Intern. J. Fuzzy Mathematical Archive
Vol. 12, No. 2, 2017, 67-73
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 30 June 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v12n2a3

International Journal of
Fuzzy Mathematical Archive

Computing Certain Degree Based Topological Indices and Coindices of E-graphs

Keerthi G. Mirajkar[1] **and** ***Pooja B***[2]

Department of Mathematics, Karnatak University's Karnatak Arts College
Dharwad-580 001, Karnataka, India.
[1]Email: keerthi.mirajkar@gmail.com,
Corresponding author. Email: bkvpooja@gmail.com

Received 20 June 2017; accepted 30 June 2017

Abstract. In this paper, we obtain the explicit formulae for general sum-connectivity index, general product-connectivity index, general Zagreb index and coindices of G-networks, extended G-networks and G_g-networks.

Keywords: degree, G-networks, extended G-networks and G_g-networks.

AMS Mathematics Subject Classification (2010): 05C76, 05C07

1. Introduction

Let G be a graph with vertex set $V(G)$, $|V(G)| = n$, and edge set $E(G)$, $|E(G)| = m$. As usual, n is order and m is size of G. If u and v are two adjacent vertices of G, then the edge connecting them will be denoted by uv. The degree of a vertex $w \in V(G)$ is the number of vertices adjacent to w and is denoted by $d_G(w)$. Any unexplained graph theoretical terminology and notation may be found in [6] or [8].

The first and second Zagreb indices, respectively, defined

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

$$\text{and } M_2(G) = \textstyle\sum_{uv \in E(G)} d_G(u) d_G(v)$$

are widely studied degree-based topological indices, that were introduced by Gutman and Trinajstic$'$ [5] in 1972.

Noticing that contribution of non adjacent vertex pairs should be taken into account when computing the weighted Wiener polynomials of certain composite graphs (see [3]) Ashrafi et al. [1], defined the first Zagreb coindex and second Zagreb coindex as $\overline{M}_1(G) = \sum_{uv \notin E(G)} [d_G(u) + d_G(v)]$ and $\overline{M}_2(G) = \sum_{uv \notin E(G)} d_G(u) d_G(v)$, respectively.

The vertex-degree-based graph invariant

$$F(G) = \textstyle\sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]$$

was encountered in [5]. Recently there has been some interest to F, called forgotten topological index or F-index [4].

Shirdel et al.[11] introduced a new Zagreb index of a graph G named hyper-Zagreb indexand is defined as:

$$HM(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))^2.$$

Recently, Veylaki et al.[12] defined the hyper-Zagreb coindex as

$$\overline{HM}(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))^2.$$

Li and Zhao [9] introduced the first general Zagreb index as follows

$$M_1^{\alpha}(G) = \sum_{u\in V(G)} [d_G(u)]^{\alpha}.$$

It is easy to write that

$$M_1^{\alpha}(G) = \sum_{uv\in E(G)} [(d_G(u))^{\alpha-1} + (d_G(v))^{\alpha-1}].$$

The general sum connectivity index [15] was introduced by Zhou et al. and is defined as

$$\chi_{\alpha}(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)]^{\alpha}.$$

The general product connectivity index [2] is defined as

$$R_{\alpha}(G) = \sum_{uv\in E(G)} [d_G(u)d_G(v)]^{\alpha}.$$

Su et al.[13] introduced the general sum-connectivity coindex as

$$\overline{\chi}_{\alpha}(G) = \sum_{uv\notin E(G)} [d_G(u) + d_G(v)]^{\alpha}.$$

The general product connectivity coindex is defined as

$$\overline{R}_{\alpha}(G) = \sum_{uv\notin E(G)} [d_G(u)d_G(v)]^{\alpha}.$$

Here we note that, $\chi_1(G) = M_1(G)$, $\overline{\chi}_1(G) = \overline{M}_1(G)$, $\chi_2(G) = HM(G)$, $\overline{\chi}_2(G) = \overline{HM}(G)$, $R_1(G) = M_2(G)$, $\overline{R}_1(G) = \overline{M}_2(G)$, $M_1^2(G) = M_1(G)$, $M_1^3(G) = F(G)$.

2.E-graphs

Let G and H be two graphs. Designate two nodes x_1 and x_2, $x_1 \neq x_2$ in H as e-nodes such that there is an automorphism $\sigma: V(H) \to V(H)$ with the property $\sigma(x_1) = x_2$; $\sigma(x_2) = x_1$. Then the symmetric edge replacement of G by H written as $G|H$, is the E-graph got by replacing every edge uv of G with a copy of H identifying u and v with x_1 and x_2 respectively [7].

