Intern. J. Fuzzy Mathematical Archive
Vol. 13, No. 2, 2017, 135-143
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 2 November 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v13n2a4

International Journal of
Fuzzy Mathematical Archive

Noiseless Coding Theorems of Generalized Useful Fuzzy Inaccuracy Measure of Order α and Type β

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Received 17 October 2017; accepted 31 October 2017

Abstract. In this paper, we present a new generalized useful fuzzy inaccuracy measure and generalized fuzzy code-word length of order α and type β. These measures are not only new but some known measures are the particular cases of our proposed measures. We have also obtained the bounds of generalized fuzzy code-word length in terms of generalized useful fuzzy inaccuracy measure.

Keywords: Fuzzy set, membership function, Kraft inequality, code-word length, uniquely decipherable codes, coding theorem, fuzzy entropy, Holder's inequality.

AMS Mathematics Subject Classification (2010): 94A17, 94A24

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [1] and developed his own theory to measure the ambiguity (uncertainty) of a fuzzy set. Fuzzy logic plays an important role in the context of information theory. Kilr and Parviz [2] first made an attempt to apply fuzzy set and fuzzy logic in information theory, later on various researchers applied the concept of fuzzy in information theoretic entropy function. The importance of fuzzy set comes from the fact that it can deal with imprecise and inexact information. Its application areas span from design of fuzzy controller to robotics and artificial intelligence. Besides above applications of fuzzy logic in information theory there is a numerous literature present on the application of fuzzy logic in information theory.

Fuzziness and uncertainty are the basic nature of human thinking and many real world objectives. Fuzziness is found in our decision, in our language and in the way of process information. The main objective of information is to remove uncertainty and fuzziness. In fact, we measure information supplied by the amount of probabilistic uncertainty removed in an experiment and the measure of uncertainty removed is also called as a measure of information, while measure of vagueness is called measure of fuzziness.

Later, many other researchers made more efforts in this particular area. For instance, Kaufmann [24] proposed fuzzy entropy of a fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set. Yager [25,26] defined an entropy measure of a fuzzy set in terms of the lack of distinction between fuzzy set and its complement. In 1989, Pal and Pal [27] proposed an entropy based on exponential function to measure the fuzziness called 'exponential fuzzy entropy'. A number of parametric generalizations of De Luca and Termini's [4] entropy measure have been studied by various researchers in last two decades. In 2007, Ding et al. [28] extended the idea of De Luca and Termini's fuzzy entropy for pairs of fuzzy sets and defined some new fuzzy information measures such as conditional fuzzy entropy, joint fuzzy entropy and fuzzy mutual information. Generalized fuzzy coding theorems by considering different generalized fuzzy information measures under the condition of uniquely decipherability codes were investigated by several authors, see for instance, the papers: Baig and Dar [10,11,12], Parkash and Sharma [13,14], Ashiq and Baig [21,22,23].

2. Preliminaries on fuzzy set theory

Let a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ then a fuzzy subset of universe X is defined as:

$$A = \{(x_i, \mu_A(x_i)) : x_i \in X, \mu_A(x_i) \in [0,1]\}$$

where $\mu_A(x_i): X \to [0,1]$ is a membership function and gives the degree of belongingness of the element x_i to the set A and is defined as follows:

$$\mu_A(x_i) = \begin{cases} 0, & \text{if } x_i \notin A \text{ and there is no ambiguity,} \\ 1, & \text{if } x_i \in A \text{ and there is no ambiguity,} \\ 0.5, & \text{if } x_i \in A \text{ or } x_i \notin A \text{ and there is maximum ambiguity,} \end{cases}$$

In fact $\mu_A(x_i)$ associates with each $x_i \in X$ gives a grade of membership function in the set A. When $\mu_A(x_i)$ takes values only 0 or 1, there is no uncertainty about it and a set is said to be a crisp (i.e. non-fuzzy) set. Some notions related to fuzzy sets which we shall need in our discussion Zadeh [1].

