing the historical data of years n-4, n-3, n-2, n-1 for framing rules to implement on fuzzy logical relation, $A_i \rightarrow A_j$, where A_i , the current state, is the fuzzified production of year n-1 and A_j , the next state, is fuzzified production of year n.

Computational Algorithm: (Forecasting production F_j for year n (i.e. 1975) and onwards by higher order method of order 4

for k= 5 to ….K (end of time series data)
Obtained fuzzy logical Relation for year k-1 to k
$A_i \rightarrow A_j$
R= 0 and S= 0
Compute
$d^2 E_i = | E_i - E_{i-1} |$
$d^3E_i = | d^2E_i - d^2E_{i-1} |$
$d^4E_i = | d^3E_i - d^3E_{i-1} |$
$X_i = E_i + d^4E_i /2$
$XX_i = E_i - d^4E_i /2$
$Y_i = E_i + d^4E_i$
$YY_i = E_i - d^4E_i$
$Pi = E_i + d^4E_i /4$
$PPi = E_i - d^4E_i /4$
$Q_i = E_i +2* d^4E_i$
$QQi = E_i - 2* d^4E_i$
If $X_i \geq L[*A_j]$ and $X_i \leq U[*A_j]$
Then $R = R + X_i$ and $S = S + 1$
If $X X_i \geq L[*A_j]$ and $XX_i \leq U[*A_j]$
Then $R = R + XX_i$ and $S = S + 1$
If $Y_i \geq L[*A_j]$ and $Y_i \leq U[*A_j]$
Then $R = R + Y_i$ and $S = S +1$
If $YY_i \geq L[*A_j]$ and $YY_i \leq U[*A_j]$
Then $R = R + YY_i$ and $S = S +1$
If $P_i \geq L[*A_j]$ and $P_i \leq U[*A_j]$
Then $R = R + P_i$ and $S = S + 1$
If $PP_i \geq L[*A_j]$ and $PP_i \leq U[*A_j]$
Then $R = R + PP_i$ and $S = S + 1$
If $Q_i \geq L[*A_j]$ and $Q_i \leq U[*A_j]$
Then $R = R + Q_i$ and $S = S +1$
If $QQ_i \geq L[*A_j]$ and $QQ_i \leq U[*A_j]$
Then $R = R + QQ_i$ and $S = S +1$
$F_j = (R+ M(*A_j))/(S +1)$
Next k

Similarly, using the d^5E_i , a fifth order model, we can forecast the production for sixth year and onwards from the above computational algorithm and so using d^6E_i , d^7E_i , d^8E_i , d^9E_i for sixth , seventh , eighth , ninth order models in the above algorithms and so on to get the forecasted enrollments by these higher order models.

4. Forecasting rice production with proposed model

In view of examining the general suitability of the proposed model, it is being implemented for forecasting the rice production. The historical time series data of rice production are of the huge farm of G.B. Pant University, Pantnagar, INDIA. The historical time series data of rice production is in terms of productivity in kg per hectare. The method has been implemented and step wise computations are as

Step 1. Universe of discourse U = [3200, 4600].

Step 2. The universe of discourse is partitioned into seven intervals of linguistic values:

u_1=[3200, 34 00], u_2=[3400, 3600], u_3=[3600, 3800],
u_4=[3800, 4000], u_5=[4000, 4200], u_6=[4200, 4400]
u_7=[4400, 4600].

Step 3. Define seven fuzzy sets A_1 ,A_2 , …,A_7 having some linguistic values on the universe of discourse U . The linguistic values to these fuzzy variables are as follows:

A_1 : poor production, A_2 : below average production
A_3 : average production A_4 : good production
A_5 : very good production A_6 : excellent production
A_7 : bumper production

The membership grades to these fuzzy sets of linguistic variables are defined as :

