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Some Properties and Theorems on Fuzzy Trident Distance

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Abstract. This paper introduces some simple properties and theorem based on fuzzy trident distance along with the help of trapezoidal fuzzy numbers. The results are discussed along with suitable illustrative example.

Keywords: trapezoidal fuzzy number, trident distance

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1. Introduction

Fuzzy set theory is introduced by Zadeh in the year 1965 [1]. Later Tran and Duckstein gave the comparison of fuzzy numbers using a fuzzy distance measure in the year 2002 [3]. Later Chen and Wang introduced the fuzzy distance of trapezoidal fuzzy numbers in the year 2008 [5]. In the year 2012, Nagoorgani [6] gave a new operation on triangular fuzzy number for solving fuzzy linear programming problem. Optimization of fuzzy production inventory model with repairable defective products under crisp or fuzzy production quantity is given by Chen, Wang and Chang in the year 2005 [4]. Arithmetic operations on generalized trapezoidal fuzzy number and its applications is given by Banerjee and Roy in the year 2012 [7]. In the year 2014, Pardhasaradhi and Shankar gave an idea on fuzzy distance measure [8]. In this paper, some simple properties and theorem based on fuzzy trident distance along with the help of trapezoidal fuzzy numbers are given. This paper consists of five sections. The preliminaries in the first section, defining trapezoidal, positive trapezoidal, negative trapezoidal fuzzy numbers in the second section, fuzzy trident distance in the third section, properties and theorem based on fuzzy trident distance in the fourth section and finally, the results are discussed with suitable numerical examples.

2. Preliminaries

The basic definitions are as follows:

Definition 1. The characteristic function $\mu_{\tilde{A}}$ of a crisp set $\tilde{A} \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$

such that the value assigned to the element of the universal set X fall within a specified range i.e., $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set $\tilde{A}$. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set [2].

Definition 2. A fuzzy set $\tilde{A}$, defined on the universal set of $\Re$, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convexi.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2 \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ $\forall x_1, x_2 \in \Re, \forall \lambda \in [0,1]$.

(ii) $\tilde{A}$ is normal i.e., $\exists x \in \Re$ such that $\mu_{\tilde{A}}(x) = 1$.

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous[2].

3. Representation of generalized (trapezoidal) fuzzy number

In general, a generalized fuzzy number $\tilde{A}$ is described at any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions: [4]

- $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\Re$ to [0,1]
- $\mu_{\tilde{A}}(x) = 0, -\infty \leq x \leq c$
- $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on [c,a]
- $\mu_{\tilde{A}}(x) = w, a \leq x \leq b$
- $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on [b,d]
- $\mu_{\tilde{A}}(x) = 0, d \leq x \leq \infty$ where $0 \prec w \leq 1$ and a, b, c and d real numbers.

We denote this type of generalized fuzzy numbers as $\tilde{A}=(c,a,b,d;w)_{LR}$. When w=1, this type of generalized fuzzy number $\tilde{A}=(c,a,b,d)_{LR}$. When L(x) and R(x) are straight line, then $\tilde{A}$ is Trapezoidal Fuzzy Number and it is denoted by (c,a, b,d).

3.1. Trapezoidal fuzzy number

A trapezoidal fuzzy number is defined as $\tilde{A} = (a_1, a_2, a_3, a_4)$, where all a_1, a_2, a_3, a_4 are real numbers and its membership function is given below:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0 & for \quad x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

3.2. Positive trapezoidal fuzzy number

A positive trapezoidal fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ where all $a_i' s > 0 \forall i = 1,2,3,4.$

Negative trapezoidal fuzzy number

A negative trapezoidal fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ where all $a_i' s < 0 \forall i = 1,2,3,4.$

4. Fuzzy trident distance

The distance between the two fuzzy numbers are calculated by using the new technique called the fuzzy trident distance as follows:

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ then the fuzzy trident distance is given by

$$F_{Tri} distance(\tilde{A}, \tilde{B}) = \left| \left\{ \frac{1}{3} \left[(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3 + (a_4 - b_4)^3 \right]^{1/3} \right\} \right|.$$

5. Properties on fuzzy trident distance

The following are the properties based on fuzzy trident distance:

Property 1. Let $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ are trapezoidal fuzzy numbers. The fuzzy trident distance of $\tilde{A}$ and $\tilde{B}$ is given by $F_{Tri} distance(\tilde{A}, \tilde{B})$ and the fuzzy trident distance of $\tilde{C}$ and $\tilde{D}$ is given by $F_{Tri} distance(\tilde{C}, \tilde{D})$ then $F_{Tri} distance(\tilde{A}, \tilde{B}) \le F_{Tri} distance(\tilde{C}, \tilde{D})$ if $\tilde{A} \le \tilde{C}$ and $\tilde{B} \le \tilde{D}$.

