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Another Theta Generalized Closed Sets in Fuzzy Topological Spaces

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Abstract. In this paper, we introduce a new class of fuzzy generalized closed sets called fuzzy $\theta g'''$-closed and its properties are investigated. Further, new concept of fuzzy $\theta g^{*}s$- closed, fuzzy $g''' \theta$- closed, fuzzy $g^{*}s\theta$-closed, and fuzzy $g'''_{\alpha}\ \theta$- closed sets are introduced using it.

Keywords: fuzzy generalized closed sets, fuzzy θg-closed sets, fuzzy θ gs-closed sets, fuzzy $\theta g'''$-closed and fuzzy θ-closed sets

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1. Introduction

Zadeh [13] introduced the concept of fuzzy sets in 1965**.** In 1968, the concept of fuzzy topological space introduced by Chang [5] and many researchers have been worked in this area. Shafei [6] introduced θ generalized closed sets in fuzzy topological space in 2005. Later Salleh [12] defined the concepts of θsemi-generalized closed sets in fuzzy topological space. Recently, Othman introduce fuzzy θ-generalized semi closed sets in fuzzy topological space [7]

In the present paper, we introduce fuzzy $\theta g'''$-closed sets in fuzzy topological space and investigate certain basic properties of these fuzzy sets.

2. Basic concepts

Throughout the present paper, (X,τ) or simply X mean fuzzy topological space (abbreviated as fts) on which no separation axioms are assumed unless otherwise mentioned. We denote and define the closure and interior for a fuzzy set A by $Cl(A) = \wedge\{\mu : \mu \geq A, 1-\mu \in \tau\}$ and $Int(A) = \vee\{\mu : \mu \leq A,\ \mu \in \tau\}$. fuzzy θ -closure of A [6] and fuzzy semi-θ closure of A [12] are denoted by $Cl_{\theta}(A) = \wedge\{cl(\mu) : A \leq \mu, \mu \in \tau\}$ and

$sCl_{\theta}(A) = \wedge\{scl(\mu) : A \leq \mu, \mu \text{ is semi-open in } \tau\}$ respectively.

Definition 2.1. A fuzzy set A of (X,τ) is called

(i) fuzzy semi-open [1] if $A \le Cl(Int(A))$

(ii) fuzzy α-open [3] if $A \le Int(Cl(Int(A)))$

(iii) fuzzy θ-open if $A = Int_\theta(A)$ [6] and fuzzy θ -closed [6] if $A = Cl_\theta(A)$

(iv) fuzzy semi-θ-open [12] if $A = sInt_\theta(A)$ and fuzzy semi-θ-closed [12] $A = sCl_\theta(A)$

The semi closure [11] (respectively α-closure [10]) of a fuzzy set A of (X,τ) is the intersection of all fs-Closed (respectively fα-closed sets) that contain A and is denoted by $sCl(A)$ (respectively $\alpha Cl(A)$).

Definition 2.2. A fuzzy set A of (X,τ) is called

(i) fuzzy generalized closed (in short, fg-closed)[4] if $Cl\,A \le H$, whenever $A \le H$ and H is fuzzy open set in X

(ii) fuzzy generalized semi closed (in short, fsg-closed)[2] if $sCl(A) \le H$, whenever $A \le H$ and H is fs-open set in X.

(iii) fuzzy generalized closed (in short, fgs-closed) [9] if $sCl(A) \le H$, whenever $A \le H$ and H is f-open set in X.

(iv) fuzzy θ-generalized closed (in short, fθg-closed) [6] if $Cl_\theta(A) \le H$, whenever $A \le H$ and H is f-open set in X.

(v) fuzzy θ-semi-generalized closed (in short, fθsg-closed) [12] if $sCl_\theta(A) \le H$, whenever $A \le H$ and H is fs-open set in X.

(vi) fuzzy θgeneralized semi closed (in short, fθgs-closed) [7] if $sCl_\theta(A) \le H$, whenever $A \le H$ and H is f-open set in X.

