Intern. J. Fuzzy Mathematical Archive
Vol. 11, No.1, 2016, 31-38
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 10 November 2016
www.researchmathsci.org
DOI: 10.22457/ijfma.v11n1a5

International Journal of
Fuzzy Mathematical
Archive

Role of Homomorphism and Cartesian Product over Fuzzy PMS-algebras

***P.M.Sithar Selvam*[1] and *K.T.Nagalakshmi*[2]**

[1]Department of Mathematics
R.V.S. School of Engineering and Technology, Dindigul– 624001, Tamilnadu, India
Corresponding author. Email: sitharselvam@gmail.com
[2]Department of Mathematics
K.L.N.College of Information and Technology, Pottapalayam- 630611, Tamilnadu, India
Email: gloryratna@yahoo.co.in

Received 16 October 2016; accepted 3 November 2016

Abstract. In this paper, we define Homomorphism and Cartesian product of PMS-algebras and discussed some of its properties in detail by using the concepts of fuzzy PMS-ideal and fuzzy PMS-sub algebra.

Keywords: fuzzy PMS-ideal, fuzzy PMS-sub algebra, homomorphism and Cartesian product.

AMS Mathematics Subject Classification (2010): 20N25, 03E72, 03F055, 06F35

1. Introduction

The concept of fuzzy set was initiated by Zadeh in 1965 [15]. Fuzzy algebra is an important branch of fuzzy mathematics from the inception of fuzzy concepts. In this path, Iseki and Tanaka [1] introduced the concept of BCK-algebras in 1978. Iseki **[2]** introduced the concept of BCI-algebras in 1980. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Selvam and Nagalakshmi [4] introduced the class of PMS-algebras, which is a generalization of BCI/ BCK/TM/KUS -algebras. In this paper, we introduce the concept of homomorphism and Cartesian product of fuzzy PMS-algebras and established some of its properties in detail.

2. Preliminaries

In this section, we give the fundamental definitions that will be used in the development of this paper.

Definition 2.1. [2] A BCK-algebra is an algebra $(X,*,0)$ of type(2,0) satisfying the following conditions:

i) $(x * y) * (x * z) \leq (z * y)$
ii) $x * (x * y) \leq y$
iii) $x \leq x$
iv) $x \leq y$ and $y \leq x \Rightarrow x=y$
v) $0 \leq x \Rightarrow x=0$,

where $x \leq y$ is defined by $x * y = 0$,for all $x, y, z \in X$.

Definition 2.2.[3] A BCI-algebra is an algebra $(X,*,0)$ of type $(2,0)$ satisfying the following conditions:

i) $(x * y) *(x * z) \leq (z*y)$
ii) $x * (x * y) \leq y$
iii) $x \leq x$
iv) $x \leq y$ and $y \leq x \Rightarrow x = y$
v) $x \leq 0 \Rightarrow x = 0$,

where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$.

Definition 2.3.[4, 5] A nonempty set X with a constant 0 and a binary operation '$*$' is called PMS – algebra if it satisfies the following axioms.

1. $0 * x = x$
2. $(y * x) * (z * x) = z * y$, $\forall$ $x, y, z \in X$.

In X, we define a binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$.

Definition 2.4. [4, 5] Let X be a PMS - algebra and I be a subset of X, then I is called a PMS - ideal of X if it satisfies following conditions:

1. $0 \in I$
2. $z * y \in I$ and $z * x \in I \Rightarrow y * x \in I$ for all $x, y, z \in X$.

Definition 2.5.[5, 6] Let X be a PMS-algebra. A fuzzy set μ in X is called a fuzzy PMS-ideal of X if it satisfies the following conditions.

i) $\mu(0) \geq \mu(x)$
ii) $\mu (y * x) \geq \min \{\mu (z *y), \mu(z * x)\}$, for all $x, y,z \in X$.

Definition 2.6.[5,6] A fuzzy set μ in a PMS-algebra X is called a fuzzy PMS- sub algebra of X if $\mu(x * y) \geq \min \{\mu(x), \mu(y)\}$, for all $x, y\in X$.

3. Homomorphism on PMS -Algebra

In this section, we discussed about PMS-ideals in PMS-algebras under homomorphism and some of its properties in detail.

