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Fuzzy Linear Regression to Overcome Assumption Violation Problems in Relationship between Rupee Exchange Rate and Gold Reserve with RBI

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Abstract. Fuzzy set theory has been applied in many fields such as engineering, control theory and life sciences etc. The fuzzy numbers and fuzzy values are widely used in statistical applications because of their suitability for representing uncertain information. In this paper, the method of least square using symmetric triangular fuzzy number instead of crisp values for both dependent and independent values and apply linear regression between rupee exchange rate and gold reserve with RBI.

Keywords: Linear regression, symmetric triangular fuzzy number and fuzzy regression

AMS Mathematics Subject Classification (2010): 03E72, 62J05, 62J12

1. Introduction

Regression analysis refers to a set of techniques for predicting an outcome variable using one (or) more explanatory variables. It is essentially about creating a model for estimating one variable based on the values of others. Simple linear regression is regression analysis in its most basic form, it is used to predict a continuous outcome variable from one continuous explanatory variable. Simple linear regression can be conceived as the process of drawing a line to represent an association between two variables on a scatterplot and using that line as a linear model for predicting the value of one variable from the value of the other (explanatory variables).

Nowadays the stock market has been called for research in many fields due to its effect on financial challenging and capacity of predicting its various aspects through different scientific methods such as genetic algorithm, Artificial Neural Network and other Meta heuristic algorithms. Many institution and academic researchers are trying to propose a method for predicting next day behaviors of stock indexes in order to be to be better than the other methods, like a research that Majhi et al. [8] did via applying bacterial foraging optimization technique for predicting stock market and S & P500 indexes in short and long terms, and they made a linear combiner model which its weights updated by BFO and comparing it with Multi-Layer Perceptron (MLP) based method showed that Majhi et al. method has less calculative complexity and more precision to MLP method. Another predicting system [17] in which counting of complex keyword topples and its transformation to predict stock market behavior periodically and doing real-time forecasting on web has been done. Some researchers used text mining approach [9], their findings investigates effects of financial news in predicting stock market. Increasing social networks and their popularity among people have been led into new ideas of investigating of effect of the popularity and application of these social networks that can have on stock market behavior.

Like a work about effect of emotions like hope, fear and worry have on increasing or decreasing amount of Dow Jones on the next day [19] on stock market. The relation between the tendencies of investors and activities of stock market found by using a new time scale which operates on updated mood of about 100 million American Facebook users between the periods of 10/09/2007 to 10/09/2010.

Since Zadeh (1965) [18] introduced fuzzy set theory, it has been widely developed in theory and application (e.g. [1,2,4,10]). Regression analysis is one of the areas in which fuzzy set theory has been used frequently. A fuzzy linear regression model was first introduced by Tanaka [11]. They formulated a linear regression model with fuzzy response data, crisp predictor data and fuzzy parameters as a mathematical programming problem. Their approach was later improved by Tanaka and others ([12, 13,14,15]). Diamond [6] proposed the fuzzy least squares approach to determine fuzzy parameters by defining a metric between two fuzzy numbers. However most of the articles on fuzzy regression analysis till now use linear programming to estimate the parameters. When using this approach, each additional observation results in several additional constraints, and the linear programming problem becomes unwieldy very quickly.

The rest of this paper is organized as follows: In Section 2, the basic concept and definitions of fuzzy numbers are presented, In Section 3, fuzzy regression model is given, In Section 4, application of a fuzzy regression model is given. Finally the conclusions are presented in Section 5.

2. Preliminaries

2.1. Fuzzy set

A fuzzy set $\hat{A}$ is defined by $\hat{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. In the pair$(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$ belong to the interval [0,1], is called membership function.

2.2. Support of fuzzy set

The support of fuzzy set $\hat{A}$ is the set of all x in X such that $\mu_{\hat{A}}(x) > 0$.

Support $(\hat{A}) = \{ x : \mu_{\hat{A}}(x) > 0 \}$

2.3. Fuzzy number

A fuzzy set $\hat{A}$ on R must possess at least the following three properties.