(vii) fuzzy g‴- closed [8] if $Cl\,A \le H$, whenever $A \le H$ and H is fuzzy gs-open set in X

(viii) fuzzy g*s- closed [8] if $sCl\,A \le H$, whenever $A \le H$ and H is fuzzy gs-open set in X

(ix) fuzzy g'''_α- closed [8]if $\alpha Cl\,A \le H$, whenever $A \le H$ and H is fuzzy gs- open set in X

3. Fuzzy θg‴-closed sets in fuzzy topological spaces

Definition 3.1. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy θg‴-closed if $Cl_\theta(A) \le H$ whenever $A \le H$ and H is fuzzy θgs –open in X.

Definition 3.2. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy θg*s- closed if $sCl_{\theta}(A) \le H$, whenever $A \le H$ and H is fuzzy θgs-open set in X.

Definition 3.3. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy g‴θ- closed if $Cl\, A \le H$, whenever $A \le H$ and H is fuzzy θgs-open set in X.

Definition 3.4. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy g*sθ- closed if $sCl\, A \le H$, whenever $A \le H$ and H is fuzzy θgs-open set in X.

Definition 3.5. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy g‴$_{\alpha}\theta$- closed if $\alpha\, Cl\, A \le H$, whenever $A \le H$ and H is fuzzy θgs-open set in X.

Theorem 3.1. Every fuzzy θ-closed set is fuzzy θg‴- closed set in a fuzzy topological space (X,τ).

Proof: Let A be fuzzy θ closed set in X. Let H be a fθgs - open set in X such that $A \le H$. Since A is fuzzy θ- closed, $Cl_{\theta}(A)=A$. Therefore $Cl_{\theta}(A) \le H$ whenever $A \le H$ and H is fuzzy θgs-open set in X. Hence A is fuzzy θg‴- closed set in X.

Remark 3.1. The converse of the above theorem need not be true as shown in the following example.

Example 3.1. Let $X = \{a\}$. Fuzzy sets A and B are defined by A (a) = 0.6 ;B(a)=0.5. Let $\tau = \{0, A, 1\}$. Then B is a fuzzy θg‴- closed set but it is not a fuzzy θ- closed set in (X,τ).

Theorem 3.2. Every fuzzy semi-θ-closed set is fuzzy θg*s- closed set in a fuzzy topological space (X,τ).

Proof: Let A be fuzzy semi-θ closed set in X. Let H be a fθgs - open set in X such that $A \le H$. Since A is fuzzy semi-θ- closed, $sCl_{\theta}(A) = A$. Therefore $sCl_{\theta}(A) \le H$ whenever $A \le H$ and H is fuzzy θgs-open set in X. Hence A is fuzzy θg*s- closed set in X.

Theorem 3.3. Every fuzzy α-closed set is fuzzy g‴$_{\alpha}\theta$- closed set in a fuzzy topological space (X,τ).

Proof: Obvious from Definition 3.5 and Definition 3.1 (ii)

Theorem 3.4. Every fuzzy closed set is fuzzy g‴θ- closed set in a fuzzy topological space (X,τ).

Proof: Obvious from Definition 3.3.

Theorem 3.5. Every fuzzy semi-closed set is fuzzy g*sθ- closed set in a fuzzy topological space (X,τ).

Proof: Obvious from Definition 3.4

Remark 3.2. The converse of the theorems 3.2, 3.3, 3.4 and 3.5 need not be true as shown in the following example.

Example 3.2. Let $X=\{a\}$. Fuzzy sets A ,B and D are defined by

A (a) = 0.5 ;B(a)=0.4 ;D(a)=0.7 .

Let τ={0, A,B,1}. Then D is

(i) fuzzy θg*s- closed set but it is not a fuzzy semi-θ-closed set in (X,τ).

(ii) fuzzy g‴$_\alpha\theta$- closed set but it is not a fuzzy α-closed set in (X,τ).

(iii) fuzzy g‴θ- closed set but it is not a fuzzy closed set in (X,τ).

(iv) fuzzy g*sθ- closed set but it is not a fuzzy semi -closed set in (X,τ).

Theorem 3.6. Every fuzzy θg‴-closed set is fuzzy θg*s- closed set in a fuzz topological space (X,τ).

Proof: Let A be a fuzzy θg‴-closed set in X. Let H be a fθgs- open set in X such that $A\le H$ .Since, $sCl_\theta(A))\le Cl_\theta(A)\le H$ .Therefore $sCl_\theta\left(A\right)\le H$, whenever $A\le H$ and H is fuzzyθgs-open set in X. Hence A is fuzzy θg*s-closed set in X.