Definition 3.1.[5,10] Let $(X,*,0)$ and $(Y,\Delta,0`)$ be PMS– algebras. A mapping $f: X \rightarrow Y$ is said to be a homomorphism if $f(x*y) = f(x) \Delta f(y)$ for all $x,y \in X$.

Remark: If $f: X \rightarrow Y$ is a homomorphism of PMS-algebra, then $f (0) = 0$.

Definition 3.2.[11, 14] Let $f: X \rightarrow X$ be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set μ^f in X as $\mu^f(x) = \mu(f(x))$ for all $x\in X$.

Definition 3.3. [11,14] For any homomorphism $f: X \rightarrow Y$, the set $\{x \in X / f(x) = 0'\}$ is called the kernel of f, denoted by Ker(f) and the set $\{ f(x) / x\in X\}$ is called the image of f, denoted by Im(f).

Theorem 3.4. Let f be an endomorphism of PMS- algebra X. If μ is a fuzzy PMS-ideal of X, then so is μ_f .

Proof: Let μ be a fuzzy PMS-ideal of X.

Now, $\mu^f(0) = \mu [f(0)]$
$\geq \mu [f(x)]$

$=\mu^f(x)$

$\Rightarrow\ \mu^f(0) \geq \mu^f(x)$.

Let x,y,z $\in$ X. Then

$$\mu^f(y * x) = \mu[f(y*x)]$$
$$\geq \min\{\mu(f(z) * f(y)), \mu(f(z) * f(x))\}$$
$$= \min\{\mu(f(z * y)), \mu(f(z * x))\}$$
$$= \min\{\mu^f(z * y), \mu^f(z * x)\}$$

$\therefore\ \mu^f(y*x) \geq \min\{\mu^f(z * y), \mu^f(z * x)\}$

Hence μ^f is a fuzzy PMS-ideal of X.

Theorem 3.5. Let f: X $\rightarrow$ Y be an epimorphism of PMS- algebra. If μ^f is a fuzzy PMS-ideal of X, then μ is a fuzzy PMS-ideal of Y.

Proof: Let μ^f be a fuzzy PMS-ideal of X and let y $\in$ Y. Then there exists x $\in$ X such that f(x) = y.

Now, $\mu(0) = \mu(f(0))$
$$= \mu^f(0)$$
$$\geq \mu^f(x) = \mu(f(x)) = \mu(y)$$

$\therefore\ \mu(0) \geq \mu(y)$

Let $y_1, y_2, y_3 \in$ Y.

$$\mu(y_2 \Delta y_1) = \mu(f(x_2) \Delta f(x_1))$$
$$= \mu(f(x_2 * x_1))$$
$$= \mu^f(x_2 * x_1)$$
$$\geq \min\{\mu^f(x_3 * x_2), \mu^f(x_3 * x_1)\}$$
$$= \min\{\mu[f(x_3 * x_2)], \mu[f(x_3 * x_1)]\}$$
$$= \min\{\mu[f(x_3) \Delta f(x_2)], \mu[f(x_3) \Delta f(x_1)]\}$$
$$= \min\{\mu[y_3 \Delta y_2], \mu[y_3 \Delta y_1]\}$$

$\therefore\ \mu(y_2 \Delta y_1) \geq \min\{\mu(y_3 \Delta y_2), \mu(y_3 \Delta y_1)\}$

$\Rightarrow\ \mu$ is a fuzzy PMS-ideal of Y.

Theorem 3.6. Let f: X $\rightarrow$ Y be a homomorphism of PMS- algebra. If μ is a fuzzy PMS-ideal of Y then μ^f is a fuzzy PMS-ideal of X.

Proof: Let μ be a fuzzy PMS-ideal of Y and let x, y, z $\in$ X.

Then $\mu^f(0) = \mu(f(0))$
$$\geq \mu(f(x))$$
$$= \mu^f(x)$$

$\Rightarrow\ \mu^f(0) \geq \mu^f(x)$.

$$\mu^f(y * x) = \mu[f(y * x)]$$
$$= \mu[f(y) \Delta f(x)]$$
$$\geq \min\{\mu(f(z) \Delta f(y)), \mu(f(z) \Delta f(x))\}$$
$$= \min\{\mu(f(z * y)), \mu(f(z * x))\}$$
$$= \min\{\mu^f(z * y), \mu^f(z * x)\}$$

$\therefore\ \mu^f(y * x) \geq \min\{\mu^f(z * y), \mu^f(z * x)\}$.