$A_1= 1/ u_1 + 0.5/ u_2 + 0/ u_3 + 0/ u_4 + 0/ u_5 + 0/ u_6 + 0/ u_7$
$A_2= 0.5/ u_1 + 1/ u_2 + 0.5/ u_3 + 0/ u_4 + 0/ u_5 + 0/ u_6 + 0/ u_7$
$A_3= 0/ u_1 + 0.5/ u_2 + 1/ u_3 + 0.5/ u_4 + 0/ u_5 + 0/ u_6 + 0/ u_7$
$A_4= 0/ u_1 + 0/ u_2 + 0.5/ u_3 + 1/ u_4 + 0.5/ u_5 + 0/ u_6 + 0/ u_7$
$A_5= 0/ u_1 + 0/ u_2 + 0/ u_3 + 0.5/ u_4 + 1/ u_5 + 0.5/ u_6 + 0/ u_7$
$A_6= 0/ u_1 + 0/ u_2 + 0/ u_3 + 0/ u_4 + 0.5/ u_5 + 1/ u_6 + 0.5/ u_7$
$A_7= 0/ u_1 + 0/ u_2 + 0/ u_3 + 0/ u_4 + 0/ u_5 + 0.5/ u_6 + 1/ u_7$

Step 4. The historical time series data are fuzzified and are placed in table 1.

Table 1: The actual and fuzzified historical data of Rice production

Year	Production Kg/hect.	Fuzzified production
1981	3552	A_2
1982	4177	A_5
1983	3372	A_1
1984	3455	A_2
1985	3702	A_3
1986	3670	A_3
1987	3865	A_4
1988	3592	A_2
1989	3222	A_1

1990	3750	A_3
1991	3851	A_4
1992	3231	A_1
1993	4170	A_5
1994	4554	A_7
1995	3872	A_4
1996	4439	A_7
1997	4266	A_6
1998	3219	A_1
1999	4305	A_6
2000	3928	A_4
2001	3978	A_4
2002	3870	A_4
2003	3727	A_3

With these fuzzified productions, the fuzzy logical relations of various orders (order 3, order 4, order 5 and so on) are constructed .

Table 2: Fourth order fuzzy logical relationships for the rice production are obtained as

$A_2,\ A_5,\ A_1,\ A_2 \rightarrow A_3$
$A_5,\ A_1,\ A_2, A_3 \rightarrow A_3$
$A_1,\ A_2,\ A_3, A_3 \rightarrow A_4$
$A_2,\ A_3,\ A_3, A_4 \rightarrow A_2$
$A_3,\ A_3,\ A_4, A_2 \rightarrow A_1$
$A_3,\ A_4,\ A_2, A_1 \rightarrow A_3$
$A_4,\ A_2,\ A_1, A_3 \rightarrow A_4$
$A_2,\ A_1,\ A_3, A_4 \rightarrow A_1$
$A_1,\ A_3,\ A_4, A_1 \rightarrow A_5$
$A_3,\ A_4,\ A_1, A_5 \rightarrow A_7$
$A_4,\ A_1,\ A_5, A_7 \rightarrow A_4$
$A_1,\ A_5,\ A_7, A_4 \rightarrow A_7$
$A_5,\ A_7,\ A_4, A_7 \rightarrow A_6$
$A_7,\ A_4,\ A_7, A_6 \rightarrow A_1$
$A_4,\ A_7,\ A_6, A_1 \rightarrow A_6$
$A_7,\ A_6,\ A_1, A_6 \rightarrow A_4$
$A_6,\ A_1,\ A_6, A_4 \rightarrow A_4$
$A_1,\ A_6,\ A_4, A_4 \rightarrow A_4$
$A_6,\ A_4,\ A_4, A_4 \rightarrow A_3$

Step 5. The forecasted values have been obtained by using the algorithms in section 3. The forecasted production of Rice obtained by these methods is placed in the Table 3.

