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Pareto Optimal Solutions of the Fuzzy Multi-Index, Bi-Criteria Fixed Charge Transportation Problem

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Abstract. In this paper, a multi-index, bi-criteria, fixed charge transportation problem (MIBCFCTP) is considered in which the parameters of cost and duration are taken as trapezoidal fuzzy numbers. An algorithm incorporating an extended VAM and an extended MODI method is developed to find Pareto optimal solutions of the problem. The algorithm is illustrated with a numerical example and solutions obtained are compared with existing methods.

Keywords: Multi-index; transportation problem; bi-criteria; fixed charge; trapezoidal fuzzy numbers; ranking function.

AMS Mathematics Subject Classification (2010): 90B06

1. Introduction

The classical transportation problem (TP) which is well studied in literature is a two-index problem. Additional indices such as source, destination, and modes of transport extend the TP to a multi index transportation problem (MITP). Haley [12] has considered the MITP with three indices as origin, destination and commodity. He further solved the MITP with North-West corner rule and an extension of the modified distribution (MODI) method. Haley [13], Haley [14], Moravek and Vlach [23], Smith [30], Vlach [34], Korsnikov [20], Bandopadhyaya and Puri [6], Junginger [16], Pandian and Anuradha [26], Bulut and Bulut [10], Zitouni [37], Djamel et al [11] are names of few researchers who considered MITP.

Sometimes in TPs there may be multiple objectives such as minimizing transportation time in addition to minimizing the transportation cost. The TP having two objective functions such as minimizing cost and time is called bi-criteria transportation problem (BCTP). Solutions of BCTP's are Pareto optimal. A solution (c, t) of bi-criteria problem is Pareto optimal if there is no other solution *(C, T)* of the problem satisfying $C \le c$ and $T \le t$ with strict inequality holding in at least one case. Bandopadhayaya [7] has studied the multi index bi-criteria transportation problem (MIBCTP).

Some transportation problems, also have fixed costs associated with the direct costs and the resultant TP is called fixed charge transportation problem (FCTP). Sandrock [29], Murty [24], Sadagopan and Ravindran [27] have presented algorithms which gave exact solutions of the FCTP's while Adlakha and Kowalski [1], Aguado [2] have presented heuristic methods to solve such problems. Khurana and Arora [17], Tuli and Chauhan [33] have given algorithms to find Pareto optimal solutions of BCTPs with fixed cost.

Following Haley [12], Ahuja and Arora [3] and Khurana and Adlakha [18] have solved multi index bi-criteria fixed charge transportation problem (MIBCFCTP) with crisp transportation parameters. Arora and Khurana [5], Yang and Feng [35], Nagarajan and Jeyaraman [25] are a few authors who have considered MIBCFCTP's.

Uncertainties in real life situations which may be due to weather changes, data unavailability or high information cost etc. necessitate the introduction of fuzzy numbers in TP's. Zadeh [36] has introduced the fuzzy concept to deal with the uncertainties. Uncertain data in TP may be represented by triangular or trapezoidal fuzzy numbers. Mohanaselvi and Ganesan [22], Samuel and Venkatachalapathy [28], Khalaf [15] have solved fuzzy transportation problem (FTP) by using triangular fuzzy numbers. Basirzadeh [8], Solaiappan and Jeyaraman [31] have solved FTPs with trapezoidal fuzzy numbers. Ritha and Vinotha [32] solved multi index fixed charge transportation problem (MIFCTP) using symmetrical trapezoidal fuzzy numbers while Kumar, Gupta and Sharma [4] proposed algorithm for bi-criteria fixed charge transportation problem (BCFCTP) using trapezoidal fuzzy number. This paper is motivated by the need of finding a better solution of the uncertain MIBCFCTP. An algorithm is developed to find Pareto optimal solutions of the MIBCFCTP with cost and duration as trapezoidal fuzzy numbers. A numerical example is given to demonstrate the algorithm and the solutions obtained are compared with existing methods by Ahuja and Arora [3] and Khurana and Adlakha [18].

The rest of the paper is organised as follows: Section 2 gives basic definition and arithmetic operations on trapezoidal fuzzy numbers, Section 3 gives the mathematical formulation of the fuzzy MIBCFCTP, Section 4 gives the steps of the proposed algorithm, Section 5 gives a numerical example to illustrate the proposed algorithm and Section 6 gives the concluding remarks.