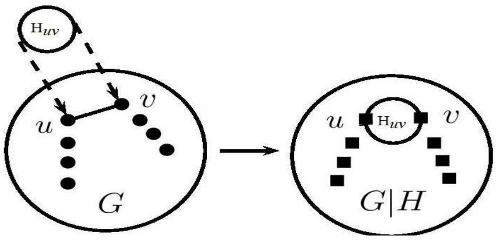

Figure1: E-graph $G|H$.

The E-graph $K_n|C_4$ where the e-nodes are two nonadjacent vertices of C_4, is called as the G-network [7].

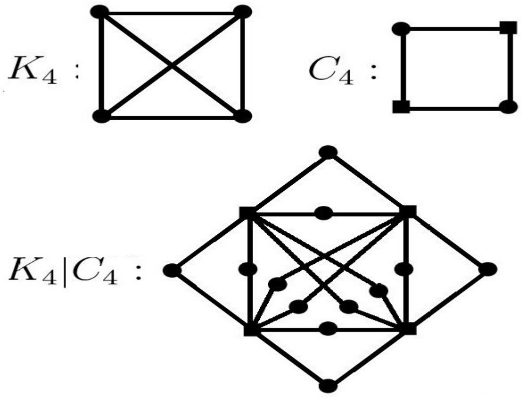

Figure 2: G–network $K_4|C_4$.

The E-graph $K_n|W_5$ where two nonadjacent vertices of degree three in H' are the e-nodes is called as the extended G-network [14].

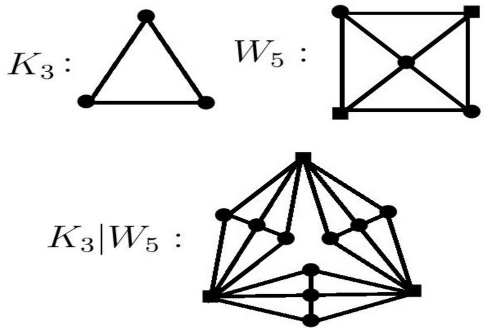

Figure 3: Extended G–network $K_3|W_5$.

The E-graph $G|C_4$ where the e-nodes are two nonadjacent vertices of C_4, is called as the G_g-network [10].The architecture of majority of computer networks is based on hypercubes Q_n. So we consider here G_g-network $Q_n|C_4$.

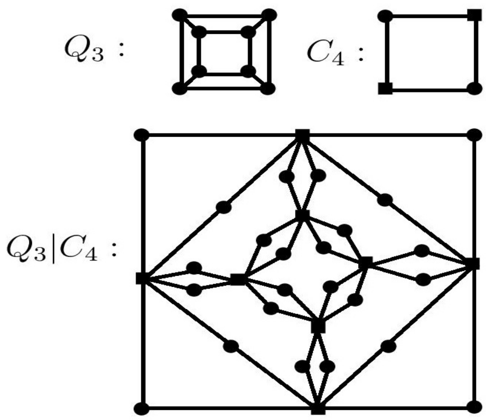

Figure 4: G_g–network $Q_3|C_4$.

3.Topological indices of G-networks, extended G-networks and G_g-network

Theorem 3.1. $(i)\ M_1^{\alpha}(K_n|C_4) = n(n-1)^{\alpha}2^{\alpha} + n(n-1)2^{\alpha}$.
$(ii) \chi_{\alpha}(K_n|C_4) = (n-1)(2n)^{\alpha+1}$.
$(iii) R_{\alpha}(K_n|C_4) = n(n-1)^{\alpha+1}2^{2\alpha+1}$.

Proof: The $G-$network $K_n|C_4$ contains n^2 vertices, out of which $n(n-1)$ vertices are of degree 2 and remaining n vertices of degree $2(n-1)$. So the $M_1^{\alpha}-$value of $K_n|C_4$ is equal to $n(n-1)^{\alpha}2^{\alpha} + n(n-1)2^{\alpha}$. This completes the proof of (i).

The $G-$network $K_n|C_4$ contains $2n(n-1)$ edges whose end vertices have degree 2 and $2(n-1)$. Hence, χ_{α} and $R_{\alpha}-$ values of $K_n|C_4$ are $(n-1)(2n)^{\alpha+1}$ and $n(n-1)^{\alpha+1}2^{2\alpha+1}$ respectively. This completes the proof of (ii) and (iii). □

By putting $\alpha = 2,3$ in Theorem 3.1 (i), $\alpha = 2$ in Theorem 3.1 (ii) and $\alpha = 1$ in Theorem 3.1 (iii), we get the following corollary.