Hence μ^f is a fuzzy PMS-ideal of X.

Theorem 3.7. If μ is a fuzzy PMS- sub algebra of X, then μ^f is also a fuzzy PMS-sub algebra of X.

Proof: Let μ be a fuzzy PMS- sub algebra of X.

Let x,y $\in$ X.

$$\text{Now, } \mu^f (x * y) = \mu [f (x * y)]$$
$$= \mu (f (x) \Delta f(y))$$
$$\geq \min \{\mu (f (x)) , \mu(f(y)) \}$$
$$= \min \{\mu^f (x), \mu^f (y)\}$$

$\Rightarrow \mu^f (x * y) \geq \min \{\mu^f (x) , \mu^f (y) \}$

Hence μ^f is a fuzzy PMS-sub algebra of X.

Theorem 3.8. Let f: X→Y be a homomorphism of a PMS-algebra X into a PMS-algebra Y, then the pre- image of μ denoted by
$f^{-1}(\mu)$ is defined as $\{f^{-1} (\mu)\}(x) = \mu(f(x)) ,\forall\ x\in X$. If μ is a fuzzy PMS- sub algebra of Y, then $f^{-1}(\mu)$ is a fuzzy PMS- sub algebra of X.

Proof: Let μ be a fuzzy PMS- sub algebra of Y. Let $x , y \in X$.

$$\text{Now, } \{ f^{-1} (\mu) \}(x * y) = \mu [f (x * y)]$$
$$= \mu [f (x) \Delta f(y)]$$
$$\geq \min \{ \mu [f (x)], \mu [f(y)] \}$$
$$= \min \{\{f^1(\mu)\}(x),\{f^1(\mu)\}(y)\}$$

$\Rightarrow \{f^{-1}(\mu)\} (x* y) \geq \min \{\{f^{-1}(\mu)\} (x), \{f^{-1}(\mu)\}(y)\}$

$\therefore f^{-1}(\mu)$ is a fuzzy PMS-sub algebra of X.

4. Cartesian product of fuzzy PMS-ideals of PMS–algebras

In this section, we discuss the Cartesian product of PMS-algebras and establish some of its properties in detail on the basis of fuzzy PMS-ideal and fuzzy PMS- sub algebra.

Definition 4.1.[5, 11] Let μ and δ be the fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta)(x, y) = \min \{\mu(x),\delta(y)\}$, for all $x, y \in X$.

Definition 4.2.[**3**] Let β be a fuzzy subset of X. The strongest fuzzy β - relation on PMS-algebra X is the fuzzy subset μ_β of X x X given by $\mu_\beta (x, y) = \min \{\beta(x), \beta(y)\}$, for all $x, y \in X$.

Theorem 4.3. If μ and δ are fuzzy PMS-ideals in a PMS– algebra X, then $\mu \times \delta$ is a fuzzy PMS-ideal in X x X.

Proof: Let $(x_1, x_2) \in X \times X$.

$$(\mu \times \delta) (0,0) = \min \{ \mu (0), \delta (0) \}$$
$$\geq \min \{ \mu (x_1) , \delta (x_2) \}$$
$$= (\mu \times \delta) (x_1,x_2)$$

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$.

$$(\mu \times \delta) [(y_1, y_2) * (x_1, x_2)] = (\mu \times \delta) [y_1 * x_1 , y_2 * x_2]$$
$$= \min \{\mu(y_1 * x_1),\delta(y_2 * x_2)\}$$
$$\geq \min \{\min \{\mu(z_1* y_1), \mu(z_1* x_1)\},\min \{\delta(z_2* y_2), \delta(z_2* x_2)\}\}$$
$$= \min \{\min \{\mu(z_1*y_1), \delta(z_2*y_2)\},\min\{ \mu(z_1*x_1) \delta(z_2* x_2)\}\}$$
$$= \min \{(\mu \times \delta) ((z_1 * y_1),(z_2* y_2)),(\mu \times \delta) ((z_1 * x_1),(z_2 * x_2))\}$$
$$= \min \{(\mu \times \delta) ((z_1 , z_2) * (y_1, y_2)), (\mu \times \delta) ((z_1 , z_2) * (x_1, x_2))\}$$

$\therefore (\mu \times \delta) [(y_1, y_2) *(x_1, x_2)] \geq \min \{(\mu \times \delta) [(z_1 , z_2) * (y_1, y_2)], (\mu \times \delta) [(z_1 , z_2) * (x_1, x_2)]\}$

Hence, $\mu \times \delta$ is a fuzzy PMS- ideal in X x X.