- ***Containment:*** If $A \subset B \Leftrightarrow \mu_A(x_i) \leq \mu_B(x_i) \ \forall \ x_i \in X$
- ***Equality:*** If $A = B \Leftrightarrow \mu_A(x_i) = \mu_B(x_i) \ \forall \ x_i \in X$
- ***Compliment:*** If $\bar{A}$ is complement of $A \Leftrightarrow \mu_A(x_i) = 1 - \mu_A(x_i) \ \forall \ x_i \in X$
- ***Union:*** If $A \cup B$ is union of A & $B \Leftrightarrow \mu_{A\cup B}(x_i) = \text{Max}\{\mu_A(x_i), \mu_B(x_i)\} \ \forall \ x_i \in X$
- ***Intersection:*** If $A \cap B$ is intersection of
  A & $B \Leftrightarrow \mu_{A\cap B}(x_i) = \text{Min}\{\mu_A(x_i), \mu_B(x_i)\} \ \forall \ x_i \in X$
- ***Product:*** If AB is product of A & $B \Leftrightarrow \mu_{AB}(x_i) = \mu_A(x_i)\mu_B(x_i) \ \forall \ x_i \in X$
- ***Sum***:
  If $A + B$ is sum of A & $B \Leftrightarrow \mu_{A+B}(x_i) = \mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i)\mu_B(x_i) \ \forall$
  $x_i \in X$

Let's consider a simple example. Later, we'll use the result of this example to provides a new method for European claim pricing. Consider a dynamic system driven by fractional noise

3. Basic concepts

If $x_1, x_2, \dots, x_n$ are members of the universe of discourse, with respective membership functions $\mu_A(x_1), \mu_A(x_2), \mu_A, \dots, \mu_A(x_n)$, then all $\mu_A(x_1), \mu_A(x_2), \mu_A(x_3), \dots, \mu_A(x_n)$ lies between 0 and 1 but these are not probabilities because their sum is not unity. $\mu_A(x_i)$

gives the element x_i the degree of belongingness to the set "A". The function $\mu_A(x_i)$ associates with each $x_i \in R^n$ a grade of membership to the set "A" and is known as membership function.

Denote

$$F.S = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ \mu_A(x_1) & \mu_A(x_2) & \dots & \mu_A(x_n) \end{bmatrix}, 0 \le \mu_A(x_i) \le 1 \ \forall \ x_i \qquad (1.1)$$

We call the scheme (1.1) as a finite fuzzy information scheme. Every finite scheme describes a state of uncertainty. Since $\mu_A(x_i)$ and $1-\mu_A(x_i)$ gives the same degree of fuzziness, therefore corresponding to entropy due to Shannon [3], De-Luca and Termini [4] suggested the following measure of fuzzy entropy.

$$H(A) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i) + \left(1-\mu_A(x_i)\right)\log\left(1-\mu_A(x_i)\right)\right] \qquad (1.2)$$

De-Luca and Termini [4] introduced a set of four properties and these properties are widely accepted as for defining new fuzzy entropy. In fuzzy set theory, the entropy is a measure of fuzziness which expresses the amount of average ambiguity in making a decision whether an element belongs to a set or not. So, a measure of average fuzziness $H(A)$ in a fuzzy set A should have the following properties to be valid fuzzy entropy:

I. (Sharpness): $H(A)$ is minimum if and only if A is a crisp set,i.e., $\mu_A(x_i) = 0$ or 1; for all x_i, $i = 1, 2, \dots, n$.
II. (Maximality): $H(A)$ is maximum if and only if A is most fuzzy set, i.e., $\mu_A(x_i) = \frac{1}{2}$; for all x_i, $i = 1, 2, \dots, n$.
III. (Resolution): $H(A^*) \le H(A)$, where A^* is sharpened version of A.
IV. (Symmetry): $H(A) = H(A^c)$, where A^c is the complement of A. i.e., $\mu_{A^c}(x_i) = 1 - \mu_A(x_i)$; for all x_i $i = 1, 2, \dots, n$

The different elements x_i depends upon the experimenters goal or upon some qualitative characteristics of the physical system taken into account; ascribe to each element x_i a non-negative number (u_i>0) directly proportional to its importance and call u_i the utility of the element x_i.Then the weighted fuzzy entropy [5] of the fuzzy set "A" is defined as:

$$H(A,U) = -\sum_{i=1}^{n} u_i\{\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\} \qquad (1.3)$$

Now let us suppose that the experimenter asserts that the membership function of the *i*th element is $\mu_B(x_i)$, where the true membership function is $\mu_A(x_i)$, thus we have two utility fuzzy information schemes:

$$F.S^* = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ \mu_A(x_1) & \mu_A(x_2) & \dots & \mu_A(x_n) \\ u_1 & u_2 & \dots & u_n \end{bmatrix}, 0 \le \mu_A(x_i) \le 1 \ \forall \ x_i, u_i > 0 \qquad (1.4)$$

Of a set of n elements after an experiment, and

$$F.S^{**} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ \mu_B(x_1) & \mu_B(x_2) & \dots & \mu_B(x_n) \\ u_1 & u_2 & \dots & u_n \end{bmatrix}, 0 \le \mu_B(x_i) \le 1 \ \forall \ x_i, u_i > 0 \qquad (1.5)$$

of the same set of n elements before the experiment. In both the schemes (1.4) and (1.5) the utility distribution is the same because we assume that the utility u_i of an element x_i is independent of its membership function $\mu_A(x_i)$, or predicted membership function

$\mu_B(x_i)$, u_i is only a 'utility' or value of the element x_i for an observer relative to some specified goal (refer to [6]).

