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A Solution of Fuzzy Multilevel Quadratic Fractional Programming Problem through Interactive Fuzzy Goal Programming Approach

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Abstract. The purpose of this paper is to study the fuzzy multilevel quadratic fractional programming problem through fuzzy goal programming procedure. A fuzzy multilevel quadratic fractional programming problem is a type of hierarchical programming problem which contains fuzzy parameters as coefficients of cost in objective function, the resources and the technological coefficients. Here, we are considering those fuzzy parameters as the triangular fuzzy numbers. Firstly, we are transferring the fuzzy multilevel quadratic fractional programming problem into a deterministic multilevel multiobjective quadratic fractional programming problem by using Zadeh extension principle. Then, an interactive fuzzy goal programming procedure is used to solve this equivalent deterministic multiobjective multilevel quadratic fractional programming problem by using respective membership functions. An illustrative numerical example for fuzzy four level quadratic fractional programming problem is provided to reveal the practicability of the proposed method.

Keywords: Quadratic programming, fractional programming, fuzzy goal programming, triangular fuzzy numbers, multilevel hierarchical optimization

AMS Mathematics Subject Classification (2010): 90C20

1. Introduction

The multi-level programming problem (MLPP) is a model set by the planner in which each level of a hierarchy has its own objective function and decision space which is not fully determined by itself but evaluated with the interference of other levels. In these types of problems, control tools of each level may enable him to impact the policies of other levels and as a result of that participation, it improves the objective function of each level. For example, in an executive board, decentralized firm, and top management, or headquarters, to build a decision such as a budget of any firm; each division governs a production planes by knowing a budget completely. In 1988, Anandalingam [2] studied mathematical programming model for decentralized bi-level programming problem (DBLPP) as well as MLPP based on Stackelberg solution concept. The multi-level fractional quadratic programming problems are special types of MLPPs. In which, the objective function of each level of MLPP is taken as the ratio of two quadratic functions. This type of model is very useful in bank balance sheet management, health care, finance corporate planning etc. Due to these applications, it attracts the keen interest of researchers in its theory. In a few decades earlier, the various researchers introduced many such problems and their solution procedures. Some important existing solution approaches for solving multilevel programming problems are such as the decent method, the extreme point search, the solution-procedure based on Karush-Kuhn Tucker (KKT) conditions, and many more. But these methods are not much successful to solve the various MLPPs rather than in solving simple types of multilevel programming problems. The concept of maximizing decision was introduced by Bellman and Zadeh [6] in fuzzy decision making problems. But Zimmermann [18] introduced firstly the use of fuzzy set theory in decision making optimization problems and theory of fuzzy linear programming was introduced by Tanaka et al. [16]. After that the various approaches were introduced in the literature of bilevel programming problems as well as in multilevel programming problems. As said earlier, there is some technical inefficiency in solving the optimization problems by using existing methods like KKT conditions or penalty functions based multilevel programming approaches. To overcome these inefficiencies, Lai [10] applied the concepts of membership function on such problems in 1996 and this concept was extended further by Shih et al [15], but this approach is lengthy one for solution procedure. To overcome this type of problem, the fuzzy goal programming approach (FGP) was proposed by Mohamed [11] and this approach was extended by Pramanik and Roy [14] to solve the multilevel linear programming problems. Pop and Stancu Minasian [13] solved the fully fuzzified linear fractional problems by representing all the variables and parameters with triangular fuzzy numbers. Baky [4] solved the various decentralized multiobjective programming problems by using the fuzzy goal programming approach. Also, Chang [7] recommended the goal programming approach for fuzzy multiobjective fractional programming problems. Pal and Gupta [12] studied the multiobjective fractional decision-making problems by formulating fuzzy goal programming with the help of a genetic algorithm. Abousina and Baky [1] suggested fuzzy goal programming procedure to solve bilevel multi-objective linear fractional programming problems. Lachhwani [9] also used fuzzy goal programming approach for multi-level linear fractional programming problems. Anusuya [3] has applied type-2 fuzzy soft sets on fuzzy decision making problems. Baskaran [5] solved a fuzzy transshipment problem with fuzziness in the preemptive goal programming formulation. Dhurai [8] proposed a fuzzy optimal solution for fully fuzzy linear programming problem by using hexagonal fuzzy numbers. Recently, Veeramany [17] used a method to solve fuzzy linear fractional programming problem by using Zadeh extension principle. Here, in this paper we are extending this approach of using Zadeh's extension principle for solving fuzzy multilevel quadratic fractional programming problems (FMLQFPP). This method works according with three characteristic features which are usually applied in the various solution procedures of decision making problems. Firstly, the fuzzy multilevel quadratic fractional programming problems (FMLQFPP) is converted into the deterministic multilevel multiobjective quadratic fractional programming problem (MLMOQFPP) by using the Zadeh's Principle. Secondly, fuzzy goals are designated by each level decision maker in the form of fractional membership functions which are linearised further by using the

Taylor series approach and finally, an interactive fuzzy goal programming procedure is adopted to solve MLMOQFPP.

2. Some basic notations

In this section, we are explaining the basic definitions of fuzzy sets, fuzzy numbers and membership functions which are given below:

Definition 2.1. A Fuzzy set $\tilde{F}_i$ on a real space R is a set of ordered pairs$\{(x, \mu_{\tilde{F}_i}(x)/x \in R)\}$, where $\mu_{\tilde{F}_i}(x): \rightarrow [0,1]$ is called as the membership function of fuzzy set.

Definition2.2. A convex fuzzy set,$\tilde{F}_i$, on a real space R is a fuzzy set in which:
$\forall\, x, y\ \in R, \forall\lambda\ \in\ [0,1]\ \mu\tilde{F}_i\ (\lambda x +\ (1\ -\ \lambda)y\ \geq\ min\ [\mu\tilde{F}_i\ (x), \mu\tilde{F}_i(y)]$.

Definition 2.3. A fuzzy set $\tilde{F}_i$, on real space R, is called positive if its membership function is such that
$\mu\tilde{F}_i(x)\ =\ 0, \forall x\ \leq\ 0$

Definition 2.4. A convex fuzzy set $\tilde{F}$ is called as triangular fuzzy number (TFN) if it can be defined as

$$\tilde{F} = (x, \mu\tilde{F}_i\ (x)) \text{ where: } \mu\tilde{F}_i = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c\ , \\ 0, & otherwise \end{cases}$$

For simplicity, we can represent TFN by three real parameters (a, b, c) which are $(a \leq b \leq c)$ will be denoted by the triangle a, b, c (Fig.1).