Theorem 4.4. Let μ and δ be fuzzy sets in a PMS-algebra X such that $\mu \times \delta$ is a fuzzy PMS-ideal of X x X. Then

(i) Either $\mu(0) \geq \mu(x)$ (or) $\delta(0) \geq \delta(x)$ for all $x \in X$.
(ii) If $\mu(0) \geq \mu(x)$ for all $x \in X$, then either $\delta(0) \geq \mu(x)$ (or) $\delta(0) \geq \delta(x)$
(iii) If $\delta(0) \geq \delta(x)$ for all $x \in X$, then either $\mu(0) \geq \mu(x)$ (or) $\mu(0) \geq \delta(x)$.

Proof: Let $\mu \times \delta$ be a fuzzy PMS-ideal of X x X.
(i)Suppose that $\mu(0) < \mu(x)$ and $\delta(0) < \delta(x)$ for some $x, y \in X$.

$$
\begin{aligned}
\text{Then } (\mu \times \delta)(x, y) &= \min\{\mu(x), \delta(y)\} \\
&> \min\{\mu(0), \delta(0)\} \\
&= (\mu \times \delta)(0, 0),
\end{aligned}
$$

which is a contradiction. Therefore, $\mu(0) \geq \mu(x)$ (or) $\delta(0) \geq \delta(x)$ for all $x \in X$.
(ii) Assume that there exists $x, y \in X$ such that $\delta(0) < \mu(x)$ and $\delta(0) < \delta(x)$.
Then $(\mu \times \delta)(0,0) = \min\{\mu(0), \delta(0)\} = \delta(0)$ and hence $(\mu \times \delta)(x, y) = \min\{\mu(x),\delta(y)\} > \delta(0) = (\mu \times \delta)(0,0)$ Which is a contradiction**.** Hence, if $\mu(0) \geq \mu(x)$ for all $x \in X$, then either $\delta(0) \geq \mu(x)$ (or) $\delta(0) \geq \delta(x)$.
Similarly, we can prove that if $\delta(0) \geq \delta(x)$ for all $x \in X$, then either $\mu(0) \geq \mu(x)$ (or) $\mu(0) \geq \delta(x)$, which yields (iii).

Theorem 4.5. Let μ and δ be fuzzy sets in a PMS-algebra X such that $\mu \times \delta$ is a fuzzy PMS-ideal of X x X. Then either μ or δ is a fuzzy PMS-ideal of X.

Proof: First we prove that δ is a fuzzy PMS-ideal of X.
Since by 4.4 (i) either $\mu(0) \geq \mu(x)$ (or) $\delta(0) \geq \delta(x)$ for all $x \in X$.
Assume that $\delta(0) \geq \delta(x)$ for all $x \in X$.
It follows from 4.4 (iii) that either $\mu(0) \geq \mu(x)$ (or) $\mu(0) \geq \delta(x)$.
If $\mu(0) \geq \delta(x)$, for any $x \in X$, then $\delta(x) = \min\{\mu(0), \delta(x)\} = (\mu \times \delta)(0, x)$

$$
\begin{aligned}
\text{Now, } \quad \delta(y * x) &= (\mu \times \delta)(0, y * x) \\
&\geq \min\{(\mu \times \delta)[(0,z) * (0,y)], (\mu \times \delta)[(0,z) * (0, x)]\} \\
&= \min\{(\mu \times \delta)[(0*0),(z*y)], (\mu \times \delta)[(0*0),(z*x)]\} \\
&= \min\{(\mu \times \delta)[0,(z*y)], (\mu \times \delta)[0, z*x]\} \\
&= \min\{\delta(z * y), \delta(z * x)\}
\end{aligned}
$$

$\Rightarrow \delta(y * x) \geq \min\{\delta(z * y), \delta(z * x)\}$.
Hence δ is a fuzzy PMS-ideal of X.
Next we will prove that μ is a fuzzy PMS-ideal of X. Let $\mu(0) \geq \mu(x)$
Since by theorem 4.4 (ii), either $\delta(0) \geq \mu(x)$ (or) $\delta(0) \geq \delta(x)$.Assume that $\delta(0) \geq \mu(x)$, then $\mu(x) = \min\{\mu(x), \delta(0)\} = (\mu \times \delta)(x,0)$.