2. Preliminaries

2.1. Trapezoidal fuzzy numbers

Trapezoidal fuzzy numbers have been studied by Bector and Chandra [9], Klir and Yuan [19], Lee [21] to name a few. These numbers are defined by $\tilde{A} = (a, b, c, d)$ and have the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a, x \ge d \\ \dfrac{x-a}{b-a}, & a \le x < b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c < x \le d \end{cases}$$

Trapezoidal fuzzy numbers can be converted to crisp numbers using various ranking approaches. Most commonly used is the average ranking approach $\Re(\tilde{A}) = \dfrac{a+b+c+d}{4}$.

2.2. Arithmetic operations on trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then arithmetic operations on trapezoidal fuzzy numbers are as follows:

i) Addition : $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

ii) Subtraction : $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

iii) Scalar Multiplication : $\lambda.\tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4) & \lambda > 0 \\ (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1) & \lambda < 0 \end{cases}$

3. Mathematical formulation of the fuzzy MIBCFCTP

The formulation of MIBCFCTP with trapezoidal fuzzy numbers is as follows.

Let there are m sources, n destinations, p types of commodities. Then objective of the problem is to

$$\text{Minimize} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \tilde{c}_{ijk} x_{ijk} + \sum_{i=1}^{m} \sum_{k=1}^{n} \tilde{F}_{ik} , \max_{\substack{1 \le i \le m \\ 1 \le j \le n \\ 1 \le k \le p}} \left[\tilde{t}_{ijk} \mid x_{ijk} > 0 \right] \right\} \tag{1}$$

subject to

$$\left.\begin{aligned}
&\sum_{i=1}^{m} x_{ijk} = A_{jk} \\
&\sum_{j=1}^{n} x_{ijk} = B_{ki} \\
&\sum_{k=1}^{p} x_{ijk} = E_{ij} \\
&x_{ijk} \ge 0; i = 1,2,..., m; j = 1,2,..., n; k = 1,2,..., p \\
&\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki} , \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{n} E_{ij} , \sum_{i=1}^{m} E_{ij} = \sum_{k=1}^{p} A_{jk} \\
&\sum_{j=1}^{n} \sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p} \sum_{i=1}^{m} B_{ki} = \sum_{i=1}^{m} \sum_{j=1}^{n} E_{ij}
\end{aligned}\right\} \tag{2}$$

where

x_{ijk} is the quantity of commodity k supplied from source i to destination j.

$\tilde{c}_{ijk}$ is the fuzzy variable transportation cost per unit quantity of commodity k supplied from source i to destination j.

$\tilde{F}_{ik}$ is the fuzzy fixed charge associated with source i and commodity k.

$$\tilde{F}_{ik} = \sum_{j=1}^{n} \tilde{f}_{ijk}\,\delta_{ijk}\,,\ i = 1,2,...,m;\, k = 1,2,..., p$$

where $\delta_{ijk} = \begin{cases} 1, & x_{ijk} > 0 \\ 0, & x_{ijk} = 0 \end{cases}$

$\tilde{t}_{ijk}$ is fuzzy time required to sent commodity k from source i to destination j.

A_{jk} is the total quantity of commodity k required at destination j.

B_{ki} is the total quantity of commodity k available at source i.

E_{ij} is the total quantity transported from source i to destination j.

$\tilde{c}_{ijk}$, $\tilde{F}_{ik}$, $\tilde{t}_{ijk}$ are trapezoidal fuzzy numbers.

The formulated problem in (1) can be divided into two sub-problems.
The first sub-problem (P_1) is

$$(P_1)\ \text{Minimize}\ \tilde{C} = \left\{ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{c}_{ijk}\,x_{ijk} + \sum_{i=1}^{m}\sum_{k=1}^{n} \tilde{F}_{ik} \right\} \quad \text{subject to (2).}$$

The second sub-problem (P_2) is

$$(P_2)\ \text{Minimize}\ \tilde{T} = \max_{\substack{1\le i\le m \\ 1\le j\le n \\ 1\le k\le p}} [\tilde{t}_{ijk} \mid x_{ijk} > 0] \quad \text{subject to (2).}$$

