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On Thorny Fuzzy Graphs

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Abstract. A thorny fuzzy graph which is analogous to the concept thorny graphs in crisp graph theory is defined. The degree of an edge in thorny fuzzy graphs is obtained. Also, the degree of an edge in fuzzy graph formed by this operation in terms of the degree of edges in the given fuzzy graphs in some particular cases is found. Moreover, it is proved that thorny fuzzy graph of effective fuzzy graph is effective.

Keywords: Thorny fuzzy graphs; thorny graphs; degree of an edge; effective fuzzy graph

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

It was Rosenfeld who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975 [11]. Later on, Bhattacharya gave some remarks on fuzzy graphs [1]. The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Moderson and Peng [5]. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using these operations were discussed by Nagoorgani and Radha [7]. Radha and Kumaravel introduced the concept of degree of an edge and total degree of an edge in fuzzy graphs [9] and studied about the degree of an edge in fuzzy graphs which are obtained from two given fuzzy graphs using the operations of union and join [10]. Sequantion join of fuzzy graphs is defined by Çolakoğlu [2].

In this paper, we have introduced the concept of thorny graph of fuzzy graphs, which are analogous to the concept thorny graph in crisp graph theory. Thorny graphs important plays in crisp graph theory. Gutman introduced thorny graphs [3]. Idrees et. al. studied on topological indices of thorny graphs [4]. Ors Yorgancıoglu and Dündar given some theorem about total coloring of thorny graphs [12] .

Let V be a nonempty set. A fuzzy graph is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ such that $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$ for all u,v in V [6]. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^{\dagger}:(\mathrm{V},\mathrm{E})$ where $E\subseteq V\times V$. $\mu(u,v)>0$ for $(\mathrm{u},\mathrm{v})\in E$, $\mu(u,v)=0$ for $(u,v)\notin E$.

Throughout this paper we assume that μ is reflexive and need not consider loops. Note that $G:(\sigma,\mu)$ denote fuzzy graphs with underlying crisp graphs $G^{\dagger}:(\mathrm{V},\mathrm{E})$, with $|V|=q$. Also, the underlying set V is assumed to be finite and σ can be chosen in any manner so as to satisfy the definition of a fuzzy graph in all the examples and all these properties are satisfied for all fuzzy graphs except null graphs. We shall denote the edge between two vertices u and v by uv.

In [7], the degree of a vertex u in G is defined by

$$d_G(u)=\sum_{u\neq v}\mu(uv)=\sum_{uv\in E}\mu(uv) \tag{1.1}$$

By Nagoorgani and Ahamed in [8], the order of a fuzzy graph G is defined by

$$O(G)=\sum_{u\in V}\sigma(\mathrm{u}). \tag{1.2}$$

The union of two fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ is defined as a fuzzy graph $G=G_1\cup \mathrm{G}_2:(\sigma_1\cup\sigma_2,\mu_1\cup\mu_2)$ on $G^{\dagger}:(\mathrm{V},E)$ where $V=V_1\cup V_2$ and $E=E_1\cup E_2$ with

$$(\sigma_1\cup\sigma_2)(u)=\begin{cases}\sigma_1(u), & u\in V_1-V_2\\ \sigma_2(u), & u\in V_2-V_1\\ \sigma_1(u)\vee\sigma_2(u), & u\in V_1\cap V_2\end{cases}$$

and

$$(\mu_1\cup\mu_2)(uv)=\begin{cases}\mu_1(uv), & uv\in E_1-E_2\\ \mu_2(uv), & uv\in E_2-E_1\\ \mu_1(uv)\vee\mu_2(uv), & uv\in E_1\cap E_2\end{cases}.$$

Assume that $V_1\cap V_2=\varnothing$. The join of G_1 and G_2 is defined as a fuzzy graph $G=G_1+G_2:(\sigma_1+\sigma_2,\mu_1+\mu_2)$ on $G^{\dagger}:(V,E)$ where $V=V_1\cup V_2$ and $E=E_1\cup E_2\cup E'$ where E' is the set of all edges joining vertices of V_1 with vertices of V_2, with

$$(\sigma_1+\sigma_2)(u)=(\sigma_1\cup\sigma_2)(u)\ \text{ for all } u\in V_1\cup V_2$$

and

$$(\mu_1+\mu_2)(uv)=\begin{cases}(\mu_1\cup\mu_2)(uv), & uv\in E_1\cup E_2\\ \sigma_1(u)\wedge\sigma_2(v), & uv\in E'\end{cases}.$$