$$
\begin{aligned}
\text{Now, } \quad \mu(y * x) &= (\mu \times \delta)(y * x, 0) \\
&\geq \min\{(\mu \times \delta)[(z,0) * (y,0)], (\mu \times \delta)[(z,0) * (x,0)]\} \\
&= \min\{(\mu \times \delta)[(z*y),(0*0)], (\mu \times \delta)[(z*x),(0*0)]\} \\
&= \min\{(\mu \times \delta)[z * y, 0], (\mu \times \delta)[z * x,0]\} \\
&= \min\{\mu(z * y), \mu(z * x)\}
\end{aligned}
$$

$\Rightarrow \mu(y * x) \geq \min\{\mu(z * y), \mu(z * x)\}$
Hence μ is a fuzzy PMS-ideal of X.

Theorem 4.6. If μ and δ are fuzzy PMS-sub algebras of a PMS-algebra X, then $\mu \times \delta$ is also a fuzzy PMS-sub algebra of X x X.

Proof: For any $x_1, x_2, y_1, y_2 \in X$.

$$
\begin{aligned}
(\mu \times \delta)((x_1, y_1) * (x_2, y_2)) &= (\mu \times \delta)(x_1 * x_2, y_1 * y_2) \\
&= \min\{\mu(x_1 * x_2), \delta(y_1 * y_2)\} \\
&\geq \min\{\min\{\mu(x_1), \mu(x_2)\}, \min\{\delta(y_1), \delta(y_2)\}\} \\
&= \min\{\min\{\mu(x_1),\delta(y_1)\}, \min\{\mu(x_2), \delta(y_2)\}\}
\end{aligned}
$$

$= \min \{(\mu \times \delta)(x_1, y_1), (\mu \times \delta)(x_2, y_2)\}$

$\Rightarrow (\mu \times \delta)((x_1, y_1) * (x_2, y_2)) \geq \min \{(\mu \times \delta)(x_1, y_1), (\mu \times \delta)(x_2, y_2)\}$

This completes the proof.

Theorem 4.7. Let μ_β be the strongest fuzzy β-relation on PMS-algebra X, where β is a fuzzy set of a PMS - algebra X. If β is a fuzzy PMS-ideal of X, then μ_β is a fuzzy PMS-ideal of X x X.

Proof: Let β be a fuzzy PMS-ideal of a PMS-algebra X.

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$.

Then $\mu_\beta(0, 0) = \min \{\beta(0), \beta(0)\}$

$\geq \min \{\beta(x_1), \beta(x_2)\}$

$= \mu_\beta(x_1, x_2)$

$\Rightarrow \mu_\beta(0, 0) \geq \mu_\beta(x_1, x_2)$

And $\mu_\beta[(y_1, y_2) * (x_1, x_2)]$

$= \mu_\beta[y_1 * x_1, y_2 * x_2]$

$= \min \{\beta(y_1 * x_1), \beta(y_2 * x_2)\}$

$\geq \min \{\min \{\beta(z_1 * y_1), \beta(z_1 * x_1)\}, \min \{\beta(z_2 * y_2), \beta(z_2 * x_2)\}\}$

$= \min \{\min \{\beta(z_1 * y_1), \beta(z_2 * y_2)\}, \min \{\beta(z_1 * x_1), \beta(z_2 * x_2)\}\}$

$= \min \{\mu_\beta[(z_1 * y_1), (z_2 * y_2)], \mu_\beta[(z_1 * x_1), (z_2 * x_2)]\}$

$= \min \{\mu_\beta[(z_1, z_2) * (y_1, y_2)], \mu_\beta[(z_1, z_2) * (x_1, x_2)]\}$

$\therefore \mu_\beta[(y_1, y_2) * (x_1, x_2)] \geq \min \{\mu_\beta[(z_1, z_2) * (y_1, y_2)], \mu_\beta[(z_1, z_2) * (x_1, x_2)]\}$

Therefore, μ_β is a fuzzy PMS-ideal of X x X.

Theorem 4.8. If μ_β is a fuzzy PMS-ideal of X x X, then β is a fuzzy PMS-ideal of a PMS-algebra X.