The sub- problem (P_1) is solved by extended Vogel's Approximation Method (eVAM) and extended Modified Distribution Method (eMODI) to get the first Pareto Optimal cost $\tilde{C}_1$ and the corresponding time $\tilde{T}_1$. Thereafter the cost matrix is modified and the second Pareto optimal cost $\tilde{C}_2$ and corresponding time $\tilde{T}_2$ are obtained satisfying $\tilde{C}_1 \le \tilde{C}_2, \tilde{T}_1 \ge \tilde{T}_2$, inequality being defined by ranking function. Proceeding similarly remaining Pareto optimal solutions $(\tilde{C}_3,\tilde{T}_3),(\tilde{C}_4,\tilde{T}_4),...$ are obtained satisfying $\tilde{C}_1 \le \tilde{C}_2 \le \tilde{C}_3 \le ...,\ \tilde{T}_1 \ge \tilde{T}_2 \ge \tilde{T}_3 \ge ...$. The process terminates when no further feasible solution with respect to cost is obtained.

4. Steps of the proposed algorithm

Let there be *m* sources, *n* destinations and *p* types of commodities.

Step 1: The fuzzy cost table (Table 1a) and fuzzy time table (Table 1b) are formed with *m* rows, *n* columns and *p* cell diagonals.

Step 2: The average ranking approach $\Re(\tilde{A}) = \frac{a+b+c+d}{4}$ is applied on fuzzy numbers in Tables (1a) and (1b) to get crisp cost table (Table 2a) and crisp time table (Table 2b).

Step 3: An eVAM is applied to the crisp cost table (Table 2a) as explained below:

(a) Calculate the penalties of all rows, columns and cell diagonals.

(b) Select the largest penalty among all rows, columns and cell diagonals in the same manner as in the Vogel's approximation method (VAM). In the row or column or cell diagonal associated with largest penalty, make the allocation in the corresponding cell of least cost.

(c) In case of tie in the largest penalty in step (3b) above, select the row/column/cell diagonal having the minimum least cost cell and make allocation in that cell.
(d)(i) If $x_{ijk}=A_{jk}$ then cross j^{th} column in k^{th} cell diagonal.
(ii) If $x_{ijk}=B_{ki}$ then cross i^{th} row in k^{th} cell diagonal.
(iii) If $x_{ijk}=E_{ij}$ then cross i^{th} row in j^{th} column.
(e) Corresponding allocations in Table (1a) are now made following step (3d) above as shown below.

(i)If $x_{ijk} = A_{jk}$ $then$ $A_{jk}(new) = 0$, $B_{ki}(new) = B_{ki}(old) - x_{ijk}$ and $E_{ij}(new) = E_{ij}(old) - x_{ijk}$

(ii) If $x_{ijk} = B_{ki}$ $then$ $A_{jk}(new) = A_{jk}(old) - x_{ijk}$, $B_{ki}(new) = 0$ and $E_{ij}(new) = E_{ij}(old) - x_{ijk}$

(ii) If $x_{ijk} = E_{ij}$ $then$ $A_{jk}(new) = A_{jk}(old) - x_{ijk}$, $B_{ki}(new) = B_{ki}(old) - x_{ijk}$ and $E_{ij}(new) = 0$

(f) Continue steps (3a) to (3e) until no more allocation is possible or $A_{jk} = B_{ki} = E_{ij} = 0$.

The initial allocations with crisp and fuzzy costs are shown in Tables 3(a) and 3 (b) respectively.

Step 4: The initial basic feasible solution is obtained from Table 3(b) as follows:
(a): The fuzzy initial feasible solution (FIFS) shown in Table 3(b) is made feasible by eliminating any negatively allocated cell by making an e-loop (closed loop in three dimensions) as explained in step 6(d) starting from that cell.

(b)For obtaining fuzzy initial basic feasible solution (FIBFS) it is checked if the number of allocated cells = *mnp-(m-1)(n-1)(p-1)*. If it is true then the feasible solution in step 4(a) is also basic feasible. Otherwise this can be made basic feasible by introducing allocations ε in non basic cells which are independent. The FIBFS obtained is shown in Table 4.

Step 5: Calculate total fuzzy fixed charge $\tilde{F} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\tilde{f}_{ijk}$ corresponding to FIBFS of step (4) .