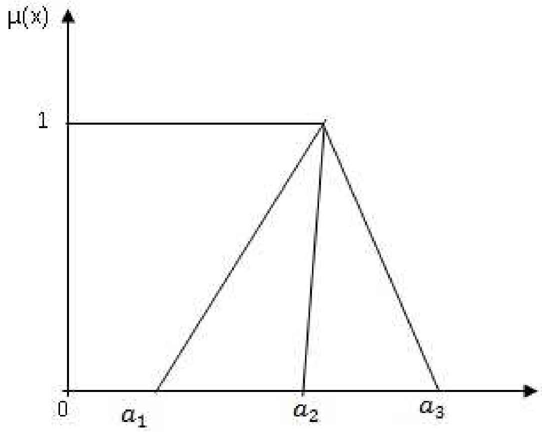
1. $\hat{A}$ must be a normal fuzzy set
2. $\hat{A}$ must be closed interval for every $\alpha \in [0,1]$
3. The support of $\hat{A}$, ${}^{0+}\hat{A}$ must be bounded.

2.4. Triangular fuzzy number

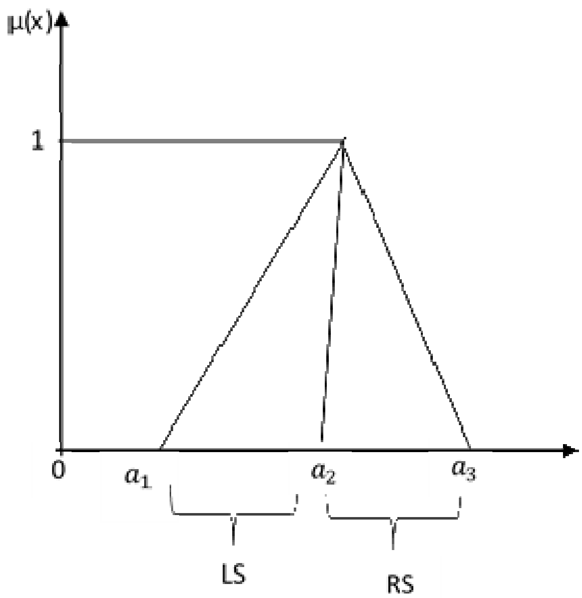
It is a fuzzy number represented with three points as follows, $\hat{A} = (a_1, a_2, a_3)$ this representation is interpreted as membership functions and holds the following conditions.

(i) a_1 to a_2 is increasing function
(ii) a_2 to a_3 is decreasing function
(iii) $a_1 \leq a_2 \leq a_3$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & for\ \ x > a_3 \end{cases}$$



2.5. Symmetric triangular fuzzy number



From the above definition the left spread (LS) and right spread (RS) are equal, then the triangular fuzzy number is called symmetric triangular fuzzy number.

2.6. Fuzzy number operations using function principle

The function principle was introduced by Chen [5] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A}$ = (a_1, a_2, a_3,) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} + \tilde{B}$ = (a_1+b_1, a_2+b_2, a_3+b_3) where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}$ x $\tilde{B}$ = (c_1, c_2, c_3), where T = {a_1b_1, a_1b_3, a_3b_1, a_3b_3}, c_1 = min T, c_2 = a_2b_2, c_3 = Max T

If a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then

$\tilde{A}$ x $\tilde{B}$ = (a_1b_1, a_2b_2, a_3b_3)

(iii) $\tilde{B}$ = (-b_3, -b_2, -b_1) then the subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} - \tilde{B}$ = (a_1-b_3, a_1-b_2, a_3-b_1), where a_1,a_2,a_3, b_1, b_2, b_3 are any real numbers

(iv) The division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$

(v) For any real number K, K$\tilde{A}$ = (Ka_1, Ka_2, Ka_3) if K > 0

K$\tilde{A}$ = (Ka_3, Ka_2, Ka_1) if K < 0

2.7. Graded mean integration representation method-defuzzification

If $\tilde{A}$ = (a_1, a_2, a_3) is a triangular fuzzy number then the graded mean integration representation of $\tilde{A}$ is given by

$$P(\tilde{A}) = [a_1 + 4a_2 + a_3]/6$$

3. Fuzzy least square fit of a straight line method

Suppose that we are given a data set (x_1, y_1), (x_2, y_2),… (x_n, y_n) of ‘n’ observations from an experiment. Now we are interested in fitting a straight line by using symmetric triangular fuzzy numbers in dependent and independent variables.

$$\text{i.e., } \tilde{y} = a\tilde{x} + b \tag{3.1}$$

From (3.1), we can find the ‘n’ residuals $\tilde{e}_i$ by

$$\tilde{e}_i = y_i - \tilde{y}_i = y_i - (a\,\tilde{x}_i + b), \quad i = 1,2,...,n. \tag{3.2}$$