Corollary 3.2. $(i)\ M_1(K_n|C_4) = 4n^2(n-1)$.
$(ii)\ F(K_n|C_4) = 8n(n-1)(n^2-2n+2)$.
$(iii)\ HM(K_n|C_4) = 8n^3(n-1)$.
$(iv) M_2(K_n|C_4) = 8n(n-1)^2$.

Theorem 3.3. $(i)\ \overline{\chi}_{\alpha}(K_n|C_4) = \binom{n(n-1)}{2}4^{\alpha} + \binom{n}{2}4^{\alpha}(n-1)^{\alpha} + \frac{2n^3-7n^2+4n}{2}(2n)^{\alpha}$.

$(ii)\ \overline{R}_{\alpha}(K_n|C_4) = \binom{n(n-1)}{2}4^{\alpha} + \binom{n}{2}4^{\alpha}(n-1)^{2\alpha} + \frac{2n^3-7n^2+4n}{2}(4(n-1))^{\alpha}$.

Proof: The $G-$network $K_n|C_4$ contains $\binom{n^2}{2} - 2n(n-1)$ non adjacent pairs of vertices, out of which $\binom{n(n-1)}{2}$ pairs of vertices of degree 2 and 2, $\binom{n}{2}$ pairs of vertices of degree $2(n-1)$ and $2(n-1)$ and remaining $\frac{2n^3-7n^2+4n}{2}$ pairs of vertices of degree 2 and $2(n-1)$.

Hence, $\overline{\chi}_{\alpha}(K_n|C_4) = \binom{n(n-1)}{2}4^{\alpha} + \binom{n}{2}4^{\alpha}(n-1)^{\alpha} + \frac{2n^3-7n^2+4n}{2}(2n)^{\alpha}$ and $\overline{R}_{\alpha}(K_n|C_4) = \binom{n(n-1)}{2}4^{\alpha} + \binom{n}{2}4^{\alpha}(n-1)^{2\alpha} + \frac{2n^3-7n^2+4n}{2}(4(n-1))^{\alpha}$. □

By putting $\alpha = 1,2$ in Theorem 3.3 (i) and $\alpha = 1$ in Theorem 3.3 (ii), we get the following corollary.

Corollary 3.4. *(i)* $\overline{M_1}(K_n|C_4) = \binom{n(n-1)}{2}4 + \binom{n}{2}4(n-1) + (2n^3-7n^2+4n)n$.
(ii) $\overline{HM}(K_n|C_4) = \binom{n(n-1)}{2}16 + \binom{n}{2}16(n-1)^2 + (2n^3-7n^2+4n)2n^2$.
(iii) $\overline{M}_2(K_n|C_4) = \binom{n(n-1)}{2}4 + \binom{n}{2}4(n-1)^2 + 2(n-1)(2n^3-7n^2+4n)$.

Theorem 3.5. (i) $M_1^{\alpha}(K_n|W_5) = 3^{\alpha}(n-1)^{\alpha}n + 3^{\alpha}n(n-1) + 2^{2\alpha-1}n(n-1)$.
(ii) $\chi_{\alpha}(K_n|W_5) = 2n(n-1)(3n)^{\alpha} + (3n+1)^{\alpha}n(n-1) + 7^{\alpha}n(n-1)$.
(iii) $R_{\alpha}(K_n|W_5) = 2n9^{\alpha}(n-1)^{\alpha+1} + 12^{\alpha}n(n-1)^{\alpha+1} + n(n-1)12^{\alpha}$.

Proof: The extended $G-$network $K_n|W_5$ contains $\frac{n(3n-1)}{2}$ vertices, out of which n vertices are of degree $3(n-1)$, $n(n-1)$ vertices of degree 3 and the remaining $\frac{n(n-1)}{2}$ vertices of degree 4. So the M_1^{α} $-$value of $K_n|W_5$ is equal to $3^{\alpha}(n-1)^{\alpha}n + 3^{\alpha}n(n-1) + 2^{2\alpha-1}n(n-1)$. This completes the proof of (i).

The extended $G-$network $K_n|W_5$ contains $4n(n-1)$ adjacent pair vertices, out of which $2n(n-1)$ pair of vertices of degree 3 and $3(n-1)$, $n(n-1)$ pair of vertices of degree 4 and $3(n-1)$, and $n(n-1)$ pair of vertices of degree 3 and 4. Hence, χ_{α} and $R_{\alpha}-$ values of $K_n|W_5$ are $2n(n-1)(3n)^{\alpha} + (3n+1)^{\alpha}n(n-1) + 7^{\alpha}n(n-1)$ and $2n9^{\alpha}(n-1)^{\alpha+1} + 12^{\alpha}n(n-1)^{\alpha+1} + n(n-1)12^{\alpha}$ respectively. This completes the proof of (ii) and (iii). □

By putting $\alpha = 2{,}3$ in Theorem 3.5 (i), $\alpha = 2$ in Theorem 3.5 (ii) and $\alpha = 1$ in Theorem 3.5 (iii), we get the following corollary.

