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Critical Path with Various Distances of Type-2 Fuzzy Numbers

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Abstract. The critical path method (CPM) is a vital tool for planning and controlling complex projects. The successful implementation of CPM requires the availability of clear determined time duration for each activity. In practical situations this requirement is usually hard to fulfill, since many of these activities are uncertain, which leads to the development of fuzzy critical path method. In this paper, we propose a new approach to find critical path and its path length using some various distances of trapezoidal type-2 fuzzy numbers. An example is included to demonstrate our proposed approach.

Keywords: Fuzzy critical path, trapezoidal type-2 fuzzy numbers, acyclic project network, metric distance, Hamming distance, normalized Hamming distance and Euclidean distance

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Network diagram plays a vital role in determining project completion time. Normally a project will consist of a number of activities. Some activities are independent and some may depend on other activities. Network analysis is a technique which determines the various sequences of activities concerning a project and its completion time. Decision making problems can be solved using the network techniques called critical path method. Since the activities in the network can be carried out in parallel, the minimum time to complete the project is the length of the longest path from the start of project to finish. The longest path is the critical path of the network. However, the vagueness of time parameters in the problem has led to the development of fuzzy CPM. The unknown problem that could occur in practical situation can be very well managed using this fuzzy CPM.

In this paper, we analyze the critical path in a general project network with fuzzy activity times. We propose distances like metric distance, Hamming distance, Normalized Hamming distance and Euclidean distance for fuzzy numbers to a critical path method for fuzzy project network, where the duration time of each activity in a fuzzy project network is represented by a trapezoidal type-2 fuzzy numbers.

The organization of the paper is as follows: In section 2, we have some basic concepts, section 3, gives some properties of total slack fuzzy time, Section 4, gives the network terminology. Section 5, gives an algorithm to find the critical path and its length combined with trapezoidal type-2 fuzzy number using metric distance ranking method. To illustrate the proposed algorithm a numerical example is solved in section 6.

2. Basic concepts

2.1. Type-2 fuzzy number

Let X be a type-2 fuzzy set defined in the universe of discourse R, if the following conditions are satisfied, then X is called a type-2 fuzzy number.

1. X is normal.
2. X is a convex set.
3. The support of X is closed and bounded.

2.2. Normal type-2 fuzzy number

A type-2 fuzzy number(T2fs), X is said to be normal if its Foot of Uncertainty (FOU) is normal interval type-2 fuzzy number (IT2FS) and it has a primary membership function.

2.3. Fuzzy critical path

In a project network a path p_c such that

$F(p_c)=\min\{F(p_n)/p_n \in P, n= 1 \text{ to } m\}$ is defined as a fuzzy critical path.

2.4. Fuzzy critical path length

The sum of the Fuzzy activity time of the corresponding path P_c is said to be the fuzzy critical path length.

2.5. Metric distance ranking

Various distance formulae between X and Y are listed below.

(i) Metric distance ranking

$$D(X,Y) = \left[\int_0^1 (g_X^L(y))^2 dy + \int_0^1 (g_X^R(y))^2 dy \right]^{\frac{1}{2}} \text{ where } Y = 0$$

The larger the value of $D(\tilde{A}, \tilde{B})$ is the better the ranking of $\tilde{A}$.

(ii) Hamming distance

$$d_H(X,Y) = \sum_{i=1}^{n} \left| X(x_i) - Y(x_i) \right|$$

(iii) Normalized Hamming distance

$$d_{nH}(X,Y) = \frac{1}{n} \sum_{i=1}^{n} \left| X(x_i) - Y(x_i) \right|$$

(iv) Euclidean distance

$$d_E(X,Y)=\sqrt{\sum_{i=1}^{n}\left(X(x_i)-Y(x_i)\right)^2}$$

2.6. Notations

t_{ij}= The activity between node i and j.

ESF_j =The earliest starting fuzzy time of node j.

LFF_i = The latest finishing fuzzy time of node i.

TSF_{ij} = The total slack fuzzy time of t_{ij}.

p_n= the n^{th} fuzzy path.

P = the set of all fuzzy paths in a project network

$F(p_n)$= The total slack fuzzy time of path p_n in a project network.

