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Secure and Inverse Secure Total Edge Domination and Some Secure and Inverse Secure Fuzzy Domination Parameters

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Abstract. A total edge dominating set F of the edge set E of $G = (V, E)$ is said to be a secure total edge dominating set of G if for every $e\in E - F$, there exists an edge $f\in F$ such that e and f are adjacent and $(F - \{f\}) \cup \{e\}$ is a total edge dominating set of G. The secure total edge domination number $\gamma_{ste}(G)$ of G is the minimum cardinality of a secure total edge dominating set of G. Let F be a minimum secure total edge dominating set of G. If $E - F$ contains a secure total edge dominating set F' of G, then F' is called an inverse secure total edge dominating set with respect to F. The inverse secure total edge domination number $\gamma_{ste}^{-1}(G)$ of G is the minimum cardinality of an inverse secure total edge dominating set of G. In this paper, we initiate a study of these parameters. Also we introduce some secure and inverse secure fuzzy domination parameters.

Keywords: secure total edge domination, inverse secure total edge domination, secure fuzzy domination, inverse secure fuzzy domination, secure fuzzy edge domination, inverse secure fuzzy edge domination.

AMS Mathematics Subject Classification (2010): 05C69, 05C78

1. Introduction

By a graph, we mean a finite, undirected without loops, multiple edges, isolated vertices and isolated edges. For definitions and notations, the reader may refer to [1]. Let $G = (V, E)$ be a graph with $|V| = p$ vertices and $|E| = q$ edges.

A set F of edges in a graph G is an edge dominating set if every edge e in $E - F$ is adjacent to at least one edge in F. The edge domination number $\gamma'(G)$ of G is the minimum cardinality of an edge dominating set of G. Recently several domination parameters are given in the books by Kulli in [2,3,4]. A secure edge dominating set of G is an edge dominating set $F\subseteq E$ with the property that for each $e\in E - F$, there exists $f\in F$ adjacent to e such that $(F - \{f\})\cup\{e\}$ is an edge dominating set. The secure edge domination number $\gamma_s'(G)$ of G is the minimum cardinality of a secure edge dominating set of G. The concept of secure edge domination was introduced by Kulli in [5]. Recently many other domination parameters were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

A set F of edges in G is a total edge dominating set in G if every edge in G is adjacent to at least one edge in F. The total edge domination number $\gamma_{te}(G)$ of G is the minimum cardinality of a total edge dominating set of G. This was introduced by Kulli and Patwari in [16] and was studied, for example, in [17].

Let F be a minimum edge dominating set of G. If $E - F$ contains an edge dominating set F', then F' is called an inverse edge dominating set of G with respect to F. The inverse edge domination number $\gamma_e^{-1}(G)$ of G is the minimum cardinality of an inverse edge dominating set of G. This concept was introduced by Kulli and Soner in [18]. Many other inverse domination parameters were studied for example, in [19, 20, 21, 22].

In this paper, we introduce some secure domination parameters in domination theory and in fuzzy domination theory.

2. Secure total edge domination

We introduce the concept of secure total edge domination in graphs.

Definition 1. A total edge dominating set F of the edge set of a graph $G = (V, E)$ is said to be a secure total edge dominating set of G if for every $e \in E - F$, there exists an edge $f \in F$ such that e and f are adjacent and $(F - \{f\}) \cup \{e\}$ is a total edge dominating set of G. The secure total edge domination number $\gamma_{ste}(G)$ of G is the minimum cardinality of a secure total edge dominating set of G.

Let γ_{ste}-set be a minimum secure total edge dominating set. Note that $\gamma_{ste}G)$ is defined only if G has no isolated vertices and isolated edges.

Proposition 2. Let G be a graph without isolated vertices and isolated edges. Then

$$\gamma_{te}(G) \leq \gamma_{ste}(G) \tag{1}$$

and this bound is sharp.

Proof: Clearly every secure total edge dominating set is a total edge dominating set. Thus (1) holds.

The graph $K_{1,4}$ achieves this bound.

Proposition 3. If F is a secure total edge dominating set of a graph G, then F is a secure edge dominating set of G.

Proof: Suppose F is a secure total edge dominating set of G. Then F is a total edge dominating set of G. Therefore F is an edge dominating set of G. Let $e \in E - F$. Then there exists $f \in F$ such that e and f are adjacent and $(F - \{f\}) \cup \{e\}$ is an edge dominating set of G. Thus F is a secure edge dominating set of G.

Proposition 4. If $K_{1,p}$ is a star with $p \geq 2$ vertices, then $\gamma_{ste}(K_{1,p}) = 2$.