By Radha and Kumaravel [9], the degree of an edge uv is defined

$$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = \sum_{\substack{uw\in E \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E \\ w\neq u}} \mu(wv) \tag{1.3}$$

2. Degree of an edge in thorny graphs

In this section, we give the definition of thorny graphs and calculated degree of an edge of fuzzy graphs that are obtained by this operation.

Thorny graph is defined in [3]. Let G^* be thorny graph of $G(V,E)$ with parameters $p_1,...,p_n$. G^* is obtained by attaching $p_i \geq 0$ new vertices of degree one to all vertices of G. Let $p_i = r, r \geq 0$. $V^* = V^*(\mathrm{G}^*) = \mathrm{V} + \mathrm{V}'$ its vertex set, whose number is $q + qr$, where V' is new vertices of degree one to each vertex i of G, $E^* = E^*(\mathrm{G}^*) = E + E'$ its edge set, whose number is $|E| + qr$, where E' is the set of all edges joining by adding r new vertices of degree one to each vertex i of G. Let σ_i, $i = 1,...,qr$ be a fuzzy subset of V' and let μ_i, $i = 1,...,qr$ be a fuzzy subset of E'. Using definition of join and union, define the fuzzy subset σ^* of V^* and μ^* of E^* as follows:

$$\sigma^*(u) = \left(\sigma \cup (\sigma_1 \cup ... \cup \sigma_r) \cup ... \cup \left(\sigma_{qr-q} \cup ... \cup \sigma_{qr}\right)\right)(u) \quad \forall u \in V^* \tag{2.1}$$

$$\mu^*(uv) = \begin{cases} \mu(uv), & uv \in E \\ \sigma(u) \wedge \sigma_i(v) \ \ (\mathrm{i} = 1,...,\mathrm{qr}), & uv \in E' \end{cases} \tag{2.2}$$

where E' is the set of all edges joining by adding r new vertices of degree one to each vertex i of G.

Theorem 2.1. Let G^* be thorny fuzzy graph of G. For any $uv \in E^*$,

$$d_{G^*}(uv) = d_G(uv) + \sum_{\substack{uw\in E' \\ w\in V', i=\overline{1,qr}}} \sigma(u) \wedge \sigma_i(w) + \sum_{\substack{wv\in E' \\ w\in V, i=\overline{1,qr}}} \sigma(w) \wedge \sigma_i(v)$$

Proof: By (1.3), we have

$$d_{G^*}(\mathrm{uv}) = \sum_{\substack{uw\in E^*-E' \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E^*-E' \\ w\neq u}} \mu(wv) + \sum_{\substack{uw\in E' \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E' \\ w\neq u}} \mu(wv) \tag{2.3}$$

Using (2.2) in (2.3) we get

$$d_{G^*}(uv) = \sum_{\substack{uw\in E \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E \\ w\neq u}} \mu(wv) + \sum_{\substack{uw\in E' \\ w\in V', i=\overline{1,qr}}} \sigma(u) \wedge \sigma_i(w) + \sum_{\substack{wv\in E' \\ w\in V, i=\overline{1,qr}}} \sigma(w) \wedge \sigma_i(v)$$

Using definition of (1.3), we completed this proof. □

In the following theorems, we find the degree of uv in G^* in some particular cases.

Nagoorgani and Radha in [6] defined the relation $\sigma_1 \geq \sigma_2$ means that $\sigma_1(u) \geq \sigma_2(v)$, for every $u \in V_1$ and for every $v \in V_2$, where σ_i is a fuzzy subset of $V_i, i = 1, 2$.

Theorem 2.2. Let G^* be thorny fuzzy graph of G.