Table 3: Actual rice production vs forecasted outputs by proposed high–order models

year	Actual production	Model of Order 4	Model of Order 5	Model of Order 6	Model of Order 7	Model of Order 8	Model of Order 9
1981	3552						
1982	4177						
1983	3372						
1984	3455						
1985	3702	3700					
1986	3670	3700	3701.78				
1987	3865	3900	3911.75	3907.75			
1988	3592	3500	3500	3500	3500		
1989	3222	3300	3300	3300	3327.5	3331.25	
1990	3750	3700	3700	3700	3700	3697.5	3700
1991	3851	3861	3867	3900	3900	3900	3916.25
1992	3231	3306.5	3300	3300	3300	3300	3300
1993	4170	4100	4100	4100	4100	4100	4100
1994	4554	4535	4500	4500	4500	4500	4500
1995	3872	3900	3900	3900	3900	3900	3900
1996	4439	4500	4500	4500	4500	4500	4500
1997	4266	4324.19	4318.67	4339.67	4331.5	4300	4334.5
1998	3219	3300	3300	3300	3300	3300	3300
1999	4305	4300	4300	4300	4300	4300	4300
2000	3928	3893.75	3925	3948.5	3900	3900	3900
2001	3978	3900	3900.38	3897.25	3924.89	3924.89	3924.89
2002	3870	3891.25	3880	3904.69	3925.92	3953.57	3963.75
2003	3727	3678.83	3700.33	3725.83	3733	3700	3700

The suitability of the proposed model in forecasting the rice production has been studied on the basis of mean square error (MSE) and average error of the forecast. The MSE is defined as

$$\text{Mean Square Error} = \frac{\sum_{i=1}^{n}(actual\,value_i - forecasted\,value_i)^2}{n}$$

Table 4: Mean square error and average error of rice production forecast of proposed models

Order of forecast	order 4	Order 5	Order 6	Order 7	Order 8	Order 9
Average Error	1.239616	1.265129	1.389997	1.46002	1.434785	1.397004
MSE	2881.2	2988.2	3442.2	3701.3	3548.9	3445.8

Further, the trends in forecast of the proposed model of these high-orders can examined with more clarity by the figure given below. One of the interesting features in the forecasting of rice production by the proposed high-order model can be visualized that the accuracy in forecasted values is of varying degree.

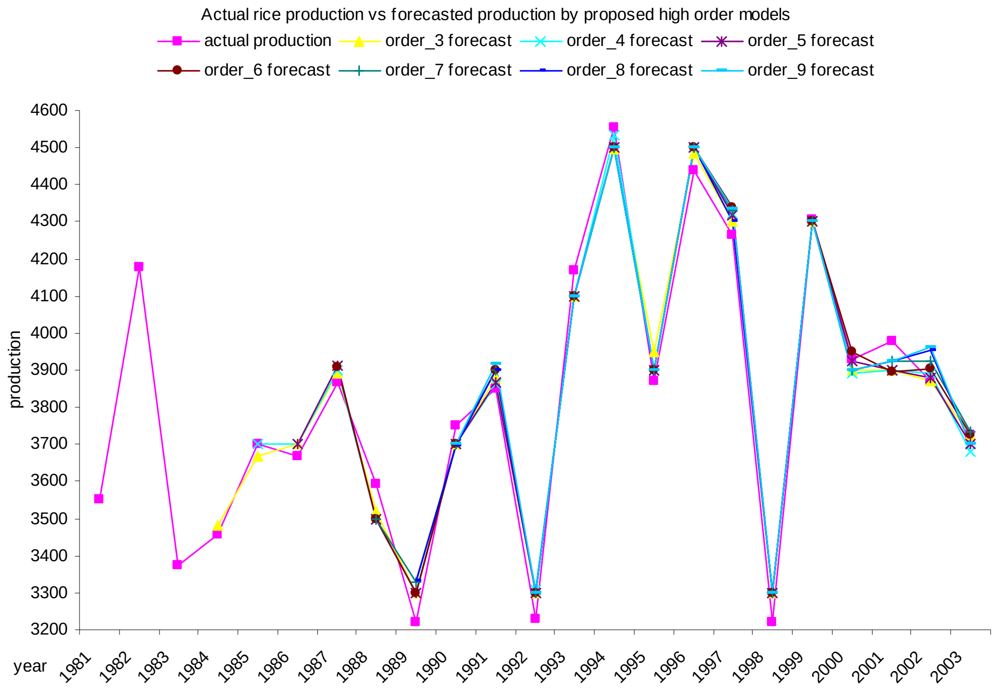

Figure 1: Actual rice production vs forecasted rice production by higher order model of proposed method.