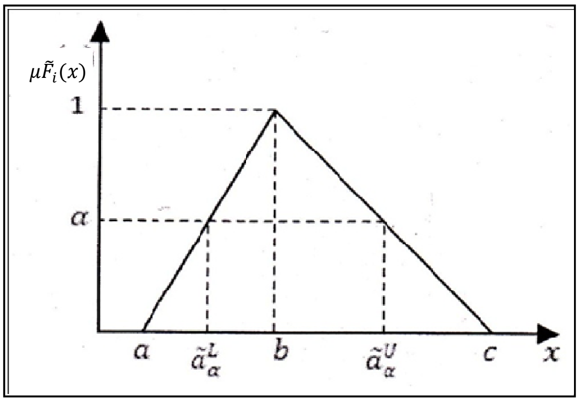

Figure1: Triangular fuzzy number

Definition 2.5. In any multilevel optimization problem, f_{ij} is objective function for decision maker of any level. Let f_{ij}^*, f_{ij}^{min} , $f_{ij}^{max}$are ideal, minimum and maximum values for f_{ij}. Then, the decision of any considered level can be formulated as follows:

Find x

So as to satisfy

$$f_{ij}\begin{pmatrix}\le\\ \cong\\ \ge\end{pmatrix}f_{ij}^*$$

Subject to $x \epsilon S$

where, f_{ij}^* is the perspective goal value for the objective function f_{ij}, $\begin{pmatrix}\le\\ \cong\\ \ge\end{pmatrix}$ represents different fuzzy relations.

Let $\left(f_{ij}{}^*, f_{ij}^{max}\right)$ be the tolerance interval selected to ij^{th} objective function f_{ij}. Thus membership function is defined as

$$\mu_{ij}\left(f_{ij}(x)\right)=\begin{cases}1, & \text{if } f_{ij}(x)\le f_{ij}{}^*\\ \dfrac{f_{ij}^{max}-f_{ij}}{f_{ij}^{max}-f_{ij}{}^*}, & f_{ij}{}^*\le f_{ij}(x)\le f_{ij}{}^{max}\\ 0, & f_{ij}\ge f_{ij}{}^{max}\end{cases}$$

where $f_{ij}{}^*$ is called an ideal value and $f_{ij}{}^{max}$ is tolerance limit for f_{ij}

Similarly, let $\left(f_{ij}{}^{min}, f_{ij}^*\right)$ be the tolerance interval selected to ijth objective function f_{ij}. Thus membership function is defined as

$$\mu_{ij}\left(f_{ij}(x)\right)=\begin{cases}1, & \text{if } f_{ij}(x)\ge f_{ij}{}^*\\ \dfrac{f_{ij}-f_{ij}{}^{min}}{f_{ij}{}^*-f_{ij}{}^{min}}, & f_{ij}{}^{min}\le f_{ij}(x)\le f_{ij}{}^*\\ 0, & f_{ij}\le f_{ij}{}^{min}\end{cases}$$

Definition 2.6. Membership functions are linearized by using Taylor series approach. The suggested procedure for fractional objectives can be continued as follow:

Obtain $\tilde{x}_i{}^* = (\tilde{x}_{i1}{}^*, \tilde{x}_{i2}{}^*, \dots\dots\dots, \tilde{x}_{ip_i}{}^*)$ which is the value that is used to maximize the ij-th membership function $\mu_{ij}\left(f_{ij}(x)\right)$ associated with ij-th objective $f_{ij}(x)$.

$$\tilde{\mu}_{ij}\left(f_{ij}(x)\right)\cong\left[\left.\frac{\mu_{ij}\left(f_{ij}(\tilde{x}_i{}^*)\right)}{\partial x_1}\right|_{\tilde{x}_i{}^*}(x_1-\tilde{x}_{i1}{}^*)+\left.\frac{\mu_{ij}\left(f_{ij}(\tilde{x}_i{}^*)\right)}{\partial x_2}\right|_{\tilde{x}_i{}^*}(x_2-\tilde{x}_{i2}{}^*)\right.$$
$$\left.+\ \dots\dots\dots\dots\dots\dots+\left.\frac{\mu_{ij}\left(f_{ij}(\tilde{x}_i{}^*)\right)}{\partial x_m}\right|_{\tilde{x}_i{}^*}(x_m-\tilde{x}_{im}{}^*)\right]$$

Definition 2.7. Multilevel programming is indicated as mathematical programming which solves the coordinating problems of decision making processes of hierarchal organization. In multilevel programming problems there are n independent decision makers. Let $x_i \in R^n$ be a vector variable which indicates the i^{th} decision variable and $f_i: R^n \to R^n$ be the i^{th} level objective function under the linear constraint vector $Ax(\le, =, \ge)b$ which is a set of m equations and its right hand side has real or fuzzy variables.

This type of programming problem is read as Multi-level Quadratic Programming Problem (MLQPP), and it can be formulated as following:

$[1^{st}\ Level]$

$$\max_{x_1} f_1 = C_1 x + \frac{1}{2} x^T D_1 x$$

where x_1 solves and x_1 is vector of decision variable

$[2^{nd}\ Level]$

$$\max_{x_2} f_2 = C_2 x + \frac{1}{2} x^T D_2 x$$

where x_2 solves and x_2 is vector of decision variable

……………………………………………

$[n^{th}\ Level]$

$$\max_{x_n} f_n = C_n x + \frac{1}{2} x^T D_n x$$

subject to

$$Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b$$

$$x \geq 0$$

where $f_1 and\ f_2$ are objective functions of the first level decision maker (FLDM), and second level decision maker (SLDM) and f_n is nth level decision maker; C_i are$(1 \times n)$ matrices and D_i are $n \times n$ real matrices for $i = 1,2, \dots \dots, n$. $A = \left(a_{pq}\right)_{m \times n}$,matrix of coefficients and $b = (b_1, b_2, \dots \dots \dots, b_m)^T$. The first-level decision maker has control over $x_1 \in R^{n_1}$, and second-level decision maker has control over the $x_2 \in R^{n_2}$ and so on.