Corollary 3.6. *(i)* $M_1(K_n|W_5) = 9n(n-1)^2 + 17n(n-1)$.
(ii)$F(K_n|W_5) = 27n(n-1)^3 + 59n(n-1)$.
(iii)$HM(K_n|W_5) = 18n^3(n-1) + n(n-1)(3n+1)^2 + 49n(n-1)$.
(iv)$M_2(K_n|W_5) = 30n(n-1)^2 + 12n(n-1)$.

Theorem 3.7.

$$(i)\ \overline{\chi}_{\alpha}(K_n|W_5) = \frac{n(n-1)}{2}6^{\alpha}(n-1)^{\alpha} + \frac{n^2(n-1)^2 - n(n-1)}{2}6^{\alpha} + \frac{n^2(n-1)^2 - 2n(n-1)}{8}8^{\alpha} + n(n-1)(n-2)(3n)^{\alpha} + \frac{n(n-1)(n-2)}{2}(3n+1)^{\alpha} + \frac{n(n-1)(n(n-1)-2)}{2}7^{\alpha}.$$

$$(ii)\ \overline{R}_{\alpha}(K_n|W_5) = \frac{n(n-1)}{2}9^{\alpha}(n-1)^{2\alpha} + \frac{n^2(n-1)^2 - n(n-1)}{2}9^{\alpha} + \frac{n^2(n-1)^2 - 2n(n-1)}{8}16^{\alpha} + n(n-1)(n-2)9^{\alpha}(n-1)^{\alpha} + \frac{n(n-1)(n-2)}{2}12^{\alpha}(n-1)^{\alpha} + \frac{n(n-1)(n(n-1)-2)}{2}12^{\alpha}.$$

Proof: The extended $G-$network $K_n|W_5$ contains $\binom{\frac{n(3n-1)}{2}}{2} - 4n(n-1)$ non adjacent pairs of vertices, out of which $\frac{n(n-1)}{2}$ pairs of vertices of degree $3(n-1)$ and $3(n-1)$, $\frac{n^2(n-1)^2-n(n-1)}{2}$ pairs of vertices of degree 3 and 3, $\frac{n^2(n-1)^2-2n(n-1)}{8}$ pairs of vertices of degree 4 and 4, $n(n-1)(n-2)$ pairs of vertices of degree 3 and $3(n-1)$, $\frac{n(n-1)(n-2)}{2}$ pairs of vertices of degree 4 and $3(n-1)$, and remaining $\frac{n(n-1)(n(n-1)-2)}{2}$ pairs of vertices of degree 4 and 3. Hence, $\overline{\chi}_{\alpha}(K_n|W_5) = \frac{n(n-1)}{2}6^{\alpha}(n-1)^{\alpha} +$

$\frac{n^2(n-1)^2-n(n-1)}{2}6^\alpha + \frac{n^2(n-1)^2-2n(n-1)}{8}8^\alpha + n(n-1)(n-2)(3n)^\alpha + \frac{n(n-1)(n-2)}{2}(3n+1)^\alpha + \frac{n(n-1)(n(n-1)-2)}{2}7^\alpha$ and

$\overline{R}_\alpha(K_n|W_5) = \frac{n(n-1)}{2}9^\alpha(n-1)^{2\alpha} + \frac{n^2(n-1)^2-n(n-1)}{2}9^\alpha + \frac{n^2(n-1)^2-2n(n-1)}{8}16^\alpha + n(n-1)(n-2)9^\alpha(n-1)^\alpha + \frac{n(n-1)(n-2)}{2}12^\alpha(n-1)^\alpha + \frac{n(n-1)(n(n-1)-2)}{2}12^\alpha$. □

By putting $\alpha = 1,2$ in Theorem 3.7 (i) and $\alpha = 1$ in Theorem 3.7 (ii), we get the following corollary.