5. Conclusions

The developed method is a computational method for various high-order forecasting based on the fuzzy time series and provides better results than the existing methods. The study reveals some interesting features of the high-order fuzzy time series forecasting that their suitability varies according to the fuzziness in the time series data. Forecasted values for some years in various high order models are invariant and may be considered a suitable forecast for the respective years. Thus an efficient and accurate forecasting also needs the computation of forecasting by various high orders, the proposed method being a computational method can be easily employed to get the forecasting of various high-orders more efficiently. Further, the implementation of fuzzy time series in crop production forecast is to support the development of decision support system in Agricultural production system, one of the real life problems falling in the category having uncertainty in known and unknown parameters. This study may be useful in the tactical and forecasting applications in agricultural decision support systems.

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REFERENCES

1. C.H.Cheng, Fuzzy time series based on adaptive expectation model for TAIEX forecasting, *Expert Systems with Applications,* 34 (2008) 1126-1132.
2. S.M.Chen, Forecasting enrollments based on fuzzy time series. *Fuzzy Sets and Systems,* 81(1996) 311-319.
3. S.M.Chen, Forecasting enrollments based on high-order fuzzy time series, *Cybernetics and systems: An International Journal,* 33 (2002) 1-16.
4. S.M.Chen and J.R.Hwang, Temperature prediction using fuzzy time series, *IEEE Transactions on Systems, man, and Cybernetics-Part B: Cybernetics* 30 (2000) 263-275.
5. S.M.Chen and C.C.Hsu, A new method to forecast enrollments using fuzzy time series, *International Journal of Appl. Sciences and Engg.,* 2 (2004) 234-244.
6. C.H.Cheng, J.R.Chang and C.A.Yeh, Entropy- based and Trapezoid fuzzification-based fuzzy time series approaches for forecasting IT project cost, *Technological Forecasting and Social Change,* 73 (2006) 524-542.
7. S.H.Cheng, S.M.Chen and W.S. Jian, Fuzzy time series forecasting based on fuzzy logical relations and similarity measures, *Information Sciences,* 327 (2016) 272-287.
8. S.S.Gangwar and S.Kumar, Partition based computational method for high order fuzzy time series forecasting, *Expert Systems with Applications,* 39 (2012) 12158-12164.
9. K.Huarng, Heuristic models of fuzzy time series for forecasting, *Fuzzy Sets and Systems,* 123 (2001) 369-386.
10. K.Huarng and T.H.K.Yu, Ratio-based lengths of intervals to improve fuzzy time series forecasting, *IEEE Trans on SMC –Part B: Cybernetics* 36 (2006) 328-340.
11. J.R.Hwang, S.M.Chen and C.H.Lee, Handling forecasting problems using fuzzy time series, *Fuzzy Sets and Systems,* 100 (1998) 217-228.
12. Z.Ismail and R Efendi, Enrollment forecasting based on modified weight fuzzy time series, *Journal of Artificial Intelligence,* 4(1) (2011) 110-118.
13. B.P.Joshi and S.Kumar, A computational method for fuzzy time series forecasting based on difference parameters, *International Journal of Modeling, Simulation and Scientific Computing,* 4(1): 1250023 (2013) 1-12.