3. Problem formulation

Multilevel quadratic programming problems are those optimizing problems which contain quadratic functions as their objective functions and in this paper, we are taking a this type of problem in such a way that its each objective function($\mathrm{f_i}$) has fractional form i.e. $\mathrm{f_i} = \frac{\mathrm{f^N}_\mathrm{i}}{\mathrm{f^D}_\mathrm{i}}$ under the linear constraint vector $\mathrm{Ax}(\leq, =, \geq)\mathrm{b}$ which is a set of m equations. This type of programming problem can be read as multi-level quadratic fractional programming problem (MLQFPP) and if its objective function, the resources and the technological coefficients are triangular numbers then it can be read as fuzzy multi-level quadratic fractional programming problem (FMLQFPP) which is formulated in the following way:

$$\left.\begin{array}{c}
[1^{st}\ \text{Level}] \\
\max_{x_1} f_1 = \frac{f^N{}_1}{f^D{}_1} = \frac{\breve{C}_{11}x+\frac{1}{2}x^T\breve{D}_{11}x}{\breve{C}_{12}x+\frac{1}{2}x^T\breve{D}_{12}x} \\
\text{where } x_1 \text{solves and } x_1 \text{is vector of decision variable} \\
[2^{nd}\ \text{Level}] \\
\max_{x_2} f_2 = \frac{f^N{}_2}{f^D{}_2} = \frac{\breve{C}_{21}x+\frac{1}{2}x^T\breve{D}_{21}x}{\breve{C}_{22}x+\frac{1}{2}x^T\breve{D}_{22}x} \\
\text{where } x_1 \text{solves and } x_1 \text{is vector of decision variable} \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. \\
[n^{th}\ \text{Level}] \\
\max_{x_n} f_n = \frac{f^N{}_n}{f^D{}_n} = \frac{\breve{C}_{n1}x+\frac{1}{2}x^T\breve{D}_{n1}x}{\breve{C}_{n2}x+\frac{1}{2}x^T\breve{D}_{n2}x} \\
\text{subject to} \\
\breve{A}x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} \breve{b} \\
x \ge 0
\end{array}\right\} \quad (1)$$

where $f_1, f_2, \dots\dots.., f_n$ are objective functions of the first level decision maker (FLDM), second level decision maker (SLDM) and n^{th} level decision maker respectively; $\breve{C}_{ij}$ are matrices of order $(1 \times n)$ and $\breve{D}_{ij}$ are $n \times n$ real matrices for $i = 1,2,3, \dots.., n$ and $j = 1,2$. $\breve{A} = (\breve{a}_{pq})_{m\times n}$ is a matrix of coefficients and $\breve{b} = (\breve{b}_1, \breve{b}_2, \dots\dots\dots.., \breve{b}_m)^T$. The matrices $\breve{C}_{ij}$, $\breve{D}_{ij}$, $\breve{A}$ and $\breve{b}$ contains triangular fuzzy numbers as their elements. The first-level decision maker has control over $x_1 \in R^{n_1}$, and second-level decision maker has control over $x_2 \in R^{n_2}$ and so on.

4. Transformation of fuzzy MLQFPP into deterministic MLMOQFPP

By using Zadeh extension principle, we can transform the above mentioned fuzzy multilevel quadratic fractional programming problem (MLQFPP) into a deterministic multilevel multiobjective quadratic fractional programming problem (MLMOQFPP) as follows:

$[1^{st}\ \text{Level}]$

$$\max_{x_1}(f_{11}, f_{12}, f_{13}) = \left(\frac{C^1{}_{11}x+\frac{1}{2}x^TD^1{}_{11}x}{C^3{}_{12}x+\frac{1}{2}x^TD^3{}_{12}x}, \frac{C^2{}_{11}x+\frac{1}{2}x^TD^2{}_{11}x}{C^2{}_{12}x+\frac{1}{2}x^TD^2{}_{12}x}, \frac{C^3{}_{11}x+\frac{1}{2}x^TD^3{}_{11}x}{C^1{}_{12}x+\frac{1}{2}x^TD^1{}_{12}x}\right)$$

where x_1 solves and x_1 is vector of decision variable

$[2^{nd}\ \text{Level}]$

$$\max_{x_2}(f_{21}, f_{22}, f_{23}) = \left(\frac{C^1{}_{21}x+\frac{1}{2}x^TD^1{}_{21}x}{C^3{}_{22}x+\frac{1}{2}x^TD^3{}_{22}x}, \frac{C^2{}_{21}x+\frac{1}{2}x^TD^2{}_{21}x}{C^2{}_{22}x+\frac{1}{2}x^TD^2{}_{22}x}, \frac{C^3{}_{21}x+\frac{1}{2}x^TD^3{}_{21}x}{C^1{}_{22}x+\frac{1}{2}x^TD^1{}_{22}x}\right)$$

where x_2 solves and x_2 is vector of decision variable

…………………………………………………………….

$[n^{th}\ \text{Level}]$

$$\max_{x_n}(f_{n1}, f_{n2}, f_{n3}) = \left(\frac{C^1{}_{n1}x+\frac{1}{2}x^TD^1{}_{n1}x}{C^3{}_{n2}x+\frac{1}{2}x^TD^3{}_{n2}x}, \frac{C^2{}_{n1}x+\frac{1}{2}x^TD^2{}_{n1}x}{C^2{}_{n2}x+\frac{1}{2}x^TD^2{}_{n2}x}, \frac{C^3{}_{n1}x+\frac{1}{2}x^TD^3{}_{n1}x}{C^1{}_{n2}x+\frac{1}{2}x^TD^1{}_{n2}x}\right)$$

subject to

$A^1x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^1,\ A^2x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^2,\ A^3x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^3$

where f_{11}, f_{12}, f_{13} are objective functions of the first level decision maker (FLDM), f_{21}, f_{22}, f_{23} for second level decision maker (SLDM) and f_{n1}, f_{n2}, f_{n3} for n^{th} level decision maker; $C^1{}_{ij}, C^2{}_{ij}, C^3{}_{ij}$ are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{C}_{ij}$ respectively and $C^1{}_{ij}, C^2{}_{ij}, C^3{}_{ij}$ are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{D}_{ij}$ respectively for $i = 1,2, \ldots\ldots, n$ and $j = 1,2$. A^1, A^2, A^3 are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{A}$ and b^1, b^2, b^3 are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{b}$.