Step 6: Apply eMODI as shown below.
(a) Calculate $\tilde{u}_{jk}, \tilde{v}_{ki}, \tilde{w}_{ij}$ $for(i,j,k) \in B(basis)$ where $\tilde{c}_{ijk} = \tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij}$

(b) Calculate $\tilde{\delta}_{ijk} = \tilde{c}_{ijk} - (\tilde{u}_{jk} + \tilde{v}_{ki} + \tilde{w}_{ij}), \forall (i,j,k) \notin B$

(c)Select a non basic cell (i,j,k) whose $\tilde{\delta}_{ijk}$ [or $\Re(\tilde{\delta}_{ijk})$] is most negative.

(d) The non-basic cell of step (6c) enters the basis by making an e-loop of 8 or more cells as explained below.

- Any two adjacent cells in the loop lie in the same row or same column or same cell diagonal.
- All cells that get a plus or a minus sign must be an allocated cell however the starting non allocated cell should always have a plus sign.

(e) After addition of the non basic cell in the basis let its allocation be I_{ijk}. Calculate difference in fuzzy direct transportation cost as $\tilde{A}_{ijk} = \tilde{\delta}_{ijk} \times I_{ijk}$.

(f) Calculate fuzzy fixed charge $\tilde{F}_{ijk}(temp)$ for the newly entered cell in the basis.

(g) Find fuzzy fixed charge difference as $\tilde{F}_{ijk}(difference) = \tilde{F}_{ijk}(temp) - \tilde{F}$.

(h) Calculate total difference in fuzzy transportation cost as $\tilde{\Delta}_{ijk} = \tilde{F}_{ijk}(difference) + \tilde{A}_{ijk}$

(i) If $\Re(\tilde{\Delta}_{ijk}) \geq 0$ then process terminates otherwise among the cells where $\Re(\tilde{\Delta}_{ijk}) < 0$ find the cell (i,j,k) whose $\Re(\tilde{\Delta}_{ijk})$ is minimum. That cell now enters the basis. Repeat steps (6a)-(6h) till all $\Re(\tilde{\Delta}_{ijk}) \geq 0$. Table 5 shows the allocations of the first fuzzy Pareto Optimal Cost.

Step 7: From Table 1(b), the fuzzy transportation time $\tilde{T}_1$ is obtained where
$\tilde{T}_1 = \text{maximum}\{\tilde{t}_{ijk} \mid x_{ijk} > 0\}$

Step 8: The first fuzzy Pareto optimal solution is obtained as $(\tilde{C}_1, \tilde{T}_1)$

Step 9: The fuzzy cost in Table 1(a) is modified by putting $\tilde{c}_{ijk} = \begin{cases} \tilde{M} \text{ if } \Re(\tilde{t}_{ijk}) \geq \Re(\tilde{T}_1) \\ \tilde{c}_{ijk} \text{ if } \Re(\tilde{t}_{ijk}) < \Re(\tilde{T}_1) \end{cases}$

where $\tilde{M}$ = (M,M,M,M) is a sufficiently large trapezoidal fuzzy number.

Step 10: Repeating steps 2-8 the second fuzzy Pareto optimal solution $(\tilde{C}_2, \tilde{T}_2)$ is obtained as shown in Table (6).

Step 11: Steps 9 and 10 are repeated to get the third Pareto optimal solution shown in Table (7). Proceeding in this manner subsequent Pareto optimal solutions $\{(\tilde{C}_1,\tilde{T}_1),(\tilde{C}_2,\tilde{T}_2),...(\tilde{C}_q,\tilde{T}_q)\}$ are obtained. The process terminates when solution becomes infeasible.

5. Numerical example illustrating the proposed algorithm

Consider a $3 \times 3 \times 3$ MIBCFCTP with trapezoidal fuzzy numbers. The fuzzy cost $\tilde{c}_{ijk}$ and fuzzy time $\tilde{t}_{ijk}$ are shown in Table 1(a) and Table 1(b) respectively.