Proof: Let E be the edge set of $K_{1,p}$, where $E = \{e_1, e_2, ..., e_p\}$. Let $F = \{e_1, e_2\} \subseteq E$. Then F is a secure total edge dominating set of $K_{1,p}$. Then $\gamma_{ste}(K_{1,p}) \leq 2$. Suppose $\gamma_{ste}(K_{1,p})$=1. Without loss of generality, F_1=$\{e_1\}$. Then F_1 is not a secure total edge dominating set, so that $\gamma_{ste}(K_{1,p}) = 1$. Without loss of generality $F_1=\{e_1\}$. Then F_1 is not a secure total edge dominating set, so that $\gamma_{ste}(K_{1,p}) \geq 2$. Thus the result follows.

The double star $S_{m,n}$ is the graph obtained from joining centers of two stars $K_{1,m}$ and $K_{1,n}$ with an edge.

Proposition 5. If $S_{m,n}$ is a double star with $1 \leq m \leq n$ vertices, then $\gamma_{ste}(S_{m,n}) = 3$.

Proof: Let E be the edge set of $S_{m,n}$ where $E = \{e, e_1, e_2, ..., e_m, f_1, f_2, ..., f_n\}$. Let $F=\{e, e_1, f_1\} \subseteq E$. Then F is a secure total edge dominating set of $S_{m,n}$. Then $\gamma_{ste}(S_{m,n}) \leq 3$. Suppose $\gamma_{ste}(S_{m,n}) = 2$. Without loss of generality, $F_1 = \{e, e_1\}$. Clearly F_1 is not a secure total edge dominating set, so that $\gamma_{ste}(S_{m,n}) \geq 3$. Thus the result follows.

3. Inverse secure total edge domination

We introduce the following concept.

Definition 6. Let $G = (V, E)$ be a graph. Let F be a minimum secure total edge dominating set of G. If $E - F$ contains a secure total edge dominating set F' of G, then F' is called an inverse secure total edge dominating set with respect to F. The inverse secure total edge domination number $\gamma_{ste}^{-1}(G)$ of G is the minimum cardinality of an inverse secure total edge dominating set of G.

Definition 7. The upper inverse secure total edge domination number $\Gamma_{ste}^{-1}(G)$ of G is the maximum cardinality of an inverse secure total edge dominating set of G.

A γ_{ste}^{-1}-set is a minimum inverse secure total edge dominating set of G.

Example 8. For the graph $K_{1,4}$, $\gamma_{ste}(K_{1,4}) = \gamma_{ste}^{-1}(K_{1,4})$.

Remark 9. Not all graphs have an inverse secure total edge dominating set.

Theorem 10. Let F be a γ_{ste}-set of a connected graph G. If a γ_{ste}^{-1}-set exists, then G has at least 4 edges.

Proof: Let F be a γ_{ste}-set of a connected graph G. Then $\gamma_{ste}(G) = |F| \geq 2$. If a γ_{ste}^{-1}-set exists, then $E - F$ contains a secure total edge dominating set with respect to F. Hence $|E - F| \geq 2$. Thus G has at least 4 edges.

Theorem 11. If $K_{1,p}$, is a star with $p \geq 4$ vertices, then $\gamma_{ste}^{-1}(K_{1,p}) = 2$.

Proof: Let F be a γ_{ste}-set of $K_{1,p}$. By Proposition 4, $|F| = 2$. Let $F = \{e, f\}$. Then $S=\{x, y\}$ is a γ_{ste}^{-1}-set of $K_{1,p}$ for $x, y \in E(K_{1,p}) - \{e, f\}$. Thus $\gamma_{ste}^{-1}(K_{1,p}) = 2$.

Proposition 12. For any star $K_{1,p}$, $p \geq 4$, $\gamma_{ste}(K_{1,p}) = \gamma_{ste}^{-1}(K_{1,p}) = 2$.

Theorem 13. If $S_{m,n}$, is a double star with $3 \leq m \leq n$ vertices, then $\gamma_{ste}^{-1}(S_{m,n}) = 4$.

Proof: Let $E(S_{m,n}) = \{e, e_1, e_2, ..., e_m, f_1, f_2, ..., f_n\}$. By Proposition 5, $F = \{e, e_1, f_1\}$ is a γ_{ste}-set of $S_{m,n}$. Then $F_1 = \{e_z, e_3, f_2, f_3\}$ is an inverse secure total edge dominating set of $S_{m,n}$ in $E - F$. Then $\gamma_{ste}^{-1}(S_{m,n}) \leq 4$. Suppose $\gamma_{ste}^{-1}(S_{m,n})=3$. Without loss of generality, $F_2 = \{e_2, e_3, f_2\}$. Clearly F_2 is not an inverse secure total edge dominating set, so that $\gamma_{ste}^{-1}(S_{m,n}) \geq 4$. Hence the result follows.

Proposition 14. If a γ_{ste}^{-1}-set exists in a graph G, then

$$\gamma_{ste}(G) \le \gamma_{ste}^{-1}(G)$$

and this bound is sharp

Proof: Clearly every inverse secure total edge dominating set of G is a secure total edge dominating set. Thus $\gamma_{ste}(G) \le \gamma_{ste}^{-1}(G)$.