5. Fuzzy goal programming procedure to solve MLMOQFPP

5.1. Construction of fractional membership functions

Let $\left(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, \ldots\ldots, x_n{}^{U_{ij}}, f_{ij}{}^{max}\right)$ and $\left(x_1{}^{L_{ij}}, x_2{}^{L_{ij}}, \ldots\ldots, x_n{}^{L_{ij}}, f_{ij}{}^{min}\right)$ be the best and worst optimal solutions of each objective function f_{ij} of every decision maker over the region S, when solved individually. Then, the fuzzy goals appear as $f_{ij} \le f_{ij}{}^{max}$ and their respective membership functions can be defined as the following:

$$\mu_{ij}\left(f_{ij}(x)\right) = \begin{cases} 1, & \text{if } f_{ij}(x) \ge f_{ij}{}^{max} \\ \dfrac{f_{ij} - f_{ij}^{min}}{f_{ij}^{max} - f_{ij}{}^{min}}, & f_{ij}{}^{min} \le f_{ij}(x) \le f_{ij}{}^{max} \\ 0, & f_{ij} \le f_{ij}{}^{min} \end{cases}$$

5.2. Linearization of fractional membership functions

Here, fractional membership functions associated with each objective function are linearized by using Taylor series approach. According to which, the fractional membership functions can be linearized at the neighborhood of the point of optimal solution $\left(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, \ldots\ldots\ldots\ldots, x_n{}^{U_{ij}}\right)$

$$\begin{aligned} \mu^*{}_{ij} = {} & \mu_{f_{ij}}\left(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, \ldots\ldots, x_n{}^{U_{ij}}\right) + \left(x_1 - x_1{}^{U_{ij}}\right) \frac{\partial \mu_{f_{ij}}}{\partial x_1}\left[\left(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, \ldots\ldots, x_n{}^{U_{ij}}\right)\right] \\ & + \left(x_2 - x_2{}^{U_{ij}}\right)\frac{\partial \mu_{f_{ij}}}{\partial x_2}\left[\left(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, \ldots\ldots, x_n{}^{U_{ij}}\right)\right] \\ & + \cdots\ldots.. + \left(x_n - x_n{}^{U_{ij}}\right)\frac{\partial \mu_{f_{ij}}}{\partial x_n}\left[\left(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, \ldots\ldots, x_n{}^{U_{ij}}\right)\right] \end{aligned}$$

5.3. Construction of membership functions for decision variables

The tolerance of decision variable which is controlled by the upper level decision maker is used to find the satisfactory solution. Thus, it is required to construct membership function for those decision variables which are controlled by the upper level decision maker after getting the optimal solution of respective level. i.e. the optimal solution at i^{th} level of MLMOQFPP is $\left(x_1{}^i, x_2{}^i, \ldots, x_n{}^i\right)$ and it controls the decision variable x_j whose positive and negative tolerance limits are $t_j{}^R$ and $t_j{}^L$ respectively, then the linear membership function for this controlled variable can be defined as the following:-

$$\mu_{x_j}(x_j) = \begin{cases} \dfrac{x_j - \left(x_j^i - t_j{}^L\right)}{t_j{}^L}, & x_j^i - t_j{}^L \le x_j \le x_j^i \\ \dfrac{\left(x_j^i + t_j{}^R\right) - x_j}{t_j{}^R}, & x_j^i \le x_j \le x_j^i + t_j{}^L \\ 0, & \text{otherwise} \end{cases}$$

Also, the decision maker may have choice to shift the range of x_j for getting desired results.

5.4. Interactive fuzzy goal programming approach

Here, the fuzzy goal programming model given by Baky [3] is used to solve MLMOQFPP by constructing linear membership functions as explained in section 5.1-5.3. Thus by using the fuzzy goal programming procedure given by Baky, we can construct the fuzzy goal programming model for first level as the following:-

Find X so as to

$\text{Min } Z = W^-{}_{11}\ d^-{}_{11} + W^-{}_{12}\ d^-{}_{12} + W^-{}_{13}\ d^-{}_{13}$

and satisfy

$\mu_{f_{11}} + d^-{}_{11} - d^+{}_{11} = 1, \quad \mu_{f_{12}} + d^-{}_{12} - d^+{}_{12} = 1, \quad \mu_{f_{13}} + d^-{}_{13} - d^+{}_{13} = 1$

$A^1x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^1, A^2x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^2, A^3x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^3$

$d^-{}_{ij}\ , d^+{}_{ij} \ge 0$ with $d^-{}_{ij}\ , d^+{}_{ij} = 0,$ (2)

where $d^-{}_{ij}$ and $d^+{}_{ij}$ represents the upper and over deviational variables

and $W^-{}_{11} = \frac{1}{f_{11}^{max} - f_{11}{}^{min}}\ , W^-{}_{12} = \frac{1}{f_{12}^{max} - f_{12}{}^{min}}, W^-{}_{13} = \frac{1}{f_{13}^{max} - f_{13}{}^{min}}$

let the solution of FGP model (2) is $(x_1{}^1, x_2{}^1, \ldots, x_n{}^1)$ and the value of each objective function f_{ij} at this point is $f_{ij}{}^1$, now we will find membership functions as $\mu_{f_{ij}}$ about the point $(x_1{}^1, x_2{}^1, \ldots, x_n{}^1)$ and positive and negative tolerance limits are $t_1{}^R$ and $t_1{}^L$ respectively. Thus, the FGP model for second level can be described as the following:

Find X so as to

$$Min\ Z = \sum_{k=1}^{3} W^-{}_{1k}\ d^-{}_{1k} + \sum_{k=1}^{3} W^-{}_{2k}\ d^-{}_{2k} + W^L{}_1\ \left(d^{L-}{}_1 + d^{L+}{}_1\right) + W^R{}_1\ \left(d^{R-}{}_1 + d^{R+}{}_1\right)$$

and satisfy

$\mu_{f_{11}} + d^-{}_{11} - d^+{}_{11} = 1, \quad \mu_{f_{12}} + d^-{}_{12} - d^+{}_{12} = 1, \quad \mu_{f_{13}} + d^-{}_{13} - d^+{}_{13} = 1$

$\mu_{f_{21}} + d^-{}_{21} - d^+{}_{21} = 1, \quad \mu_{f_{22}} + d^-{}_{22} - d^+{}_{22} = 1, \quad \mu_{f_{23}} + d^-{}_{23} - d^+{}_{23} = 1$