Remark 3.3. The converse of the above theorem need not be true as shown in the following example.

Example 3.3. Let $X=\{a,b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.1 , A(b)=0.2 ; B(a)=0.5 , B(b) = 0.4 . Let τ ={0, B,1}. Then A is fuzzy θg*s-closed but it is not fθg‴-closed.

Theorem 3.7. Every fuzzy θg‴-closed set is fuzzy g‴θ- closed set in a fuzzy topological space (X,τ).

Proof: Let A be a fuzzy θg‴-closed set in X. Let H be a fθgs- open set in X such that $A\le H$ .Since, $Cl(A))\le Cl_\theta(A)\le H$ .Therefore $Cl\,A\le H$, whenever $A\le H$ and H is fuzzyθgs-open set in X. Hence A is fuzzy g‴θ-closed set in X.

Remark 3.4. The converse of the above theorem need not be true as shown in the following example

Example 3.4. Let $X=\{a,b\}$ and the fuzzy sets A, B and D be defined as follows A (a) = 0.4 , A(b)=0.4 ; B(a)=0.5 , B(b) = 0.4 ; $D(a)=0.5, D(b)=0.6$.

Let τ $=\{0, \text{A,B},1\}$. Then A is fuzzy g‴ θ-closed but it is not f θ g‴-closed.

Theorem 3.8. Every fuzzy θ g‴-closed set is fuzzy g‴$_{\alpha}$ θ- closed set in a fuzzy topological space (X,τ).

Proof: Let A be a fuzzy θ g‴-closed set in X. Let H be a f θ gs- open set in X such that $A \le H$. Since, $\alpha Cl(A)) \le Cl_{\theta}(A) \le H$. Therefore $\alpha\ Cl\ A \le H$, whenever $A \le H$ and H is fuzzy θ gs-open set in X. Hence A is fuzzy g‴$_{\alpha}$ θ-closed set in X.

Remark 3.5. The converse of the above theorem need not be true as shown in the following example.

Example 3.5. Let $X=\{a,b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.3 , A(b)=0.4 ; B(a)=0.7 , B(b) = 0.6

Let τ $=\{0, \text{A},1\}$. Then B is fuzzy g‴$_{\alpha}$ θ-closed but it is not f θ g‴-closed.

Theorem 3.9. Every fuzzy θ g*s -closed set is fuzzy g*s θ- closed set in a fuzzy topological space (X,τ).

Proof: Let A be a fuzzy θ g*s -closed set in X. Let H be a f θ gs- open set in X such that $A \le H$. Since, $sCl(A) \le sCl_{\theta}\left(A\right) \le H$. Therefore $s\,Cl\ A \le H$, whenever $A \le H$ and H is fuzzy θ gs-open set in X. Hence A is fuzzy g*s θ-closed set in X.

Remark 3.6. The converse of the above theorem need not be true as shown in the following example.

Example 3.6. Let X={a} and the fuzzy sets A , B and D be defined as follows A (a) = 0.6; $B(a)=0.5$; $D(a)=0.1$. Let τ $=\{0, \text{A,B},1\}$. Then D is fuzzy g*s θ -closed but it is not f θ g^* s -closed.

Theorem 3.11. Every fuzzy θ g‴ -closed set is fuzzy g*s θ- closed set in a fuzzy topological space (X,τ).

Proof: It is clear from Theorem 3.6 and Theorem 3.10

Remark 3.7. The converse of the above theorem need not be true as shown in the following example.

Example 3.7. $X=\{a,b\}$. Consider the fuzzy topology τ as in Example 3.3 where A is defined by A (a) = 0.1 , A(b)=0.2 . Clearly A is fuzzy g*s θ- closed set but not fuzzy θ g‴ -closed set.

Theorem 3.8. Every fuzzy g‴ θ-closed set is fuzzy g‴$_{\alpha}$ θ- closed set in a fuzzy topological space (X,τ).

Proof: Let A be a fuzzy g‴ θ-closed set in X. Let H be a f θgs- open set in X such that $A \le H$. Since, $\alpha Cl(A)) \le Cl(A) \le H$. Therefore $\alpha\ Cl\, A \le H$, whenever $A \le H$ and H is fuzzy θgs-open set in X. Hence A is fuzzy g‴$_\alpha$ θ-closed set in X.