Property 2. If the trapezoidal fuzzy numbers are positive, then the fuzzy trident distance $F_{Tri} distance(\tilde{A}, \tilde{B})$ is positive.

Property 3. If the trapezoidal fuzzy numbers are negative, then the fuzzy trident distance $F_{Tri}distance(\tilde{A},\tilde{B})$ is positive.

6. Theorem on fuzzy trident distance

The theorem based on fuzzy trident distance is as follows:

Theorem 1. The fuzzy trident distance $F_{Tri}dis(\tilde{A},\tilde{B})$ where $\tilde{A},\tilde{B}$ are trapezoidal fuzzy number then the following conditions hold:

$$(i)\,F_{Tri}dis(\tilde{A},\tilde{B}) \geq 0,\ for\ \tilde{A},\tilde{B} > 0.$$

$$(ii)F_{Tri}dis(\tilde{A},\tilde{B}) = F_{Tri}dis(\tilde{B},\tilde{A}).$$

$$(iii)\,F_{Tri}dis(\tilde{A},\tilde{B}) = 0 \Leftrightarrow \tilde{A} = \tilde{B}.$$

$$(iv)F_{Tri}dis(\tilde{A},\tilde{C}) \leq F_{Tri}dis(\tilde{A},\tilde{B}) + F_{Tri}dis(\tilde{B},\tilde{C}),$$

$where A, B, C\ are\ trapezoidal\ fuzzy\ numbers.$

Proof. $(i)\ To\ prove\ F_{Tri}\,dis(\tilde{A},\tilde{B})\geq 0.$

Let us consider $\tilde{A}=(a_1,a_2,a_3,a_4),\tilde{B}=(b_1,b_2,b_3,b_4)$ are trapezoidal fuzzy numbers.
The proof is obvious from the definition of fuzzy trident distance is given by

Thus for all values of $\tilde{A},\tilde{B} > 0, F_{Tri}dis(\tilde{A},\tilde{B}) \geq 0.$
Hence the proof.

$(ii)\ To\ prove\ F_{Tri}dis(\tilde{A},\tilde{B})=F_{Tri}dis(\tilde{B},\tilde{A})$

Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ are trapezoidal fuzzy numbers.

$$F_{Tri}\,dis(\tilde{A},\tilde{B})=\left|\left\{\frac{1}{3}\left[(a_1-b_1)^3+(a_2-b_2)^3+(a_3-b_3)^3+(a_4-b_4)^3\right]^{1/3}\right\}\right|.$$

$$=\left|\left\{\frac{1}{3}\left[(b_1-a_1)^3+(b_2-a_2)^3+(b_3-a_3)^3+(b_4-a_4)^3\right]^{1/3}\right\}\right|.$$

$$=F_{Tri}\,dis(\tilde{B},\tilde{A}).$$

Thus $F_{Tri}dis(\tilde{A},\tilde{B})=F_{Tri}dis(\tilde{B},\tilde{A})$
Hence the proof.

$(iii)\ To\ prove\ F_{Tri}dis(\tilde{A},\tilde{B})=0\Leftrightarrow\tilde{A}=\tilde{B}$

Let us consider $\tilde{A}=(a_1,a_2,a_3,a_4),\tilde{B}=(b_1,b_2,b_3,b_4)$ are trapezoidal fuzzy numbers.