$\dfrac{x_1 - (x_1^1 - t_1{}^L)}{t_1{}^L} + d^{L-}{}_1 - d^{L+}{}_1 = 1, \quad \dfrac{(x_1^1 + t_1{}^R) - x_1}{t_1{}^R} + d^{R-}{}_1 - d^{R+}{}_1 = 1$

$$A^1x\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}b^1 \quad , \quad A^2x\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}b^2 \quad , \quad A^3x\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}b^3 \quad , \quad d^-{}_{ij}\ , d^+{}_{ij} \geq 0 \quad \text{with} \quad d^-{}_{ij}\ , d^+{}_{ij} = 0, \tag{3}$$

where $d^-{}_{ij}$ *and* $d^+{}_{ij}$ represents the upper and over deviational variables
and $W^-{}_{ij} = \frac{1}{f_{ij}^{max} - f_{ij}}$, $W^L{}_1 = \frac{1}{t_1{}^L}$, $W^R{}_1 = \frac{1}{t_1{}^R}$

Let the solution of FGP model (3) is $(x_1{}^2, x_2{}^2, \ldots, x_n{}^2)$ and the value of each objective function f_{ij} at this point is $f_{ij}{}^2$, now we will find membership functions as $\mu_{f_{ij}}$ about $(x_1{}^2, x_2{}^2, \ldots, x_n{}^2)$ and new positive and negative tolerance limits for x_1 are $t_1{}^R$and $t_1{}^L$ respectively; also consider the positive and negative tolerance limits for $x_2$are $t_2{}^R$and $t_2{}^L$ respectively. Similarly, we can formulate the FGP model for third level whose solution will come as $(x_1{}^3, x_2{}^3, \ldots, x_n{}^3)$ and this process continues till $n-1^{\text{th}}$ level whose solution will come as $(x_1{}^{n-1}, x_2{}^{n-1}, \ldots, x_n{}^{n-1})$. With the help of this point, we can formulate the FGP model for n$^{\text{th}}$ level as the following:

Find X so as to

$$Min\, Z = \sum_{k=1}^{3} W^-{}_{1k}\ d^-{}_{1k} + \sum_{k=1}^{3} W^-{}_{2k}\ d^-{}_{2k} + \ \ldots\ldots + \sum_{k=1}^{3} W^-{}_{nk}\ d^-{}_{nk}$$
$$+ \sum_{k=1}^{n-1}\left(W^L{}_k\ (d^{L-}{}_k + d^{L+}{}_k) + W^R{}_k\ (d^{R-}{}_k + d^{R+}{}_k)\right)$$

and satisfy

$$\mu_{f_{11}} + d^-{}_{11} - d^+{}_{11} = 1, \quad \mu_{f_{12}} + d^-{}_{12} - d^+{}_{12} = 1, \quad \mu_{f_{13}} + d^-{}_{13} - d^+{}_{13} = 1$$
$$\mu_{f_{21}} + d^-{}_{21} - d^+{}_{21} = 1, \quad \mu_{f_{22}} + d^-{}_{22} - d^+{}_{22} = 1, \quad \mu_{f_{23}} + d^-{}_{23} - d^+{}_{23} = 1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\mu_{f_{n1}} + d^-{}_{n1} - d^+{}_{n1} = 1, \quad \mu_{f_{n2}} + d^-{}_{n2} - d^+{}_{n2} = 1, \quad \mu_{f_{n3}} + d^-{}_{n3} - d^+{}_{n3} = 1$$
$$\frac{x_1 - (x_1^{n-1} - t_1{}^L)}{t_1{}^L} + d^{L-}{}_1 - d^{L+}{}_1 = 1, \quad \frac{(x_1^{n-1} + t_1{}^R) - x_1}{t_1{}^R} + d^{R-}{}_1 - d^{R+}{}_1 = 1$$
$$\frac{x_2 - (x_2^{n-1} - t_2{}^L)}{t_2{}^L} + d^{L-}{}_2 - d^{L+}{}_2 = 1, \quad \frac{(x_2^{n-1} + t_2{}^R) - x_2}{t_2{}^R} + d^{R-}{}_2 - d^{R+}{}_2 = 1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\frac{x_{n-1} - (x_2^{n-1} - t_{n-1}{}^L)}{t_{n-1}{}^L} + d^{L-}{}_{n-1} - d^{L+}{}_{n-1} = 1,$$
$$\frac{(x_{n-1}^{n-1} + t_{n-1}{}^R) - x_{n-1}}{t_{n-1}{}^R} + d^{R-}{}_{n-1} - d^{R+}{}_{n-1} = 1$$
$$A^1x\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}b^1 \quad , \quad A^2x\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}b^2 \quad , \quad A^3x\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}b^3, \quad d^-{}_{ij}\ , d^+{}_{ij} \geq 0 \quad \text{with} \quad d^-{}_{ij}\ , d^+{}_{ij} = 0, \tag{4}$$

where $d^-{}_{ij}$ and $d^+{}_{ij}$ represents the upper and over deviational variables
and $W^-{}_{ij} = \frac{1}{f_{ij}^{max} - f_{ij}}$, $W^L{}_1 = \frac{1}{t_1{}^L}$, $W^R{}_1 = \frac{1}{t_1{}^R}, \ldots\ldots, W^L{}_{n-1} = \frac{1}{t_{n-1}{}^L}, W^R{}_{n-1} = \frac{1}{t_{n-1}{}^R}$
and solution of this FGP model (4) is $(x_1{}^n, x_2{}^n, \ldots\ldots, x_n{}^n)$ and the value of each objective function f_{ij} at this point is $f_{ij}{}^n$, which is the required satisfactory solution.