Corollary 3.8. $(i)\ \overline{M_1}(K_n|W_5) = 3n(n-1)^2 + 4n^2(n-1)^2 - 5n(n-1) + 3n^2(n-1)(n-2) + \frac{n(n-1)(n-2)(3n+1)}{2} + \frac{7n(n-1)(n(n-1)-2)}{2}$.

$$(ii)\overline{HM}(K_n|W_5) = 18n(n-1)^3 + 26n^2(n-1)^2 - 34n(n-1) + 9n^3(n-1)(n-2) + \frac{n(n-1)(n-2)(3n+1)^2}{2} + \frac{49n(n-1)[n(n-1)-2]}{2}.$$

$$(iii)\overline{M}_2(K_n|W_5) = \frac{9n(n-1)^3}{2} + \frac{25n^2(n-1)^2}{2} - \frac{41n(n-1)}{2} + 15n(n-1)^2(n-2).$$

Theorem 3.9. $(i)\ M_1^\alpha(Q_n|C_4) = 2^{n+\alpha}(n^\alpha + n)$.
$(ii)\ \chi_\alpha(Q_n|C_4) = 2^{n+\alpha+1}(n+1)^\alpha n$.
$(iii) R_\alpha(Q_n|C_4) = 2^{2\alpha+n+1}n^{\alpha+1}$.

Proof: The G_g −network $Q_n|C_4$ contains $2^n(n+1)$ vertices, out of which 2^n vertices are of degree $2n$, and remaining $n2^n$ vertices of degree 2. So the M_1^α −value of $Q_n|C_4$ is equal to $2^{n+\alpha}(n^\alpha + n)$. This completes the proof of (i).

The G_g −network $Q_n|C_4$ contains $n2^{n+1}$ edges whose end vertices have degree 2 and $2n$. Hence, χ_α and $R_\alpha-$ values of $Q_n|C_4$ are $2^{n+\alpha+1}(n+1)^\alpha n$ and $2^{2\alpha+n+1}n^{\alpha+1}$ respectively. This completes the proof of (ii) and (iii). □

By putting $\alpha = 2,3$ in Theorem 3.9 (i), $\alpha = 2$ in Theorem 3.9 (ii) and $\alpha = 1$ in Theorem 3.9 (iii), we get the following corollary.

Corollary 3.10. $(i)\ M_1(Q_n|C_4) = 2^{n+2}(n^2 + n)$.
$(ii)\ F(Q_n|C_4) = 2^{n+3}(n^3 + n)$.
$(iii)\ HM(Q_n|C_4) = 2^{n+3}n(n+1)^2$.
$(iv) M_2(Q_n|C_4) = 2^{n+3}n^2$.

Theorem 3.11. $(i)\overline{\chi}_\alpha(Q_n|C_4) = 2^{2\alpha+1}\binom{2^{n-1}n}{2} + 4^\alpha n^\alpha\binom{2^n}{2} + 2^{\alpha+n+1}(n+1)^\alpha n$.
$(ii)\overline{R}_\alpha(Q_n|C_4) = 2^{2\alpha+1}\binom{2^{n-1}n}{2} + 4^\alpha n^{2\alpha}\binom{2^n}{2} + 2^{2\alpha+n+1}n^{\alpha+1}$.

Proof: The G_g −network $Q_n|C_4$ contains $\binom{2^n(n+1)}{2} - 2^{n+1}n$ non adjacent pairs of vertices, out of which $2\binom{2^{n-1}n}{2}$ pairs of vertices of degree 2 and 2, $\binom{2^n}{2}$ pairs of vertices of degree $2n$ and $2n$, and remaining $2^{n+1}n$ pairs of vertices of degree 2 and

$2n$. Hence, $\overline{\chi}_\alpha(Q_n|C_4) = 2^{2\alpha+1}\binom{2^{n-1}n}{2} + 4^\alpha n^\alpha \binom{2^n}{2} + 2^{\alpha+n+1}(n+1)^\alpha n$ and $\overline{R}_\alpha(Q_n|C_4) = 2^{2\alpha+1}\binom{2^{n-1}n}{2} + 4^\alpha n^{2\alpha}\binom{2^n}{2} + 2^{2\alpha+n+1}n^{\alpha+1}$. □

By putting $\alpha = 1,2$ in Theorem 3.11 (i) and $\alpha = 1$ in Theorem 3.11 (ii), we get the following corollary.

Corollary 3.12. $(i)\overline{M_1}(Q_n|C_4) = 8\binom{2^{n-1}n}{2} + 4n\binom{2^n}{2} + 2^{n+2}(n+1)n.$

$(ii)\overline{HM}(Q_n|C_4) = 32\binom{2^{n-1}n}{2} + 16n^2\binom{2^n}{2} + 2^{n+3}(n+1)^2 n.$

$(iii)\overline{M}_2(Q_n|C_4) = 8\binom{2^{n-1}n}{2} + 4n^2\binom{2^n}{2} + 2^{n+3}n^2.$

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