The quantitative-qualitative measure of fuzzy inaccuracy corresponding to Taneja and Tuteja measure of inaccuracy [7] with the above schemes is:

$$I(A;B;U) = -\sum_{i=1}^{n} u_i\{\mu_A(x_i)\log\mu_B(x_i) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\} \tag{1.6}$$

Guiasu and Picard [8] considered the problem of encoding the letter output by the source (1.4) by means of a single prefix code with code-words $c_1, c_2, \dots, c_n$ having lengths $l_1, l_2, \dots, l_n$ satisfying Kraft [9] inequality:

$$\sum_{i=1}^{n} D^{-l_i} \leq 1 \tag{1.7}$$

where D being the size of the code alphabet. Corresponding to Guiasu and picard [8] useful mean code-word length we have the following useful fuzzy mean length of the code

$$L(A;U) = \frac{\sum_{i=1}^{n} u_i\{\mu_A(x_i)+(1-\mu_A(x_i))\}l_i}{\sum_{i=1}^{n} u_i\{\mu_A(x_i)+(1-\mu_A(x_i))\}} \tag{1.8}$$

and obtain bounds for it in terms of (1.6) under the condition:

$$\sum_{i=1}^{n}\{\mu_A(x_i)\mu_B^{-1}(x_i) + (1-\mu_A(x_i))(1-\mu_B(x_i))^{-1}\}\, D^{l_i} \leq 1 \tag{1.9}$$

where D is the size of code alphabet .Inequality (1.9) is generalized fuzzy Kraft's inequality.

A code satisfying generalized fuzzy Kraft's inequality is known as a personal fuzzy code. It is easy to see that for $\mu_A(x_i) = \mu_B(x_i) \forall x_i, i = 1,2,3,\dots,n$ (1.9) reduces to Kraft [9] inequality.

In this particular paper generalized useful fuzzy code-word mean length are considered and bounds have been obtained in terms of generalized useful fuzzy inaccuracy measure of order α and type β .The main aim of these results is that it generalizes some well-known fuzzy measures already existing in the literature.

4. Coding theorems of generalized useful fuzzy inaccuracy measure

Consider a function:

$$I_\alpha^\beta(A;B;U) = \frac{1}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n} u_i\left[\mu_A^\beta(x_i)\mu_B^{\beta(1-\alpha)}(x_i) + (1-\mu_A(x_i))^\beta(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}{\sum_{i=1}^{n} u_i\left[\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\right]}\right] \tag{2.1}$$

where $\alpha > 0 (\neq 1)$, $\beta > 0$, $\mu_A(x_i) \geq 0$, $\mu_B(x_i) \geq 0\ \forall\, x_i, i = 1, 2, 3, \dots, n$

Remarks of (2.1)

(i) When $\beta = 1$ (2.1) reduces to useful fuzzy information measure of order α corresponding to Bhatia [15] information measure of order α.

(ii) When $\beta = 1, u_i = 1\ \forall\, i = 1, 2, 3, \dots, n$ (2.1) reduces to fuzzy inaccuracy measure of corresponding to Nath [16], further it reduces to fuzzy entropy corresponding to Renyi's [17] entropy by taking $\mu_A(x_i) = \mu_B(x_i)\ \forall\, x_i$, $i = 1, 2, 3, \dots, n$

(iii) When $\beta = 1, u_i = 1\ \forall\, i = 1, 2, 3, \dots, n$ and $\alpha \to 1$ (2.1) reduces to the fuzzy measure corresponding to Kerridge [18]

We call (2.1) the generalized useful fuzzy inaccuracy measure of order α and type β.

Further we define a parametric fuzzy code-word mean length credited with utilities and membership functions as:

$$L_\beta^t(A;U)=\frac{1}{t}\log_D\left[\frac{\sum_{i=1}^n u_i^{t+1}\left[\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\right]D^{tl_i}}{\left(\sum_{i=1}^n u_i\left[\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\right]\right)^{t+1}}\right], -1<t<\infty, t\neq 0, \beta>0 \quad (2.2)$$

Remarks of (2.2)

(i) When $\beta=1$, (2.2), reduces to useful fuzzy code-word mean length corresponding to code given by Bhatia [15].