Remark 3.8. The converse of the above theorem need not be true as shown in the following example.

Example 3.8. Let $X = \{a\}$ and the fuzzy sets A, B and D be defined as follows A (a) = 0.5; $B(a) = 0.3$; $D(a) = 0.6$. Let $\tau = \{0, A, B, 1\}$. Then D is fuzzy g‴$_\alpha$ θ-closed but it is not f g‴ θ closed.

Remark 3.9. The following examples shows that the fuzzy θg‴-closed set is independent of fuzzy g‴-closed, fuzzy g*s-closed, fuzzy g‴$_\alpha$ – closed, fuzzy closed, fuzzy semi-θ-closed, fuzzy θ-g-closed, fuzzy θ-gs closed, fuzzy θsg –closed.

Example 3.9. Let $X = \{a, b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.6 , A(b)=0.5 ; B(a)=0.6 , B(b) = 0.6 .

Let $\tau = \{0, A, 1\}$. Then B is fuzzy θg‴--closed but it is not fuzzy g‴-closed.

Example 3.10. Let $X = \{a, b\}$ Consider the Fuzzy topology τ as in Example 3.4 where D is defined by D (a) = 0.5 , D(b)=0.6 .Clearly D is both fuzzy closed and fuzzy g‴--closed but it is not fuzzy θg‴-closed.

Example 3.11. Let $X = \{a, b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.6 , A(b)=0.6 ; B(a)=0.7 , B(b) = 0.8 .

Let $\tau = \{0, A, 1\}$. Then B is fuzzy θg‴--closed but it is neither g‴$_\alpha$ –closed nor fuzzy g*s-closed.

Example 3.12. Let $X = \{a, b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.6 , A(b)=0.6 ; B(a)=0.3 , B(b) = 0.4 .

Let $\tau = \{0, A, 1\}$. Then B is both fuzzy g‴$_\alpha$ -closed and fuzzy g*s-closed but it is not fuzzy θg‴--closed.

Example 3.13. Let $X = \{a, b\}$ Consider the Fuzzy topology τ as in Example 3.4 where E is defined by E (a) = 0.6 , E(b)=0.6 . Clearly E is fuzzy semi-θ- closed but it is not fuzzy θg‴-closed and D is defined by $D(a) = 0.5, D(b) = 0.6$ is fuzzy closed but it is not fuzzy θg‴--closed.

Example 3.14. Let $X = \{a, b\}$ and the fuzzy sets A and B be defined as follows A (a) = 1 , A(b)=0.2 ; B(a)=0.5 , B(b) = 0.9 .

Let τ $=\{0, \text{A},1\}$. Then B is fuzzy θg‴--closed but it is neither fuzzy closed nor fuzzy semi-θ- closed.

Example 3.15. Let $X=\{a\}$ and the fuzzy sets A, B and D be defined as follows A (a) = 0.7; $B(a)=0.8$; $D(a)=0.4$. Let τ $=\{0, \text{A,B},1\}$. Then D is fuzzy θg‴-closed but it is not fuzzy θg -closed.

Example 3.16. Let $X=\{a,b\}$ and the fuzzy sets A and B be defined as follows A (a) = 0.5 , A(b)=0.5; B(a)=0.2 , B(b) = 0.3.

Let τ $=\{0, \text{A},1\}$. Then B is fuzzy θg -closed but it is not fuzzy θg‴--closed.

Example 3.17. Let $X=\{a\}$ and the fuzzy sets A, B be defined as follows A (a) = 0.4; $B(a)=0.3$. Let τ $=\{0, \text{A},1\}$. Then B is both fuzzy θsg –closed and fuzzy θgs –closed, but it is not fuzzy θg‴-closed.

Example 3.18. Let $X=\{a\}$ and the fuzzy sets A , B be defined as follows A (a) = 0.7; $B(a)=0.9$. Let τ $=\{0, \text{A},1\}$.Then B is fuzzy θg‴-closed but it is not fuzzy θsg -closed.

Example 3.19. Let $X=\{a\}$ and the fuzzy sets A, B be defined as follows A (a) = 0.6; $B(a)=0.5$. Let τ $=\{0, \text{A},1\}$. Then B is fuzzy θg‴-closed but it is not fuzzy θgs -closed.