The graph $K_{1,4}$ achieves the lower bound.

Proposition 15. Let G be a graph with p vertices. If a γ_{ste}^{-1}-set exists, then

$$\gamma_{ste}(G) + \gamma_{ste}^{-1}(G) \le p$$

and this bound is sharp.

Proof: This follows from the definition of $\gamma_{ste}^{-1}(G)$.

The graph $K_{1,4}$ realizes the sharp lower bound.

We establish lower and upper bounds for the inverse secure total edge domination number of G.

Theorem 16. Let G be a graph with p vertices. It a γ_{ste}^{-1}-set exists in G, then

$$2 \le \gamma_{ste}^{-1}(G) \le p-2$$

and these bounds are sharp.

Proof: By Proposition 14, $\gamma_{ste}(G) \le \gamma_{ste}^{-1}(G)$ and since $2 \le \gamma_{ste}(G)$, we have

$$2 \le \gamma_{ste}^{-1}(G).$$

By Proposition 15, $\gamma_{ste}^{-1}(G) \le p - \gamma_{ste}(G)$ and since $2 \le \gamma_{ste}(G)$, we have

$$\gamma_{ste}^{-1}(G) \le p-2.$$

Thus the result follows.

The graph $K_{1,4}$ achieves both the lower and upper bounds.

Problem 1. Characterize graphs G for which $\gamma_{ste}(G) = \gamma_{ste}^{-1}(G)$.

Problem 2. Characterize graphs G for which $\gamma_{ste}(G) + \gamma_{ste}^{-1}(G) = p$.

4. Some secure fuzzy domination parameters

In this section, we present some secure fuzzy domination parameters in fuzzy domination theory.

4.1. Secure and inverse secure fuzzy domination

A fuzzy graph G = (V, σ, μ) is a nonempty V together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V\times V \rightarrow [0, 1]$ such that $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. We say that u dominates v in G if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$.

Definition 17. Let G be a fuzzy graph. Let $u, v\in V$. A subset D of V is called a fuzzy dominating set if for every $v \in V - D$, there exists a vertex $u \in D$ such that u dominates v. The minimum cardinality of a fuzzy dominating set of G is called the fuzzy domination number of G and is denoted by $\gamma_f(G)$, see [23, 24].

We now define the secure fuzzy domination in fuzzy graphs.

Definition 18. Let G be a fuzzy graph. A secure fuzzy dominating set of a fuzzy graph G is a fuzzy dominating set $D\subseteq V$ with the property that for each $u \in V - D$ there exists $v\in D$

adjacent to u such that $(D - \{v\})\cup\{u\}$ is a fuzzy dominating set. The secure fuzzy domination number $\gamma_{sf}(G)$ of G is the minimum cardinality of a secure fuzzy dominating set of G.

Next, we define the inverse secure fuzzy domination number.

Definition 19. Let G be a fuzzy graph. Let D be a minimum secure fuzzy dominating set of a fuzzy graph G. If $V - D$ contains a secure fuzzy dominating set D' of G, then D' is called an inverse secure fuzzy dominating set with respect to D. The inverse secure fuzzy domination number $\gamma_{sf}^{-1}(G)$ of G is the minimum cardinality of an inverse secure fuzzy dominating set in G.

4.2 Secure and inverse secure fuzzy edge domination

An edge $e = uv$ of a fuzzy graph is called an effective edge if $\mu(uv)= \sigma(u)\wedge\sigma(v)$.

Definition 20. Let G be a fuzzy graph on (V, E). A subset F of E is called a fuzzy edge dominating set if for every edge in $E - F$ is adjacent to at least one effective edge in F. The minimum cardinality of a fuzzy edge dominating set of G is called the fuzzy edge domination number of G and it is denoted by $\gamma_{fe}(G)$, see [25].

We now define the secure fuzzy edge domination in fuzzy graphs.

Definition 21. Let G be a fuzzy graph on (V, E). A secure fuzzy edge dominating set of a fuzzy graph G is a fuzzy edge dominating set $F\subseteq E$ with the property that for each edge $e\in E - F$, there exists $f\in F$ adjacent to e such that $(F - \{f\}) \cup \{e\}$ is a fuzzy edge dominating set. The secure fuzzy edge domination number $\gamma_{sfe}(G)$ of G is the minimum cardinality of a secure fuzzy edge dominating set of G.

We also define the inverse secure fuzzy edge domination number as follows:

Definition 22. Let G be a fuzzy graph on (V, E). Let F be a minimum secure fuzzy edge dominating set of a fuzzy graph G. If $E - F$ contains a secure fuzzy edge dominating set F' of G, then F' is called an inverse secure fuzzy edge dominating set with respect to F. The secure fuzzy edge domination number $\gamma_{sfe}^{-1}(G)$ of G is the minimum cardinality of an inverse secure fuzzy edge dominating set in G.

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