3. Properties of total slack fuzzy time

Property 3.1. (Forward pass calculation)

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie) $ESF_1 = (0.0,0.0,0.0,0.0)$

$ESF_j = \max_i\{ESF_j + TSF_{ij}\}, j \neq i, j \in N,$ i=number of preceding nodes.(ESF_j =The earliest starting fuzzy time of node j).

Ranking value is utilized to identify the maximum value. Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.

Property 3.2. (Backward pass calculation)

To calculate the latest finishing time in the project network set $LFF_n = ESFn$. $LFF_j = \min_j\{LFF_j(-)SET_{ij}\}, i \neq n, i \in N,$ j = number of succeeding nodes. Ranking value is utilized to identify the minimum value. Latest starting fuzzy time= Latest finishing

Fuzzy time (-) Fuzzy activity time.

Property 3.3.

For the activity t_{ij}, i<j

Total fuzzy slack:

$SFT_{ij} = LFF_j(-)(ESF_i(+)SFT_{ij})$ (or) $(LFF_j(-)SFT_{ij})(-)ESF_i$, $1 \leq i \leq j \leq n$; $i, j \in N,$

Property 3.4.

$F(p_n)=\sum_{\substack{1\le i\le j\le n\\ i,j\in p_k}} SFT_{ij}$, $p_k \in P,$ p_n is the possible paths in a network from source node to the destination node, k=1 to m.

4. Network terminology

Consider a directed acyclic project network G(V,E) consisting of a finite set of nodes V={1,2,…..n} and a set of m directed edges $E \subseteq V \times V$. Each edge is denoted by an ordered pair (i,j) where $i.j \in V$ and $i.\neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path p_{ij} as a sequence p_{ij}=[i=i_1,(i_1,i_2),i_2,….i_{i-1},i_l=j] of alternating nodes and edges. The existence of at least one path p_{si} in G(V,E) is assumed for every node $i \in V - \{S\}$. $\tilde{d}_{ij}$ denotes a trapezoidal type–2 fuzzy number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy length along the path P is denoted by $\tilde{d}(P)$ and is defined as $\tilde{d}(P) = \sum_{(i,j \in p)} \tilde{d}_{ij}$

5. Algorithm (for finding critical path)

Step 1: Estimate the fuzzy activity time with respect to each activity.

Step 2: Let $ESF_1 = (0.0,0.0,0.0,0.0)$ and calculate ESF_j , j =2,3,…. n by using property 1.

Step 3: Let $LFF_n = ESFn$ and calculate LFF_i ,i=n-1, n-2,….. 2,1. By using property 2.

Step 4: Calculate SFT_{ij} with respect to each activity in a project network by using property 3.

Step 5: Find all the possible paths and calculate F(p_n) by using property 4.

Step 6: Identify the critical path by using definition 2.7.

6. Numerical example

The problem is to find the critical path and critical path length between the source node to the destination node in the acyclic fuzzy project network having 6 vertices and 7 edges with trapezoidal type-2 fuzzy number.

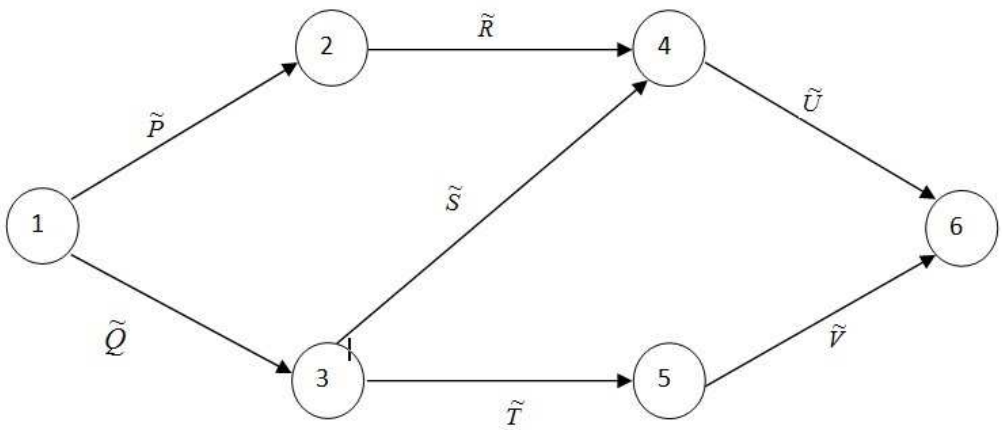

Figure 6.1: A network

The edge lengths are

$$\tilde{P} = ((2.5,2.8,3.0,3.5),(2.2,2.5,3.0,3.2),(2.1,2.3,2.5,3.0),(2.0,2.3,2.5,3.0))$$

$$\tilde{Q} = ((2.6,2.7,2.8,3.0),(2.5,2.6,3.0,3.5),(2.3,2.5,3.0,3.3),(2.2,2.4,2.5,2.8))$$