1) For $\sigma_i \geq \sigma$ $(\mathrm{i}=1,...,\mathrm{qr})$ the following equality holds:

$$d_{G^*}(uv) = d_G(uv) + 2O(\mathrm{G})$$

2) If $\sigma \geq \sigma_i$ $(\mathrm{i}=1,...,\mathrm{qr})$ then

$$d_{G^*}(uv) = d_G(uv) + \sum_{\substack{uw \in E' \\ w \in V', i=\overline{1,qr}}} \sigma_i(w).$$

Proof: Assume that $\sigma_i \geq \sigma$ $(\mathrm{i}=1,...,\mathrm{qr})$ the. From Theorem 2.1, we have

$$d_{G^*}(uv) = d_G(uv) + \sum_{u \in V} \sigma(u) + \sum_{w \in V} \sigma(w).$$

By using equation (1.2), we have

$$d_{G^*}(uv) = d_G(uv) + 2O(\mathrm{G}).$$

Now, for $\sigma \geq \sigma_i$ $(\mathrm{i}=1,...,\mathrm{qr})$, from Theorem 2.1, we have

$$d_{G^*}(uv) = d_G(uv) + \sum_{\substack{uw \in E' \\ w \in V', i=\overline{1,qr}}} \sigma_i(w) + \sum_{\substack{wv \in E' \\ v \in V', i=\overline{1,qr}}} \sigma_i(v)$$

$v \in V$ *and* $v \notin V'$. Hence, poof of Theorem 2.2 (2) is completed. □

Theorem 2.3. Thorny fuzzy graph of effective fuzzy graphs is an effective fuzzy graph.

Proof: Let $G(\sigma, \mu)$ be effective fuzzy graphs. Then $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for $uv \in E$. Let G^* be thorny fuzzy graph of G. By (2.2), the fuzzy subset μ^* of E^* is

$$\mu^*(uv) = \begin{cases} \mu(uv) = \sigma(u) \wedge \sigma(v), & uv \in E \\ \sigma(u) \wedge \sigma_i(v) \ (\mathrm{i}=1,...,\mathrm{qr}), & uv \in E' \end{cases}$$

Thus, the proof of the theorem is completed. □

REFERENCES

1. A.Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recognition Letters*, 9 (1987) 159-162.
2. Ö.Çolakoğlu, On Sequential Join of Fuzzy Graphs, *Intern. J. Fuzzy Mathematical Archive*, 8 (2015) 19-27.
3. I.Gutman, Distance of Thorny graphs, *Publications De L'Institüt Mathematique*, 63 (1998) 31-36.
4. N.Idrees, M.J.Saif, A.Rauf, S.Mustafa, First and second zagreb eccentricity indices of thorny graphs, *Symmetry*, 9 (2017) 1-9.

5. J.N.Mordeson and C.S.Peng, Operation on fuzzy graphs, *Information Sciences*, 79 (1994) 159-170.
6. J.N.Mordeson and P.S.Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica Verlag, Heidelberg (2000).
7. A.Nagoorgani and K.Radha, The degree of a vertex in some fuzzy graphs, *International Journal of Algorithms Computing and Mathematics*, 2 (2009) 107-116.
8. A.Nagoorgani and M.Basheer Ahamed, Order and size in fuzzy graph, *Bulletin of Pure and Applied Sciences*, 22E (1) (2003) 145-148.
9. K.Radha and N.Kumaravel, The degree of an edge in Cartesian product and composition of two fuzzy graphs, *Intern. J. Applied Mathematics & Statistical Sciences*, 2(2) (2013) 65-78.
10. K.Radha and N.Kumaravel, The degree of an edge in union and join of two fuzzy graphs, *International Journal of Fuzzy Mathematical Archive*, 4(1) (2014) 8-19.
11. A.Roseneld, fuzzy graphs, in: L.A.Zadeh, K.S. Fu. K .Tanaka, M.Shimura (Eds), *Fuzzy sets and Their Applications to Cognitive and Decision Processes*, Academic Press, New York, (1975) 77-95.
12. Z.Ors Yorgancıoglu and P.Dündar, Total coloring and total coloring of thorny graphs, *American International Journal of Contemporary Research*, 3 (2013) 11-15.