Proof: Let μ_β is a fuzzy PMS-ideal of X x X.

Then for all $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$.

$\min \{\beta(0), \beta(0)\} = \mu_\beta(0,0) \geq \mu_\beta(x_1, x_2) = \min \{\beta(x_1), \beta(x_2)\}$

$\Rightarrow \min \{\beta(0), \beta(0)\} \geq \min \{\beta(x_1), \beta(x_2)\} \Rightarrow \beta(0) \geq \beta(x_1)$ or $\beta(0) \geq \beta(x_2)$

Also, $\min \{\beta(y_1 * x_1), \beta(y_2 * x_2)\}$

$= \mu_\beta[y_1 * x_1, y_2 * x_2]$

$= \mu_\beta[(y_1, y_2) * (x_1, x_2)]$

$\geq \min \{\mu_\beta[(z_1, z_2) * (y_1, y_2)], \mu_\beta[(z_1, z_2) * (x_1, x_2)]\}$

$= \min \{\mu_\beta[(z_1 * y_1), (z_2 * y_2)], \mu_\beta[(z_1 * x_1), (z_2 * x_2)]\}$

$= \min \{\min \{\beta(z_1 * y_1), \beta(z_2 * y_2)\}, \min \{\beta(z_1 * x_1), \beta(z_2 * x_2)\}\}$

$= \min \{\min \{\beta(z_1 * y_1), \beta(z_1 * x_1)\}, \min \{\beta(z_2 * y_2), \beta(z_2 * x_2)\}\}$

Put $x_2 = y_2 = z_2 = 0$, We get $\beta(y_1 * x_1) \geq \min \{\beta(z_1 * y_1), \beta(z_1 * x_1)\}$

Hence, β is a fuzzy PMS-ideal of a PMS-algebra X.

Theorem 4.9. If β is a fuzzy PMS-sub algebra of a PMS-algebra X, then μ_β is a fuzzy PMS-sub algebra of X x X.

Proof: Let β be a fuzzy PMS-sub algebra of a PMS-algebra X.

Let $x_1, x_2, y_1, y_2 \in X$.

Then $\mu_\beta((x_1, y_1) * (x_2, y_2)) = \mu_\beta(x_1 * x_2, y_1 * y_2)$

$= \min \{\beta(x_1 * x_2), \beta(y_1 * y_2)\}$

$\geq \min \{\min \{\beta(x_1), \beta(x_2)\}, \min \{\beta(y_1), \beta(y_2)\}\}$

$= \min \{\min \{\beta(x_1), \beta(y_1)\}, \min \{\beta(x_2), \beta(y_2)\}\}$

$= \min \{\mu_\beta(x_1, y_1), \mu_\beta(x_2, y_2)\}$

$\Rightarrow\ \mu_\beta\,((x_1, y_1) * (x_2, y_2)) \geq \min\{\mu_\beta(x_1, y_1), \mu_\beta(x_2, y_2)\}$.
Therefore μ_β is a fuzzy PMS-sub algebra of X x X.

Theorem 4.10. If μ_β is a fuzzy PMS-sub algebra of X x X, then β is a fuzzy PMS- sub algebra of a PMS - algebra X.

Proof: Let x, y $\in$ X.

Now, $\beta(x * y) = \min\{\beta(x * y), \beta(x * y)\}$

$= \mu_\beta((x * y) * (x * y))$

$\geq \min\{\mu_\beta(x * y), \mu_\beta(x * y)\}$

$= \min\{\min\{\beta(x), \beta(y)\}, \min\{\beta(x), \beta(y)\}\}$

$= \min\{\beta(x), \beta(y)\}$

$\Rightarrow \beta(x * y) \geq \min\{\beta(x), \beta(y)\}$

$\therefore$ β is a fuzzy PMS - sub algebra of a PMS - algebra X.

5. Conclusion

In this article, we have been discussed homomorphism and Cartesian product on PMS-algebras. It adds another dimension to the defined PMS-algebras. This concept can further be generalized to Intuitionistic fuzzy set, interval valued fuzzy sets for new results in our future work.

Acknowledgement

Authors wish to thank Dr.T.Priya, Faculty of Mathematics, Omen and to anonymous referees, Editor-in-Chief for their comments, suggestions and ideas to improve and make this paper as successful one.

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