$\tilde{R} = ((2.8,2.9,3.2,3.6),(2.6,2.9,3.2,3.6),(2.5,2.7,3.1,3.3),(2.2,2.8,3.3,3.6))$

$\tilde{S} = ((2.3,2.5,.2.9,3.2),(2.3,2.4,3.3,3.4,),(2.2,2.6,3.3,3.8),(2.0,2.5,2.9,3.3))$

$\tilde{T} = ((2.8,2.9,3.3,3.5),(2.6,2.8,3.3,3.4,),(2.5,2.6,2.9,3.3),(2.3,2.6,2.9,3.5))$

$\tilde{U} = ((2.5,2.7,3.0,3.5),(2.3,2.6,3.2,3.5),(2.2,2.6,3.3,3.6),(2.1,2.5,2.8,3.0))$

$\tilde{V} = ((2.9,3.0,3.3,3.4),(2.8,3.1,3.3,3.5),(2.5,2.7,3.0,3.3),(2.4,2.7,3.0,3.5))$

Using the properties of total slack fuzzy time, the fuzzy durations and total slack fuzzy time for each activities are calculated and tabulated in table 6.1. We calculate the fuzzy durations and total slack fuzzy time for each activity as shown in table 6.2.

Table 6.1: Activities, fuzzy durations and total slack fuzzy time for each activity

Activity (i-j) $i<j$	Duration TSF_{ij}	ESF_i	LFF_j	SET_{ij}
1-2	((2.5,2.8,3.0,3.5), (2.2,2.5,3.0,3.2), (2.1,2.3,2.5,3.0), (2.0,2.3,2.5,3.0))	(0.0,0.0,0.0,0.0)	((1.2,2.4,3.8,4.6), (0.8,2.1,4.1,5.5), (0.4,1.4,3.6,5.2), (0.3,1.6,3.1,5.5))	((-2.3,-0.6,1.0,2.1), (-2.4,-0.9,1.6,3.3), (-2.6,-1.1,1.3,3.1), (-2.7,-0.9,0.8,3.5))
1-3	((2.6,2.7,2.8,3.0), (2.5,2.6,3.0,3.5), (2.3,2.5,3.0,3.3), (2.2,2.4,2.5,2.8))	(0.0,0.0,0.0,0.0)	((1.4,2.0,3.5,4.2), (1.0,1.9,3.7,5.0), (0.7,1.9,3.6,4.9), (-0.1,1.8,3.1,5.1))	((-1.6,-0.8,0.8,1.6), (-2.5,-1.1,1.1,2.5), (-2.6,-1.1,1.1,2.6), (-2.9,-0.7,0.7,2.9))
2-4	((2.8,2.9,3.2,3.6), (2.6,2.9,3.2,3.6), (2.5,2.7,3.1,3.3), (2.2,2.8,3.3,3.6))	((2.5,2.8,3.0,3.5), (2.2,2.5,3.0,3.2), (2.1,2.3,2.5,3.0), (2.0,2.3,2.5,3.0))	((4.8,5.6,6.7,7.4), (4.4,5.3,7.0,8.1), (3.7,4.5,6.3,7.7), (3.9,4.9,5.9,7.7))	((-2.3,-0.6,1.0,2.1), (-2.4,-0.9,1.6,3.3), (-2.6,-1.1,1.3,3.1), (-2.7,-0.9,0.8,3.5))
3-4	((2.3,2.5,2.9,3.2), (2.3,2.4,3.3,3.4), (2.2,2.6,3.3,3.8), (2.0,2.5,2.9,3.3))	((2.6,2.7,2.8,3.0), (2.5,2.6,3.0,3.5), (2.3,2.5,3.0,3.3), (2.2,2.4,2.5,2.8))	((4.8,5.6,6.7,7.4), (4.4,5.3,7.0,8.1), (3.7,4.5,6.3,7.7), (3.9,4.9,5.9,7.7))	((-1.4,-0.1,1.5,2.5), (-2.5,-1.0,2.0,3.3), (-3.4,-1.8,1.2,3.2), (-2.2,-0.5,1.0,3.5))
3-5	((2.8,2.9,3.3,3.5), (2.6,2.8,3.3,3.4), (2.5,2.6,2.9,3.3), (2.3,2.6,2.9,3.5))	((2.6,2.7,2.8,3.0), (2.5,2.6,3.0,3.5), (2.3,2.5,3.0,3.3), (2.2,2.4,2.5,2.8))	((4.9,5.3,6.4,7.0), (4.4,5.2,6.5,7.6), (4.0,4.8,6.2,7.4), (3.4,4.7,5.7,7.4))	((-1.6,-0.8,0.8,1.6), (-2.5,-1.1,1.1,2.5), (-2.6,-1.1,1.1,2.6), (-2.9,-0.7,0.7,2.9))
4-6	((2.5,2.7,3.0,3.5), (2.3,2.6,3.2,3.5), (2.2,2.6,3.3,3.6), (2.1,2.5,2.8,3.0))	((5.3,5.7,6.2,7.1), (4.8,5.4,6.2,6.8), (4.6,5.0,5.6,6.3), (4.2,5.1,5.8,6.6))	((8.3,8.6,9.4,9.9), (7.9,8.5,9.6,10.4), (7.3,7.8,8.9,9.9), (6.9,7.7,8.4,9.8))	((-2.3,-0.6,1.0,2.1), (-2.4,-0.9,1.6,3.3), (-2.6,-1.1,1.3,3.1), (-2.7,-0.9,0.8,3.5))
5-6	((2.9,3.0,3.3,3.4), (2.8,3.1,3.3,3.5), (2.5,2.7,3.0,3.3), (2.4,2.7,3.0,3.5))	((5.4,5.6,6.1,6.5), (5.1,5.4,6.3,6.9), (4.8,5.1,5.9,6.6), (4.5,5.0,5.4,6.3))	((8.3,8.6,9.4,9.9), (7.9,8.5,9.6,10.4), (7.3,7.8,8.9,9.9), (6.9,7.7,8.4,9.8))	((-1.6,-0.8,0.8,1.6), (-2.5,-1.1,1.1,2.5), (-2.6,-1.1,1.1,2.6), (-2.9,-0.7,0.7,2.9))