(ii) When $\beta=1, u_i=1\ \forall\ i=1,2,\dots,n.$ (2.2), reduces to fuzzy code-word mean length corresponding to Cambell [19] mean length.

(iii) When $\beta=1, u_i=1\ \forall\ i=1,2,\dots,n.$ and $t\to 0$ (2.2), reduces to optimal fuzzy code length corresponding to Shannon [3] optimal code length.

(iv) When $u_i=1\ \forall\ i=1,2,\dots,n.$ (2.2), reduces to the fuzzy code-word mean length corresponding to Khan and Haseen [20] code length.

Now we found the bounds for (2.2) in terms of (2.1) under the condition

$$\sum_{i=1}^n\left[\mu_A^\beta(x_i)\mu_B^{-\beta}(x_i)+\left(1-\mu_A(x_i)\right)^\beta(1-\mu_B(x_i))^{-\beta}\right]D^{-l_i}\leq 1 \quad (2.3)$$

where D is the size of code alphabet, also (2.3) is fuzzy generalization of Kraft [9] inequality. It is easy to see that for $\beta=1$ *and* $\mu_A(x_i)=\mu_B(x_i)\ \forall\ x_i,\ i=1,2,3,\dots,n,$ inequality (2.3) reduces to Kraft [9] inequality.

Theorem 4.1. For all integers D $(D>1)$. Let l_i satisfies the the condition (2.3), then the generalized parametric useful fuzzy code-word mean length satisfies

$$L_\beta^t(A;U)\geq I_\alpha^\beta(A;B;U) \quad (2.4)$$

where $\alpha=\frac{1}{1+t}$, equality holds iff

$$l_i=-\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^\beta(x_i)\mu_B^{\beta(\alpha-1)}(x_i)+\left(1-\mu_A(x_i)\right)^\beta(1-\mu_B(x_i))^{\beta(\alpha-1)}\right]}\right] \quad (2.5)$$

Proof: By Holder's inequality we have

$$\sum_{i=1}^n x_iy_i\geq\left(\sum_{i=1}^n x^p\right)^{\frac{1}{p}}\left(\sum_{i=1}^n x^q\right)^{\frac{1}{q}} \quad (2.6)$$

For all $x_i,y_i>0,\ i=1,2,3,\dots,n$ and $\frac{1}{p}+\frac{1}{q}=1,\ p<1(\neq 0), q<0$ or $q<1(\neq 0), p<0.$

We see the equality holds iff there exists a positive constant c such that

$$x_i^p=cy_i^q \quad (2.7)$$

Making the substitution

$$x_i=u_i^{-\left(\frac{t+1}{t}\right)}\left(\mu_A^{\frac{-\beta}{t}}(x_i)+\left(1-\mu_{A(x_i)}\right)^{\frac{-\beta}{t}}\right)\left[\frac{1}{\sum_{i=1}^n u_i\left[\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\right]}\right]^{-\left(\frac{t+1}{t}\right)}D^{-l_i} \quad (2.8)$$

$$y_i=u_i^{\left(\frac{t+1}{t}\right)}\left[\mu_A^{\beta\left(\frac{t+1}{t}\right)}(x_i)\mu_B^{-\beta}(x_i)+\left(1-\mu_A(x_i)\right)^{\beta\left(\frac{t+1}{t}\right)}(1-\mu_B(x_i))^{-\beta}\right]\left[\frac{1}{\sum_{i=1}^n u_i\left[\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\right]}\right]^{\left(\frac{t+1}{t}\right)} \quad (2.9)$$

$p=-t=\frac{\alpha-1}{\alpha}$ and $q=\frac{t}{1+t}=1-\alpha$

in (2.6) and after suitable simplification, we get

$$\sum_{i=1}^{n}\left[\mu_A^{\beta}(x_i)\mu_B^{-\beta}(x_i)+\left(1-\mu_A(x_i)\right)^{\beta}(1-\mu_B(x_i))^{-\beta}\right]D^{-l_i}$$

$$\geq\left[\frac{\sum_{i=1}^{n}u_i^{t+1}\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]D^{tl_i}}{\left(\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]\right)^{t+1}}\right]^{\frac{-1}{t}}$$

$$\left[\frac{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]}\right]^{\frac{1}{1-\alpha}}$$