Table 1: (a) (Fuzzy Cost Table)

Destinations → / Sources ↓	j=1			j=2			j=3			B_{ik}		
i=1	(3,5,8,16)			(1,5,6,8)			(3,4,7,14)			6		
		(3,4,7,14)			(2,4,6,12)			(1,2,3,6)			9	
	$E_{11}=10$		(2,4,6,12)	$E_{122}=6$		(4,6,10,20)	$E_{13}=9$		(5,6,11,22)			10
i=2	(5,6,11,22)			(4,5,9,18)			(6,7,13,26)			13		
		(3,5,8,16)			(7,8,15,30)			(3,4,7,14)			14	
	$E_{21}=21$		(6,7,13,26)	$E_{22}=9$		(5,7,12,24)	$E_{23}=14$		(3,5,8,16)			17
i=3	(1,5,6,8)			(3,5,8,16)			(4,6,10,20)			15		
		(2,4,6,12)			(4,5,9,18)			(2,4,6,12)			13	
	$E_{31}=21$		(3,4,7,14)	$E_{32}=13$		(3,4,7,14)	$E_{33}=12$		(5,7,12,24)			18
A_{jk}	15			8			11			34		
		17			11			8			36	
			20			9			16			45

Table 1:(b) (Fuzzy Time Table)

Destinations → Sources ↓	j=1			j=2			j=3		
i=1	(1,2,3,6)			(3,5,8,16)			(3,4,7,14)		
		(2,3,5,10)			(2,4,6,12)			(1,3,4,8)	
			(1,3,4,8)			(3,5,8,16)			(0,0.5,1.5,2)
i=2	(0,1,2,5)			(1,3,4,8)			(2,4,6,12)		
		(0,0.5,1.5,2)			(0,1,2,5)			(0,0.5,1.5,2)	
			(2,4,6,12)			(0,1,2,5)			(0,0.5,1.5,2)
i=3	(1,3,4,8)			(1,2,3,6)			(1,3,4,8)		
		(3,5,8,16)			(0,1,2,5)			(0,1,2,5)	
			(0,0.5,1.5,2)			(0,0.5,1.5,2)			(3,5,8,16)

Table 2(a) and Table 2(b) shows the crisp cost table and crisp time table after applying Step 2 on Table 1(a) and Table1(b) respectively.

Table 2:(a) (Crisp Cost Table)

Destinations → Sources ↓	j=1			j=2			j=3			B_{kj}		
i=1	8			5			7			6		
		7			6			3			9	
	$E_{11}=10$		6	$E_{12}= 6$		10	$E_{13}=9$		11			10
i=2	11			9			13			13		
		8			15			7			14	
	$E_{21}=21$		13	$E_{22}=9$		12	$E_{23}=14$		8			17
i=3	5			8			10			15		
		6			9			6			13	
	$E_{31}=21$		7	$E_{32}=13$		7	$E_{33}=12$		12			18
A_{ik}	15			8			11			34		
		17			11			8			36	
			20			9			16			45

Table 2:(b) (Crisp Time Table)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	3 5 4	8 6 8	7 4 1
i=2	2 1 6	4 2 2	6 1 1
i=3	4 8 1	3 2 1	4 2 8

On applying Step 3 of the proposed algorithm the initial allocations with crisp and fuzzy costs are shown in Tables 3(a) and 3 (b) respectively.

Table 3: (a) (Crisp Initial Solution)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	8 7 6 [10]	5 [5] 6 [1] 10	7 [1] 3 [8] 11
i=2	11 [10] 8 [14] 13 [-3]	9 [3] 15 12 [6]	13 7 8 [14]
i=3	5 [5] 6 [3] 7 [13]	8 9 [10] 7 [3]	10 [10] 6 12 [2]

Table 3: (b) (Fuzzy Initial Solution)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	(3,5,8,16)	(1,5,6,8) [5]	(3,4,7,14) [1]
	(3,4,7,14)	(2,4,6,12) [1]	(1,2,3,6) [8]
	(2,4,6,12) [10]	(4,6,10,20)	(5,6,11,22)
i=2	(5,6,11,22) [10]	(4,5,9,18) [3]	(6,7,13,26)
	(3,5,8,16) [14]	(7,8,15,30)	(3,4,7,14)
	(6,7,13,26) [-3]	(5,7,12,24) [6]	(3,5,8,16) [14]
i=3	(1,5,6,8) [5]	(3,5,8,16)	(4,6,10,20) [10]
	(2,4,6,12) [3]	(4,5,9,18) [10]	(2,4,6,12)
	(3,4,7,14) [13]	(3,4,7,14) [3]	(5,7,12,24) [2]

From Table 3(b) it is observed that the solution obtained is not feasible as cell (2, 1, 3) has a negative allocation of -3 units. To make it feasible an e-loop as explained in step 6d is made and the solution is made feasible as shown in Table 4. Also number of allocated cells is 18 which is less than $mnp-(m-1)(n-1)(p-1)=19$. Hence an allocation of ε is made to the cell (1,1,2) to get the 19 allocations of the initial basic feasible solution.