6. Numerical example

We are taking the following fuzzy four level quadratic fractional programming problem (FLQFPP) whose objective function of each level contains fractional function with numerator and denominator as the quadratic one:

FLQFPP

$[1^{st}$ Level$]$

$$\mathrm{Max}_{x_1} f_1 = \frac{[6]x_1{}^2 + [4]x_2x_3 + [8]}{[5]x_1x_4 + [12]}$$

$\left[2^{nd}\text{ Level}\right]$

$$\mathrm{Max}_{x_2} f_2 = \frac{[2]x_3{}^2 + [4]x_4 + [5]}{[2]x_1{}^2 + [10]x_2{}^2 + [6]}$$

$\left[3^{rd}\text{ Level}\right]$

$$\mathrm{Max}_{x_3} f_3 = \frac{[4]x_3x_2 + [6]x_3x_1 + [2]x_1 + [5]}{[2]x_4x_1 + [10]x_3 + [8]}$$

$\left[4^{th}\text{ Level}\right]$

$$\mathrm{Max}_{x_4} f_4 = \frac{[10]x_3 + [4]x_4x_1 + [2]}{[6]x_4{}^2 + [5]x_1x_2 + [12]x_3 + [2]}$$

$$[3]x_1 + [5]x_2 - [2]x_3 + [4]x_4 \le [20],$$
$$[8]x_1 - [12]x_2 + [6]x_3 - [12]x_4 \le [10], \quad x_1, x_2, x_3 \ge 0$$

Let us assume that the various fuzzy numbers used in above problem are

$[2] = (1,2,3)$, $[3] = (1,3,5)$, $[4] = (4,4,5)$, $[5] = (3,5,7)$, $[6] = (5,6,7)$,
$[8] = (7,8,9)$, $[10] = (8,10,12)$, $[12] = (11,12,13)$, $[20] = (15,20,25)$

Thus, its equivalent deterministic fourlevel three-objective quadratic fractional programming problem (FLTOQFPP) can be formed as the following:

FLTOQFPP

$[1^{st}$ Level$]$

$$\mathrm{Max}_{x_1} f_{11} = \frac{5x_1{}^2 + 4x_2x_3 + 7}{7x_1x_4 + 13}, \qquad f_{12} = \frac{6x_1{}^2 + 4x_2x_3 + 8}{5x_1x_4 + 12},$$
$$f_{13} = \frac{7x_1{}^2 + 5x_2x_3 + 9}{3x_1x_4 + 11}$$

$\left[2^{nd}\text{ Level}\right]$

$$\mathrm{Max}_{x_2} f_{21} = \frac{1x_3{}^2 + 4x_4 + 3}{3x_1{}^2 + 12x_2{}^2 + 7}, \qquad f_{22} = \frac{2x_3{}^2 + 4x_4 + 5}{2x_1{}^2 + 10x_2{}^2 + 6}, \quad f_{23} = \frac{3x_3{}^2 + 5x_4 + 7}{1x_1{}^2 + 8x_2{}^2 + 5}$$

$\left[3^{rd}\text{ Level}\right]$

$$\mathrm{Max}_{x_3} f_{31} = \frac{4x_3x_2 + 5x_3x_1 + 1x_1 + 3}{3x_4x_1 + 12x_3 + 9}, f_{32} = \frac{4x_3x_2 + 6x_3x_1 + 2x_1 + 5}{2x_4x_1 + 10x_3 + 8}, f_{33}$$
$$= \frac{5x_3x_2 + 7x_3x_1 + 3x_1 + 7}{1x_4x_1 + 8x_3 + 7}$$

$\left[4^{th}\text{ Level}\right]$

$$\mathrm{Max}_{x_4} f_{41} = \frac{8x_3 + 4x_4x_1 + 1}{7x_4{}^2 + 7x_1x_2 + 13x_3 + 1}, f_{42} = \frac{10x_3 + 4x_4x_1 + 2}{6x_4{}^2 + 5x_1x_2 + 12x_3 + 2}, f_{43}$$
$$= \frac{12x_3 + 5x_4x_1 + 3}{5x_4{}^2 + 3x_1x_2 + 11x_3 + 3}$$

$1x_1 + 3x_2 - 1x_3 + 4x_4 \le 15,\ \ 3x_1 + 5x_2 - 2x_3 + 4x_4 \le 20,$
$5x_1 + 7x_2 - 3x_3 + 5x_4 \le 25,$
$7x_1 - 11x_2 + 5x_3 - 11x_4 \le 8, 8x_1 - 12x_2 + 6x_3 - 12x_4 \le 10,$
$9x_1 - 13x_2 + 7x_3 - 13x_4 \le 12, \quad x_1, x_2, x_3 \ge 0$

Optimize solution of each level's decision maker, when taken individually, is as following:

The various results are listed in the following table:

Decision variables	f_{ij}^{max}	$\left(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}}\right)$	f_{ij}^{min}	$\left(x_1^{L_{ij}}, x_2^{L_{ij}}, x_3^{L_{ij}}\right)$
f_{11}	130.48	(0,14.63,28.88,0)	0.20	(0.83,0,13.49,6.92)
f_{12}	141.43	(0,14.63,28.88,0)	0.30	(0.76,0,13.70,6.98)
f_{13}	192.77	(0,14.63,28.88,0)	0.48	(0.66,0,14.05,7.10)
f_{21}	42.42	(0,0.03,16.2,7.79)	0.018	(0,3.72,0.35,0)
f_{22}	93.64	(0,0.03,16.2,7.79)	0.036	(0,3.69,0.29,0)
f_{23}	166.91	(0,0.03,16.2,7.79)	0.063	(0,3.69,0.27,0)
f_{31}	4.76	(0,14.63,28.88,0)	0.015	(0,0,16.2,7.8)
f_{32}	5.71	(0,14.63,28.88,0)	0.029	(0,0,16.2,7.8)
f_{33}	8.90	(0,14.63,28.88,0)	0.051	(0,0,16.2,7.8)
f_{41}	1.81	(1.7,0,0,0.36)	0.010	(0,0,0,3.75)
f_{42}	1.68	(2.08,0,0,0.60)	0.023	(0,0,0,3.75)
f_{43}	2.08	(2.92,0,01.13)	0.041	(0,0,0,3.75)

Table 1: Optimized values of all objective functions at optimized points by taking them individually in the optimization procedure

Software LINGO 15 is used to find the optimize solution of each type of optimizing problem in this numerical example.