$$F_{Tri}\,distance(\tilde{A},\tilde{B})=0$$
$$\Leftrightarrow\left|\left\{\frac{1}{3}\left[(a_1-b_1)^3+(a_2-b_2)^3+(a_3-b_3)^3+(a_4-b_4)^3\right]^{1/3}\right\}\right|=0$$
$$\Leftrightarrow(a_1-b_1)^3+(a_2-b_2)^3+(a_3-b_3)^3+(a_4-b_4)^3=0$$
$$\Leftrightarrow(a_1-b_1)^3=0,(a_2-b_2)^3=0,(a_3-b_3)^3=0,(a_4-b_4)^3=0$$
$$\Leftrightarrow a_1-b_1=0,a_2-b_2=0,a_3-b_3=0,a_4-b_4=0.$$
$$\Leftrightarrow a_1=b_1,a_2=b_2,a_3=b_3,a_4=b_4.$$
$$\Leftrightarrow\tilde{A}=\tilde{B}$$

Thus $F_{Tri}\,dis(\tilde{A},\tilde{B})=0\Leftrightarrow\tilde{A}=\tilde{B}$

Hence the proof.

(iv) *To prove* $F_{Tri}\,dis(\tilde{A},\tilde{C})\leq F_{Tri}\,dis(\tilde{A},\tilde{B})+F_{Tri}\,dis(\tilde{B},\tilde{C})$.

Let us consider $\tilde{A}=(a_1,a_2,a_3,a_4),\tilde{B}=(b_1,b_2,b_3,b_4),\tilde{C}=(c_1,c_2,c_3,c_4)$ are trapezoidal fuzzy numbers.

$$F_{Tri}\,distance(\tilde{A},\tilde{C})=\left|\left\{\frac{1}{3}\left[(a_1-c_1)^3+(a_2-c_2)^3+(a_3-c_3)^3+(a_4-c_4)^3\right]^{1/3}\right\}\right|.$$
$$\leq\left|\left\{\frac{1}{3}\begin{bmatrix}(a_1-b_1+b_1-c_1)^3+(a_2-b_2+b_2-c_2)^3+\\(a_3-b_3+b_3-c_3)^3+(a_4-b_4+b_4-c_4)^3\end{bmatrix}^{1/3}\right\}\right|.$$
$$\leq\left|\left\{\frac{1}{3}\begin{bmatrix}(a_1-b_1)^3+(b_1-c_1)^3+(a_2-b_2)^3+(b_2-c_2)^3+\\(a_3-b_3)^3+(b_3-c_3)^3+(a_4-b_4)^3+(b_4-c_4)^3\end{bmatrix}^{1/3}\right\}\right|$$
$$\leq\left|\left\{\frac{1}{3}\left[(a_1-b_1)^3+(a_2-b_2)^3+(a_3-b_3)^3+(a_4-b_4)^3\right]^{1/3}\right\}\right|$$
$$+\left|\left\{\frac{1}{3}\left[(b_1-c_1)^3+(b_2-c_2)^3+(b_3-c_3)^3+(b_4-c_4)^3\right]^{1/3}\right\}\right|$$
$$\leq F_{Tri}\,dis(\tilde{A},\tilde{B})+F_{Tri}\,dis(\tilde{B},\tilde{C}).$$

Thus $F_{Tri}\,dis(\tilde{A},\tilde{C})\leq F_{Tri}\,dis(\tilde{A},\tilde{B})+F_{Tri}\,dis(\tilde{B},\tilde{C})$.

Hence the proof.

Example 1. Consider the following example:

Let $\tilde{A}=(0.2,0.4,0.6,0.8),\tilde{B}=(0.3,0.5,0.7,0.9)\, and\, \tilde{C}=(0.1,0.2,0.3,0.4)$

(i)To prove $F_{Tri}dis(\tilde{A},\tilde{B})\geq 0.$

$Solution:$

$$F_{Tri}dis(\tilde{A},\tilde{B})=\left|\left\{\frac{1}{3}\left[(0.2-0.3)^3+(0.4-0.5)^3+(0.6-0.7)^3+(0.8-0.9)^3\right]^{1/3}\right\}\right|$$

$$=\left|\frac{1}{3}(-0.1587)\right|=0.05291\geq 0.$$

Hence proved.