Table 4: (Fuzzy Initial Basic Feasible Solution)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	(3,5,8,16)	(1,5,6,8) [5]	(3,4,7,14) [1]
	(3,4,7,14) [ε]	(2,4,6,12) [1]	(1,2,3,6) [8]
	(2,4,6,12) [10]	(4,6,10,20)	(5,6,11,22)
i=2	(5,6,11,22) [10]	(4,5,9,18) [3]	(6,7,13,26)
	(3,5,8,16) [11]	(7,8,15,30) [3]	(3,4,7,14)
	(6,7,13,26)	(5,7,12,24) [3]	(3,5,8,16) [14]
i=3	(1,5,6,8) [5]	(3,5,8,16)	(4,6,10,20) [10]
	(2,4,6,12) [6]	(4,5,9,18) [7]	(2,4,6,12)
	(3,4,7,14) [10]	(3,4,7,14) [6]	(5,7,12,24) [2]

The fuzzy fixed charges are given as

f_{111}=(4,6,10,20) f_{121}=(14,16,30,60) f_{131}=(9,11,20,40)
f_{112}=(9,11,20,40) f_{122}=(9,11,20,40) f_{132}=(9,11,20,40)
f_{113}=(14,16,30,60) f_{123}=(9,11,20,40) f_{133}=(4,6,10,20)
f_{211}=(4,6,10,20) f_{221}=(9,11,20,40) f_{231}=(9,11,20,40)
f_{212}=(4,6,10,20) f_{222}=(4,6,10,20) f_{232}=(14,16,30,60)
f_{213}=(19,21,40,80) f_{223}=(4,6,10,20) f_{233}=(4,6,10,20)
f_{311}=(4,6,10,20) f_{321}=(19,21,40,80) f_{331}=(9,11,20,40)
f_{312}=(9,11,20,40) f_{322}=(4,6,10,20) f_{332}=(14,16,30,60)
f_{313}=(9,11,20,40) f_{323}=(4,6,10,20) f_{333}=(4,6,10,20)

The total fuzzy fixed charge for fuzzy initial basic feasible solution is calculated from Table 4. Then eMODI method as explained in step (6) is applied to Table 4 which gives the allocations corresponding to the first Pareto Optimal cost as shown in Table 5.

Table 5: (First Fuzzy Optimal Solution)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	(3,5,8,16)	(1,5,6,8) [2]	(3,4,7,14) [4]
	(3,4,7,14) [ε]	(2,4,6,12) [4]	(1,2,3,6) [5]
	(2,4,6,12) [10]	(4,6,10,20)	(5,6,11,22)
i=2	(5,6,11,22) [7]	(4,5,9,18) [6]	(6,7,13,26)
	(3,5,8,16) [14]	(7,8,15,30)	(3,4,7,14)
	(6,7,13,26)	(5,7,12,24) [3]	(3,5,8,16) [14]
i=3	(1,5,6,8) [8]	(3,5,8,16)	(4,6,10,20) [7]
	(2,4,6,12) [3]	(4,5,9,18) [7]	(2,4,6,12) [3]
	(3,4,7,14) [10]	(3,4,7,14) [6]	(5,7,12,24) [2]

From Table 5 the first fuzzy Pareto Optimal cost is obtained as
$\tilde{C}_1 = (137, 173, 310, 620) + (339, 544, 883, 1726) = (476, 717, 1193, 2346)$ where $\Re(\tilde{C}_1) = 1183$
and corresponding fuzzy time $\tilde{T}_1 = (3, 5, 8, 16)$ where $\Re(\tilde{T}_1) = 8.$ The first Pareto optimal solution is $(\tilde{C}_1, \tilde{T}_1) = ((476, 717, 1193, 2346), (3, 5, 8, 16)).$

To obtain the 2nd Pareto Optimal solution the cost Table 1(a) is modified by applying Step 9. Thereafter Steps 2 -8 are applied to get the allocations of the 2nd Pareto optimal solution in the fuzzy cost table as shown in Table 6.