14. I.Kim and S.R. Lee, A fuzzy time series prediction method based on consecutive values, *IEEE International Fuzzy Systems Conference, August 22-25, Seol Korea, Proceedings* II : (1999) 703-707.
15. H.S.Lee and M.T.Chou, Fuzzy forecasting based on fuzzy time series, *International Journal of Computer Mathematics,* 81(2004) 781-789.
16. L.W.Lee, L.H.Wang and S.M. Chen, Handling forecasting problems based on two-factor high-order fuzzy time series, *IEEE Transactions on Fuzzy Systems,* 14(3) (2006) 468-477.
17. M.H.Lee, R.Efendi and Z.Ismail, Modified weighted for enrollment forecasting based on fuzzy time series, *MATEMATIKA,* 25(1) (2009) 67-78.
18. S.T.Li and Y.P.Chen, Natural partitioning-based forecasting model for fuzzy time series, *FUZZ-IEEE -2004 Hungry,* IEEE (2004) 1355-1359.
19. C.M.Own and P.T.Yu, Forecasting fuzzy time series on a heuristic high-order model, *Cybernetics and Systems:An International Journal,* 36 (2005) 705-717.
20. W.Qiu, X.Liu and H.Li, A generalized method for forecasting based on fuzzy time series, *Expert Systems with Applications*, 38 (2011) 10446-10453.
21. P.Singh and B.Borah, An efficient time series forecasting model based on fuzzy time series, *Engineering Applications of Artificial Intelligence,* 26 (2013) 2443-2457.
22. S.R.Singh, A simple method of forecasting based on fuzzy time series, *Applied Mathematics and Computation,*186 (2007) 330-339.
23. S.R.Singh, A robust method of forecasting based on fuzzy time series, *Applied Mathematics and Computation* , 188 (2007) 472-484.
24. S.R.Singh, A simple time variant method for fuzzy time series forecasting, *Cybernetics and Systems:An International Journal*, 38 (2007) 1-17.
25. Q.Song and B.S.Chissom, Fuzzy time series and its models, *Fuzzy Sets and Systems,* 54 (1993) 269-277.
26. Q.Song and B.S.Chissom, Forecasting enrollments with fuzzy time series–Part I, *Fuzzy Sets and Systems*, 54 (1993) 1-9.
27. Q.Song and B.S.Chissom, Forecasting enrollments with fuzzy time series–Part II, *Fuzzy Sets and Systems*, 64 (1994) 1-8.
28. J.Sullivan and W.H.Woodall, A comparison of fuzzy forecasting and Markov modeling, *Fuzzy Sets and Systems*, 64 (1994) 279- 293.
29. C.C.Tsai and S.J.Wu, A study for second-order modeling of fuzzy time series, *IEEE International Fuzzy Systems Conference Proceedings Aug 22-25,1999 Seoul, Korea, IEEE:*II (1999) 719-725.
30. C.C.Tsai and S.J.Wu, Forecasting local region data with fuzzy time series, *ISIE 2001,Pusan, KOREA, Proceedings, IEEE* (2001) 122-133.
31. C.C.Tsai, S.J.Wu and W.H.Ting, On the effect of membership function in a high-order fuzzy time series, *IEEE International Fuzzy Systems Conference Proceeding, IEEE* (2002) 437-442.
32. R.C.Tsaur, J.C.O.Yang and H.F.Wang, Fuzzy relation analysis in fuzzy time series model , *Computer and Mathematics with Applications,* 49(2005) 539-548.
33. V.R.Uslu, E.Bas, U.Yolcu and E.Egrioglu, A fuzzy time series approach based on weights determined by the number of recurrences of fuzzy relations, *Swarm and Evolutionary Computation*, 15 (2014) 19-26.
34. W.K.Wong, E.Bai and A.W.C.Chu, Adaptive time variant model for fuzzy time

series forecasting, *IEEE Transactions on System, Man, and Cybernetics-Part B; Cybernetics*, 40(6) (2010) 1531-1542.

35. S.Xihao and L.Yimin, Averaged–based fuzzy time series models for forecasting Shanghai compound index, *World Journal of Modelling and Simulation*, 4(2) (2008) 104-111.
36. H.K.Yu, A refined fuzzy time series model for forecasting, *PHYSICA A,* 346 (2005) 657-681.
37. H.K.Yu, Weighted fuzzy time series model for TAIEX forecasting, *PHYSICA A,* 349 (2005) 609-624.
38. L.A.Zadeh, Fuzzy Set, *Information and Control*, 8 (1965) 338-353.
39. L.A.Zadeh, The concept of a linguistic variable and its application to approximate reasoning-Part I, *Information Sciences*, 8 (1975) 199–249.