Remark 3.10. From the above results and examples we have the following diagram of implications.

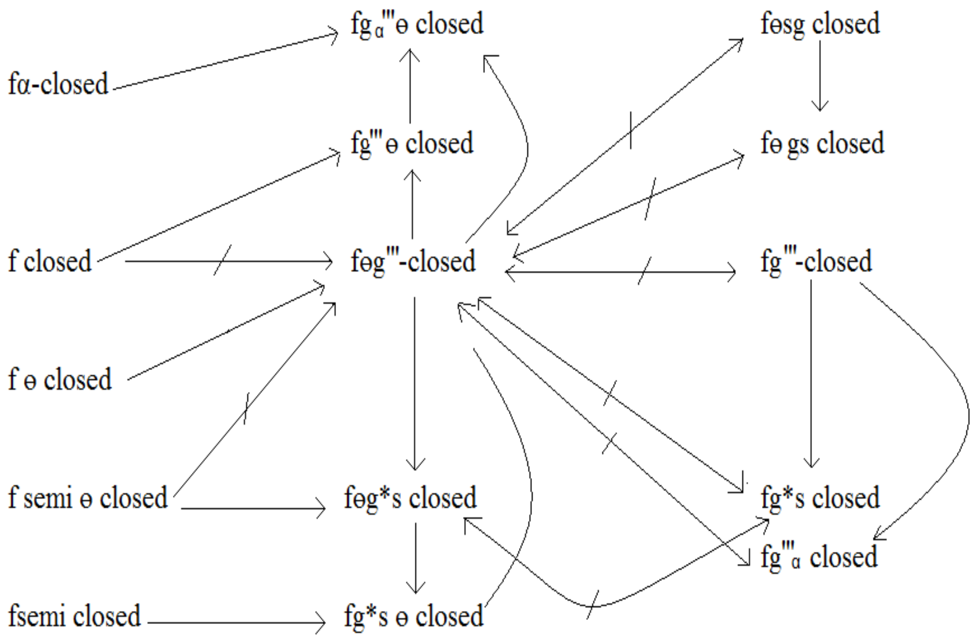

Figure 3.1:

4. Properties of fuzzy θg''' –closed sets

Theorem 4.1 Let X be a fts, then the union of two fuzzy θg''' - closed sets is fuzzy θg'''-closed set.

Proof: Suppose that A and B are fuzzy θg''' - closed sets in X and let H be a fuzzy θgs open set in X such that $A \vee B \leq H$. Since A and B are fuzzy θg'''-closed sets, we have $Cl_\theta(A) \vee Cl_\theta(B) \leq H$.

Since $Cl_\theta(A \vee B) \leq Cl_\theta(A) \vee Cl_\theta(B)$,therefore $Cl_\theta(A \vee B) \leq H$, whenever $A \vee B \leq H$ and H is fuzzy θgs open set in X.

Theorem 4.2. If A is a fuzzy θg'''- closed set in (X, τ) and $A \leq B \leq Cl_\theta(A)$, then B is fuzzy θg'''- closed set in (X, τ).

Proof: Let A be a fuzzy θg'''- closed set in (X, τ). Let $B \leq H$ where H is a fuzzy θgs -open set in X. Then $A \leq H$. Since A is fuzzy θg'''- closed set in, it follows that $Cl_\theta(A) \leq H$.

Now $B \leq Cl_\theta(A)$ implies $Cl_\theta(B) \leq Cl_\theta(Cl_\theta(A)) = Cl_\theta(A)$. We get, $Cl_\theta(B) \leq H$. Hence, B is fuzzy θg'''- closed set in (X, τ).

Theorem 4.3. If a fuzzy set A of a fuzzy topological space X is both fuzzy θgs-open and fuzzy θg'''- closed then it is fuzzy θ- closed.

Proof: Suppose that a fuzzy set A of X is both fuzzy θgs -open and fuzzy $\theta g'''$- closed. Now $A \geq Cl_{\theta}(A)$ whenever $A \geq A$ and A is fuzzy θgs -open .Since $A \leq Cl_{\theta}(A)$. We get $A = Cl_{\theta}(A)$. Hence A is fuzzy θ- closed in X.

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