Now using the inequality (2.3) we get

$$\left[\frac{\sum_{i=1}^{n}u_i^{t+1}\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]D^{tl_i}}{\left(\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]\right)^{t+1}}\right]^{\frac{1}{t}}$$

$$\geq\left[\frac{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]}\right]^{\frac{1}{1-\alpha}}$$

Taking logarithm to both with base D we get

$$\frac{1}{t}\log_D\left[\frac{\sum_{i=1}^{n}u_i^{t+1}\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]D^{tl_i}}{\left(\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]\right)^{t+1}}\right]$$

$$\frac{1}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]}\right]$$

Or equivalently, we can write

$$L_{\beta}^{t}(A;U)\geq I_{\alpha}^{\beta}(A;B;U)$$

Theorem 4.1. For every code with lengths $l_1,l_2,\ldots,l_n$ satisfies the condition (2.3), $L_{\beta}^{t}(A;U)$ can be made to satisfy the inequality

$$L_{\beta}^{t}(A;U)<I_{\alpha}^{\beta}(A;B;U)+1 \tag{2.10}$$

Proof: Let l_i be the positive integer satisfying the inequality

$$\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]\leq l_i<$$

$$-\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^{n}u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]+1 \tag{2.11}$$

Consider the interval

$$\delta_i = \begin{bmatrix} -\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right], \\ -\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]+1 \end{bmatrix} \tag{2.12}$$

of length 1, in every δ_i, there lies exactly one positive integer l_i, such that

$$0 < -\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right] \le l_i <$$

$$-\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]+1 \tag{2.13}$$

We will first show that the sequence $l_1, l_2, \ldots, l_n$ thus defined above satisfies (2.3).
Subsequently from the left inequality of (2.13) we have

$$-\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right] \le l_i$$

Or equivalently, we can write

$$\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right] \ge D^{-l_i} \tag{2.14}$$

Multiply both sides of equation (2.14) by

$$\left[\mu_A^{\beta}(x_i)\mu_B^{-\beta}(x_i)+\left(1-\mu_A(x_i)\right)^{\beta}(1-\mu_B(x_i))^{-\beta}\right]$$

and then summing over $i = 1, 2, \ldots, n$ on both sides to the result that we obtain we get the required result (2.3).

Now take the last inequality of (2.13) we have

$$l_i < -\log_D\left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]+1$$

Or equivalently, we can write above as

$$D^{l_i} < \left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]^{-1} D \tag{2.15}$$

Raising power , $t = \frac{1-\alpha}{\alpha}$ on both sides to equation (2.15), we get

$$D^{tl_i} < \left[\frac{u_i\left[\mu_B^{\alpha\beta}(x_i)+(1-\mu_B(x_i))^{\alpha\beta}\right]}{\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}\right]^{-t} D^t \tag{2.16}$$

Multiply both sides of equation (2.16) by

$$\frac{u_i^{t+1}\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]}{\left(\sum_{i=1}^n u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]\right)^{t+1}}$$

And then summing over $i = 1, 2, \ldots, n$ to the resulted expression, and after making suitable operations, we get

$$\frac{\sum_{i=1}^{n} u_i^{t+1}\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]D^{tl_i}}{\left(\sum_{i=1}^{n} u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]\right)^{t+1}} <$$

$$\left[\frac{\sum_{i=1}^{n} u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}{\sum_{i=1}^{n} u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]}\right]^{t+1} D^t \qquad (2.17)$$

Taking logarithms to both sides with base D to equation (2.17), and then dividing both sides by $t = \frac{1-\alpha}{\alpha}$, to the resulted expression and after suitable operations, we get

$$\frac{1}{t}\log_D\left[\frac{\sum_{i=1}^{n} u_i^{t+1}\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]D^{tl_i}}{\left(\sum_{i=1}^{n} u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]\right)^{t+1}}\right] <$$

$$\frac{1}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n} u_i\left[\mu_A^{\beta}(x_i)\mu_B^{\beta(1-\alpha)}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\beta(1-\alpha)}\right]}{\sum_{i=1}^{n} u_i\left[\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right]}\right]+1$$

Or equivalently, we can write

$L_{\beta}^{t}(A;U) < I_{\alpha}^{\beta}(A;B;U)+1.$

5. Conclusion

In this article, we present a new generalized useful (weighted) fuzzy inaccuracy measure and its corresponding generalized fuzzy code-word length and show that these measures generalizes some well-known measures that already exists in the literature of fuzzy information measures. Also we obtain the bounds of generalized fuzzy code-word length in terms of generalizes useful (weighted) fuzzy inaccuracy measure.

Acknowledgements. We thank the Editor of this journal and referee for their valuable suggestions to improve the quality of this paper.

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