Firstly, we take the point (0,14.63,28.88,0)at which the membership functions of each objective functions for first level are formed and thus we get the following FGP model for the first level as the following:

[1st level]

$\min = 0.0077d^-_{11} + 0.0071d^-_{12} + 0.0052d^-_{13};$

subject to

$0.00000x_1 + 0.06818x_2 + 0.03454x_3 + 0.00000x_4 + d^-_{11} - d^+_{11} = 1.99483;$

$$0.00000x_1 + 0.06821x_2 + 0.03455x_3 + 0.00000x_4 + d^-{}_{12} - d^+{}_{12} = 1.99533;$$
$$0.00000x_1 + 0.06827x_2 + 0.03458x_3 + 0.00000x_4 + d^-{}_{13} - d^+{}_{13} = 1.99700;$$
$$1x_1 + 3x_2 - 1x_3 + 4x_4 \le 15, \quad 3x_1 + 5x_2 - 2x_3 + 4x_4 \le 20,$$
$$5x_1 + 7x_2 - 3x_3 + 5x_4 \le 25,$$
$$7x_1 - 11x_2 + 5x_3 - 11x_4 \le 8, 8x_1 - 12x_2 + 6x_3 - 12x_4 \le 10,$$
$$9x_1 - 13x_2 + 7x_3 - 13x_4 \le 12, \quad x_1, x_2, x_3 \ge 0$$

Its solution comes as $x_1 = 0, \quad x_2 = 14.63, \quad x_3 = 28.88, \quad x_4 = 0$

Thus, we find new membership functions of each objective functions for first and second level decision makers by using the values of decision variables as $(x_1, x_2, x_3, x_4) = (0,14.63,28.88,0)$ and we take positive limit for first decision variable as $3.0(t_1{}^R = 3.0)$ and negative limit for first decision variable as $0.5(t_1{}^L = 0.5)$and get the FGP model for second level decision maker as the following:-

[2nd level]

$$\min = 0.0077d^-{}_{11} + 0.0071d^-{}_{12} + 0.0052d^-{}_{13} + 0.0236d^-{}_{21} + 0.0107d^-{}_{22} + 0.0060d^-{}_{23} + 2d^{L-}{}_1 + 2d^{L+}{}_1 + 0.33d^{R-}{}_1 + 0.33d^{R+}{}_1;$$

subject to

$$0.00000x_1 + 0.06818x_2 + 0.03454x_3 + 0.00000x_4 + d^-{}_{11} - d^+{}_{11} = 1.99483;$$
$$0.00000x_1 + 0.06821x_2 + 0.03455x_3 + 0.00000x_4 + d^-{}_{12} - d^+{}_{12} = 1.99533;$$
$$0.00000x_1 + 0.06827x_2 + 0.03458x_3 + 0.00000x_4 + d^-{}_{13} - d^+{}_{13} = 1.99700;$$
$$0.0000x_1 + -0.0010x_2 + 0.0005x_3 + 0.0000x_4 + d^-{}_{21} - d^+{}_{21} = 0.992746174;$$
$$0.0000x_1 + -0.0011x_2 + 0.0006x_3 + 0.0000x_4 + d^-{}_{22} - d^+{}_{22} = 0.992053678;$$
$$0.0000x_1 + -0.0012x_2 + 0.0006x_3 + 0.0000x_4 + d^-{}_{23} - d^+{}_{23} = 0.991622519;$$
$$2x_1 + d^{L-}{}_1 - d^{L+}{}_1 = 0; -0.33x_1 + d^{R-}{}_1 - d^{R+}{}_1 = 0;$$
$$1x_1 + 3x_2 - 1x_3 + 4x_4 \le 15, \quad 3x_1 + 5x_2 - 2x_3 + 4x_4 \le 20,$$
$$5x_1 + 7x_2 - 3x_3 + 5x_4 \le 25,$$
$$7x_1 - 11x_2 + 5x_3 - 11x_4 \le 8, \quad 8x_1 - 12x_2 + 6x_3 - 12x_4 \le 10,$$
$$9x_1 - 13x_2 + 7x_3 - 13x_4 \le 12, \quad x_1, x_2, x_3 \ge 0$$

Its solution comes as $x_1 = 0, \quad x_2 = 14.63, \quad x_3 = 28.88, \quad x_4 = 0$

Thus, we find new membership functions of each objective functions for first, second and third level decision makers by using the values of decision variables as $(x_1, x_2, x_3, x_4) = (0,14.63,28.88,0)$ and we take positive limits for first and second decision variable as $(t_1{}^R = 3.0, t_2{}^R = 0.5)$ and negative limits for first and second decision variable as $(t_1{}^L = 0.5, t_2{}^L = 10)$ and forms the FGP model for third level decision maker as the following:-

[3rd level]

$$\min = 0.0077d^-{}_{11} + 0.0071d^-{}_{12} + 0.0052d^-{}_{13} + 0.0236d^-{}_{21} + 0.0107d^-{}_{22} + 0.0060d^-{}_{23} + 0.2107\,d^-{}_{31} + 0.1760\,d^-{}_{32} + 0.1130d^-{}_{33} + 2d^{L-}{}_1 + 2d^{L+}{}_1 + 0.33d^{R-}{}_1 + 0.33d^{R+}{}_1 + 0.1d^{L-}{}_2 + 0.1d^{L+}{}_2 + 2d^{R-}{}_2 + 2d^{R+}{}_2 + 0.1d^{L-}{}_3;$$

subject to

$$0.00000x_1 + 0.06818x_2 + 0.03454x_3 + 0.00000x_4 + d^-{}_{11} - d^+{}_{11} = 1.99483;$$
$$0.00000x_1 + 0.06821x_2 + 0.03455x_3 + 0.00000x_4 + d^-{}_{12} - d^+{}_{12} = 1.99533;$$
$$0.00000x_1 + 0.06827x_2 + 0.03458x_3 + 0.00000x_4 + d^-{}_{13} - d^+{}_{13} = 1.99700;$$
$$0.0000x_1 + -0.0010x_2 + 0.0005x_3 + 0.0000x_4 + d^-{}_{21} - d^+{}_{21} = 0.99274;$$
$$0.0000x_1 + -0.0011x_2 + 0.0006x_3 + 0.0000x_4 + d^-{}_{22} - d^+{}_{22} = 0.99205;$$