(ii)To prove $F_{Tri}dis(\tilde{A},\tilde{B})=F_{Tri}dis(\tilde{B},\tilde{A})$

$Solution:$

$L.H.S:$

$$F_{Tri}dis(\tilde{A},\tilde{B})=\left|\left\{\frac{1}{3}\left[(0.2-0.3)^3+(0.4-0.5)^3+(0.6-0.7)^3+(0.8-0.9)^3\right]^{1/3}\right\}\right|$$

$$=\left|\frac{1}{3}(-0.1587)\right|=0.05291\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$R.H.S:$

$$F_{Tri}dis(\tilde{B},\tilde{A})=\left|\left\{\frac{1}{3}\left[(0.3-0.2)^3+(0.5-0.4)^3+(0.7-0.6)^3+(0.9-0.8)^3\right]^{1/3}\right\}\right|$$

$$=\left|\frac{1}{3}(0.1587)\right|=0.05291\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From equations (1) and (2)

L.H.S = R.H.S.

Thus $F_{Tri}dis(\tilde{A},\tilde{B})=F_{Tri}dis(\tilde{B},\tilde{A}).$

Hence proved.

(iii) To prove $F_{Tri}dis(A,B)=0\Leftrightarrow A=B.$

$Solution:$

Let us consider $\tilde{A}=(0.2,0.4,0.6,0.8),\tilde{B}=(0.2,0.4,0.6,0.8)$ be two trapezoidal fuzzy numbers. If $\tilde{A}=\tilde{B}$ then,

$$F_{Tri}dis(\tilde{A},\tilde{B})=\left|\left\{\frac{1}{3}\left[(0.2-0.2)^3+(0.4-0.4)^3+(0.6-0.6)^3+(0.8-0.8)^3\right]^{1/3}\right\}\right|$$

$=0.$

Thus $\tilde{A}=\tilde{B}\Rightarrow F_{Tri}dis(\tilde{A},\tilde{B})=0.$

Conversely, if $F_{Tri}dis(\tilde{A},\tilde{B})=0$ then,

$F_{Tri}dis(\tilde{A},\tilde{B})=0$

$$\Rightarrow\left|\left\{\frac{1}{3}\left[(0.2-0.2)^3+(0.4-0.4)^3+(0.6-0.6)^3+(0.8-0.8)^3\right]^{1/3}\right\}\right|=0$$

$$\Rightarrow (0.2-0.2)^3+(0.4-0.4)^3+(0.6-0.6)^3+(0.8-0.8)^3=0$$

$$\Rightarrow (0.2-0.2)^3=0, (0.4-0.4)^3=0, (0.6-0.6)^3=0, (0.8-0.8)^3=0$$

$$\Rightarrow 0.2=0.2, 0.4=0.4, 0.6=0.6, 0.8=0.8.$$

$$\Rightarrow A=B.$$

Thus $F_{Tri}dis(\tilde{A},\tilde{B})=0\Rightarrow\tilde{A}=\tilde{B}.$

Hence proved.

(iv) To prove $F_{Tri}dis(\tilde{A},\tilde{C})\le F_{Tri}dis(\tilde{A},\tilde{B})+F_{Tri}dis(\tilde{B},\tilde{C}).$

Solution

L.H.S :

$$F_{Tri}dis(\tilde{A},\tilde{C})=\left|\left\{\frac{1}{3}\left[(0.2-0.1)^3+(0.4-0.2)^3+(0.6-0.3)^3+(0.8-0.4)^3\right]^{1/3}\right\}\right|$$

$$=\frac{1}{3}(0.1)^{1/3}=\frac{0.4642}{3}=0.15472\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

R.H.S:

$F_{Tri}dis(\tilde{A},\tilde{B})+F_{Tri}dis(\tilde{B},\tilde{C})$

$$=\left|\left\{\frac{1}{3}\left[(0.2-0.3)^3+(0.4-0.5)^3+(0.6-0.7)^3+(0.8-0.9)^3\right]^{1/3}\right\}\right|+$$

$$\left|\left\{\frac{1}{3}\left[(0.3-0.1)^3+(0.5-0.2)^3+(0.7-0.3)^3+(0.9-0.4)^3\right]^{1/3}\right\}\right|$$

$=0.05291+0.2024$

$=0.25534\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$

From equations (3) and (4)
L.H.S $\leq$ R.H.S.

Thus $F_{Tri}\, dis(\tilde{A},\tilde{C}) \leq F_{Tri}\, dis(\tilde{A},\tilde{B}) + F_{Tri}\, dis(\tilde{B},\tilde{C})$.

Hence proved.

7. Conclusion

The main aim of this paper is to introduce new properties and the theorem based on fuzzy trident distance. The advantage of this paper is simple and easy to apply and to solve transportation problems.

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