Now consider the sum of the squares of residuals ($\tilde{e}_i$),

$$E = \sum_{i=1}^{n} \tilde{e}_i^2$$

$$E = \sum_{i=1}^{n} [\,\tilde{y}_i - (a\,\tilde{x}_i + b)]^2 \tag{3.3}$$

Note that E is a function of parameters ‘*a*’ and ‘*b*’. we need to find ‘*a*’ and ‘*b*’ such that E is minimum. The necessary condition for E to be minimum is given by

$$\frac{\partial E}{\partial a} = 0 = \frac{\partial E}{\partial b}. \tag{3.4}$$

The condition $\frac{\partial E}{\partial a} = 0$ yields: $\frac{\partial E}{\partial a} = \sum_{i=1}^{n} 2\tilde{x}_i[\,\tilde{y}_i - (a\,\tilde{x}_i + b)] = 0$

$$\text{i.e., } a\sum_{i=1}^{n}\tilde{x}_i^2 + b\sum_{i=1}^{n}\tilde{x}_i = \sum_{i=1}^{n}\tilde{x}_i\tilde{y}_i. \tag{3.5}$$

Similarly $\frac{\partial E}{\partial b}$ = 0 yields: a $\sum_{i=1}^{n} \tilde{x}_i$+ nb = $\sum_{i=1}^{n} \tilde{y}_i$. (3.6)

Equations (3.5) and (3.6) are called as fuzzy normal equations, which are to be solved to get desired values for *a* and *b*.

4. Application of fuzzy regression model

There are verbal statements and theoretical arguments relating gold reserves with exchange values of the currencies of the countries. These statements are to be quantitatively verified. The connection between real exchange rates and real gold prices has not been explored in the extent empirical literature. Gold is regarded as a hedge against the collapse of fiat money [16].

There is a strong correlation between value of gold and the strength of currencies trading on foreign exchanges [7]. The speech of the Governor of the Bank of Italy explains the strong preference for gold by the central banks of the countries that in periods of crisis gold can constitute a sort of reserve or guarantee "of last resort" for a country [3] The long term relationship between exchange rate of national currencies and gold reserves with the central banks of the countries can be studied by a model with annual average exchange rate of a country as response variable and percentage of gold reserves in the total foreign exchange reserves of the country.

Now we are going to fit a straight line between percentage gold in the total foreign exchange reserves of India (X) and Exchange rate of Indian rupee for 1 US dollar (Y).

X	3.7848	3.4061	3.2411	3.8004	6.4452	7.5364	9.1792	8.7974	7.0893	5.5743
Y	44.273	45.249	40.260	45.993	47.443	45.562	47.922	54.409	60.501	61.143

The straight line equation is Y = a + bX

Now applying the Symmetric triangular fuzzy numbers on both X and Y values, then the required equation is $\tilde{y}$ = a+ $b\tilde{x}$.

The STFN is $(\tilde{y}_i, \tilde{y}_{i+1}, \tilde{y}_{i+2})$ = a + b $(\tilde{x}_i, \tilde{x}_{i+1}, \tilde{x}_{i+2})$ and the corresponding normal equations are

$$a\sum_{i=1}^{n}(\tilde{x}_i, \tilde{x}_{i+1}, \tilde{x}_{i+2}) + b\sum_{i=1}^{n}(\tilde{x}_i^2, \tilde{x}_{i+1}^2, \tilde{x}_{i+2}^2) = \sum_{i=1}^{n}(\tilde{x}_i\tilde{y}_i, \tilde{x}_{i+1}\tilde{y}_{i+1}, \tilde{x}_{i+2}\tilde{y}_{i+2}). \quad (4.1)$$

$$n\,a + b\sum_{i=1}^{n}(\tilde{x}_i, \tilde{x}_{i+1}, \tilde{x}_{i+2}) = \sum_{i=1}^{n}(\tilde{y}_i, \tilde{y}_{i+1}, \tilde{y}_{i+2}). \quad (4.2)$$

Consider the left and right spread values are -1 and +1 on both X and Y, then the equations (4.1) and (4.2) are

a(48.8542,58.854,68.8542)+b(284.4864,392.1948,519.9032)
= (2422.8251, 2964.4405, 3526.0559)

10a + b (48.8542, 58.854, 68.8542) = (482.7612, 492.7612, 502.7612)

Solving these two equations we get,

a= (41.011, 41.011, 41.011)
b= (1.4042, 1.4042, 1.4042)

Using the ranking principle for defuzzifying, we get

a= 41.011 and b= 1.4042.

Hence the required equation is Y = 41.011 + 1.4042X.

5. Conclusion

In this paper, a fuzzy regression method using symmetric triangular fuzzy number on both dependent and independent variables in real application problem are discussed.

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