Table 6: (Second Fuzzy Optimal Solution)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	(3,5,8,16)	(M,M,M,M)	(3,4,7,14) [6]
	(3,4,7,14) [3]	(2,4,6,12) [6]	(1,2,3,6) [ε]
	(2,4,6,12) [7]	(M,M,M,M)	(5,6,11,22) [3]
i=2	(5,6,11,22) [4]	(4,5,9,18) [8]	(6,7,13,26) [1]
	(3,5,8,16) [14]	(7,8,15,30)	(3,4,7,14)
	(6,7,13,26) [3]	(5,7,12,24) [1]	(3,5,8,16) [13]
i=3	(1,5,6,8) [11]	(3,5,8,16)	(4,6,10,20) [4]
	(M,M,M,M)	(4,5,9,18) [5]	(2,4,6,12) [8]
	(3,4,7,14) [10]	(3,4,7,14) [8]	(M,M,M,M)

From Table 6 the econd Pareto optimal solution obtained is $(\tilde{C}_2, \tilde{T}_2) = ((489, 726, 1215, 2386), (3, 4, 7, 14))$. Again on applying Steps (9) and (10) the allocations of the 3rd Pareto optimal solution obtained in the fuzzy cost table are shown in Table 7.

Table 7: (Third Fuzzy Optimal Solution)

Destinations → Sources ↓	j=1	j=2	j=3
i=1	(3,5,8,16) [6]	(M,M,M,M)	(M,M,M,M)
	(3,4,7,14) [3]	(2,4,6,12) [6]	(1,2,3,6)
	(2,4,6,12) [1]	(M,M,M,M)	(5,6,11,22) [9]
i=2	(5,6,11,22)	(4,5,9,18) [6]	(6,7,13,26) [7]
	(3,5,8,16) [14]	(7,8,15,30)	(3,4,7,14) [ε]
	(6,7,13,26) [7]	(5,7,12,24) [3]	(3,5,8,16) [7]
i=3	(1,5,6,8) [9]	(3,5,8,16) [2]	(4,6,10,20) [4]
	(M,M,M,M)	(4,5,9,18) [5]	(2,4,6,12) [8]
	(3,4,7,14) [12]	(3,4,7,14) [6]	(M,M,M,M)

From Table 7 the 3rd fuzzy Pareto optimal solution obtained is
$(\tilde{C}_3, \tilde{T}_3) = ((545, 774, 1319, 2602), (2, 4, 6, 12))$.
The procedure terminates as no further feasible solutions can be obtained.
The solutions obtained by the proposed algorithm are compared by solutions obtained by existing methods as shown in Table 8.

Table 8 (Comparative study between existing methods and proposed method)

Pareto Optimal solutions Cost Time Trade off pair	Solutions by				Number of Iterations		
	Existing method [3]	Existing method [18]	Proposed method (Fuzzy)	Proposed method (Crisp using average ranking)	Existing method [3]	Existing method [18]	Proposed method (Fuzzy)
I	(1185, 8)	(1183,8)	((476,717,1193,2346), (3,5,8,16))	(1183, 8)	10	13	7
II	(1204, 7)	(1204,7)	((489,726,1215,2386), (3,4,7,14))	(1204, 7)	7	7	5
III	(1310, 6)	(1310,6)	((545,774,1319,2602), (2,4,6,12))	(1310, 6)	5	6	5
		Total Number of Iterations			22	26	17

It is observed from Table 8 that the proposed method gives better first Pareto optimal solution than one of the existing methods [3] while second and third Pareto optimal solutions are the same as in existing methods. There is also a drastic reduction in the percentage of iterations for the proposed method by nearly 23% (vis-à-vis existing method).

6. Concluding remarks

In the paper, MIBCFCTP is studied with cost and time taken as trapezoidal fuzzy numbers which makes the problem more robust than existing crisp problems. It has been observed that the proposed method using eVAM and eModi (using e-loop) requires far less iterations than existing methods of Ahuja and Arora [3] and Khurana and Adlakha [18] thus providing a better alternative to the existing methods. The proposed method can also be extended to more complex fuzzy transportation problems where availabilities, requirements and actual quantity transported are also fuzzy numbers.

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