Find all the possible paths and calculate $F(p_n)$ by using property 3.4. The possible paths are (1-2-4-6), (1-3-4-6), (1-3-5-6), which are denoted by p_1, p_2, p_3.

For path p_1,
$F(p_1)$=((-6.9,-1.8,3.0,6.3),(-7.2,-2.7,4.8,9.9),(-7.8,-3.3,3.9,9.3),(-8.1,-2.7,2.4,10.5))

For path p_2,
$F(p_2)$=((-5.3,-1.5,3.3,6.2),(-7.4,-3.0,4.7,9.1),(-8.6,-4.0,3.6,8.9), (-7.8,-2.1,2.5,9.9))

For path p_3,
$F(p_3)$=((-4.8,-2.4,2.4,4.8),(-7.5,-3.3,3.3,7.5),(-7.8,-3.1,3.1,7.8),(-8.7,-2.1,2.1,8.7))

The possible paths p_1, p_2, p_3 for the given fuzzy project network are calculated using metric distance, Hamming distance, Normalized Hamming distance and Euclidean distances. They are tabulated below:

Table 2:

Path	Metric Distance	Hamming Distance	Normalized Hamming Distance	Euclidean Distance	Ranking
p_1:1-2-4-6	1.4032	90.6	5.6625	25.34	3
p_2:1-3-4-6	1.5380	87.9	5.4938	24.49	2
p_3:1-3-5-6	1.5718	79.4	4.9628	22.2031	1

The path having maximum rank is decided as a critical path. Hence the path p_3 :1-3-5-6 is a required path for the given project network.

6. Conclusion

Fuzzy critical path and fuzzy critical path length are significant for the project managers to take decision in planning and scheduling the complex projects. In this work, an attempt is made to find fuzzy critical path and its length in a acyclic project network using various distances with trapezoidal type-2 fuzzy numbers.

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