$$0.0000x_1 + -0.0012x_2 + 0.0006x_3 + 0.0000x_4 + d^-{}_{23} - d^+{}_{23} = 0.99162;$$
$$0.0862x_1 + 0.0685x_2 + 0.0008x_3 + 0.0000x_4 + d^-{}_{31} - d^+{}_{31} = 1.02500;$$
$$0.1040x_1 + 0.0685x_2 + 0.0008x_3 + 0.0000x_4 + d^-{}_{32} - d^+{}_{32} = 1.02627;$$
$$0.0974x_1 + 0.0686x_2 + 0.0009x_3 + 0.0000x_4 + d^-{}_{33} - d^+{}_{33} = 1.02870;$$
$$2x_1 + d^{L-}{}_1 - d^{L+}{}_1 = 0; -0.33x_1 + d^{R-}{}_1 - d^{R+}{}_1 = 0;$$
$$0.1x_2 + d^{L-}{}_2 - d^{L+}{}_2 = 1.463; -2x_2 + d^{R-}{}_2 - d^{R+}{}_2 = -27.26;$$
$1x_1 + 3x_2 - 1x_3 + 4x_4 \le 15, \quad 3x_1 + 5x_2 - 2x_3 + 4x_4 \le 20,$
$5x_1 + 7x_2 - 3x_3 + 5x_4 \le 25,$
$7x_1 - 11x_2 + 5x_3 - 11x_4 \le 8, \quad 8x_1 - 12x_2 + 6x_3 - 12x_4 \le 10,$
$9x_1 - 13x_2 + 7x_3 - 13x_4 \le 12, \quad x_1, x_2, x_3 \ge 0$
Its solution comes as $x_1 = 0, \quad x_2 = 13.63, \quad x_3 = 28.01, \quad x_4 = 0.53,$

Thus, we find new membership functions of each objective functions for first, second, third and fourth level decision makers by using the values of decision variables as $(x_1, x_2, x_3, x_4) = (0,13.63,28.01,0.53)$ and we take positive limits for first, second and third decision variable as $(t_1{}^R = 3.0, t_2{}^R = 0.5, t_3{}^R = 0.5)$ and negative limits for first, second and third decision variable as $(t_1{}^L = 0.5, t_2{}^L = 10, t_3{}^L = 10)$ and forms the FGP model for fourth level decision maker as the following:

[4th level]

$$\begin{aligned}\min = {} & 0.0077d^-{}_{11} + 0.0071d^-{}_{12} + 0.0052d^-{}_{13} + 0.0236d^-{}_{21} + 0.0107d^-{}_{22} \\ & + 0.0060d^-{}_{23} + 0.2107\, d^-{}_{31} + 0.1760\, d^-{}_{32} + 0.1130d^-{}_{33} \\ & + 0.5556d^-{}_{41} + 0.6035d^-{}_{42} + 0.4904d^-{}_{43} + 2d^{L-}{}_1 + 2d^{L+}{}_1 \\ & + 0.33d^{R-}{}_1 + 0.33d^{R+}{}_1 + 0.1d^{L-}{}_2 + 0.1d^{L+}{}_2 + 2d^{R-}{}_2 + 2d^{R+}{}_2 \\ & + 0.1d^{L-}{}_3 + 0.1d^{L+}{}_3 + 2d^{R-}{}_3 + 2d^{R+}{}_3;\end{aligned}$$

subject to

$$-0.2584x_1 + 0.0661x_2 + 0.0322x_3 + 0.0000x_4 + d^-{}_{11} - d^+{}_{11} = 1.89865;$$
$$-0.2002x_1 + 0.0662x_2 + 0.0322x_3 + 0.0000x_4 + d^-{}_{12} - d^+{}_{12} = 1.89911;$$
$$-0.1311x_1 + 0.0662x_2 + 0.0322x_3 + 0.0000x_4 + d^-{}_{13} - d^+{}_{13} = 1.90070;$$
$$0.0000x_1 - 0.0012x_2 + 0.0006x_3 + 0.0000x_4 + d^-{}_{21} - d^+{}_{21} = 0.99206;$$
$$0.0000x_1 - 0.0013x_2 + 0.0006x_3 + 0.0000x_4 + d^-{}_{22} - d^+{}_{22} = 0.99133;$$
$$0.0000x_1 - 0.0014x_2 + 0.0007x_3 + 0.0000x_4 + d^-{}_{23} - d^+{}_{23} = 0.99087;$$
$$0.0818x_1 + 0.0684x_2 + 0.0008x_3 + 0.0000x_4 + d^-{}_{31} - d^+{}_{31} = 1.02386;$$
$$0.1005x_1 + 0.0685x_2 + 0.0008x_3 + 0.0000x_4 + d^-{}_{32} - d^+{}_{32} = 1.02498;$$
$$0.0952x_1 + 0.0685x_2 + 0.0009x_3 + 0.0000x_4 + d^-{}_{33} - d^+{}_{33} = 1.02729;$$
$$-0.0853x_1 + 0.0000x_2 + 0.0000x_3 - 0.0069x_4 + d^-{}_{41} - d^+{}_{41} = 0.66251;$$
$$-0.0968x_1 + 0.0000x_2 + 0.0001x_3 - 0.0094x_4 + d^-{}_{42} - d^+{}_{42} = 0.50977;$$
$$-0.0655x_1 + 0.0000x_2 + 0.0001x_3 - 0.0090x_4 + d^-{}_{43} - d^+{}_{43} = 0.48592;$$
$$2x_1 + d^{L-}{}_1 - d^{L+}{}_1 = 0; -0.33x_1 + d^{R-}{}_1 - d^{R+}{}_1 = 0; 0.1x_2 + d^{L-}{}_2 - d^{L+}{}_2 = 1.463;$$
$$-2x_2 + d^{R-}{}_2 - d^{R+}{}_2 = -27.26; 0.1x_3 + d^{L-}{}_3 - d^{L+}{}_3 = 2.801; -2x_3 + d^{R-}{}_3 - d^{R+}{}_3 = -56;$$
$1x_1 + 3x_2 - 1x_3 + 4x_4 \le 15, \quad 3x_1 + 5x_2 - 2x_3 + 4x_4 \le 20,$
$5x_1 + 7x_2 - 3x_3 + 5x_4 \le 25,$
$7x_1 - 11x_2 + 5x_3 - 11x_4 \le 8, \quad 8x_1 - 12x_2 + 6x_3 - 12x_4 \le 10,$
$9x_1 - 13x_2 + 7x_3 - 13x_4 \le 12, \quad x_1, x_2, x_3 \ge 0$

Its solution comes as $x_1 = 0, \quad x_2 = 13.63, \quad x_3 = 28, \ x_4 = 0.52$
Thus, we get the satisfactory solution: $x_1 = 0, \quad x_2 = 13.63, \quad x_3 = 28, \quad x_4 = 0.52,$

7. Conclusion

In this paper, we solve the fuzzy multilevel quadratic fractional programming problem by using interactive fuzzy goal programming procedure and this study can be extended to solve nonlinear multilevel and nonlinear multiobjective programming problems and it is wished that the approach presented in this paper can contribute to future study of hierarchical optimization problems.

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