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On Some Structural Properties of Fuzzy Soft Topological Spaces

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Abstract. This paper deals with some structural properties of fuzzy soft topological spaces. Fuzzy soft closure and fuzzy soft interior of a fuzzy soft set are studied and investigated. Fuzzy soft exterior and fuzzy soft boundary of a fuzzy soft set are introduced and some properties related to these structures are established.

***Keywords*:** Soft set, fuzzy set, fuzzy soft set, fuzzy soft topological spaces, fuzzy soft open set, fuzzy soft closed set, fuzzy soft closure, fuzzy soft interior, fuzzy soft exterior, fuzzy soft boundary.

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1. Introduction

Arena of engineering, physics, computer science, economics, social science, medical science, and many other diverse fields deal with the uncertain data which may not be successfully modeled by the classical mathematics. There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets etc. which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets, introduced by L.A.

Zadeh [44] in 1965. This theory brought a paradigmatic change in mathematics. But there exists difficulty, how to set the membership function in each particular case. The theory of intuitionistic fuzzy sets is more generalized concept than the theory of fuzzy sets, but this theory has the same difficulties. All the above mentioned theories are successful to some extent in dealing with problems arising due to vagueness present in the real world. But there are also cases where these theories failed to give satisfactory results, possibly due to inadequacy of the parameterization tool in them. As a necessary supplement to the existing mathematical tools for handling uncertainty, in 1999, Molodtsov [31] initiated the concept of soft set via a set-valued mapping. The theory of soft sets is free from the difficulties mentioned above. Since its introduction, the concept of soft set has gained considerable attention and this concept has resulted in a series of works [3], [11], [16], [17], [18], [19], [20], [25], [27], [30], [41], [42], [43], [45] including some successful applications in information processing [14], [15], [32], [47], decision-making [8], [12], [26], [29], [35], demand analysis [10], forcasting [40], relations [9], algebraic structures of the set theory [1], [3], [4], [11], [17], [18], [19], [20], [21], [22], [23], [24], [34], [38] , topology [6], [11], [37], [39], [46], theory of BCK/BCI-algebra [17], operation research [5], [13], [16] etc. In recent times, researchers have contributed a lot towards fuzzification of theory soft sets. Maji et al. [28], introduced the concept of fuzzy soft set and some properties regarding fuzzy soft union, fuzzy soft intersection, complement of fuzzy soft set, De Morgan Law etc. In section 3, properties of fuzzy soft closure, fuzzy soft interior are studied and investigated. Also in this section, concepts of fuzzy soft exterior, fuzzy soft boundary are introduced and some properties related to these structures are established.

2. Preliminaries

Definition 2.1. [31**]** Let $A \subseteq E$. A pair (F, A) is called a soft set over U if and only if F is a mapping given by $F : A \rightarrow \mathrm{P}(U)$ such that $F(e) = \varphi$ if $e \notin A$ and $F(e) \neq \varphi$ if $e \in A$, where U is an initial universe set and E be the set of parameters, $\mathrm{P}(U)$ be the set of all subsets of U. Here F is called approximate function of the soft set (F, A) and the value $F(e)$ is a set called e–element of the soft set. In other words, the soft set is a parameterized family of subsets of the set U.

Definition 2.2. [44] A fuzzy set A in any arbitrary set U is defined by the mapping $\mu_A : U \rightarrow [0,1] = I$ [or by $A : U \rightarrow [0,1] = I$], where $\mu_A(u)$ or $A(u)$ states the grade of belongingness (membership) of u in A. That is, a fuzzy set A in U can be represented by the set of ordered pairs $A = \{ (u, \mu_A(u)) : u \in U \}$. The family of all fuzzy sets in U is denoted by I^U.

Definition 2.3. [44] Let A and B be two fuzzy sets in U. That is, $A, B \notin I^U$

(a) A is contained in B, denoted by $A \leq B$, *if and only if* $A(u) \leq B(u)$ for all $u \in U$.

(b) The union of A and B, denoted by $A \vee B$, is a fuzzy set C and defined as $C(u) = (A \vee B)(u) = Max\{ A(u), B(u) \}$ for each $u \in U$.

(c) The intersection of A and B, denoted by $A \wedge B$, is a fuzzy set D and defined as $D(u) = (A \wedge B)(u) = Min\{ A(u), B(u) \}$ for each $u \in U$.
(d) The complement of a fuzzy set A is a fuzzy set, denoted by A', and is defined as $A'(u) = 1 - A(u)$ for every $u \in U$.

Definition 2.4. [36, 28] Let U be an initial universe set and let E be a set of parameters. Let I^U (I=[0,1]) denotes the set of all fuzzy sets of U. Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U if and only if F is a mapping given by $F : A \rightarrow I^U$ such that $F(e) = 0_U$ if $e \notin A$ and $F(e) \neq 0_U$ if $e \in A$, where $0_U(u) = 0$ for all $u \in U$. Here F is called approximate function of the fuzzy soft set (F, A) and the value $F(e)$ is a fuzzy set called e–element of the fuzzy soft set (F, A). Thus a fuzzy soft set (F, A) over U can be represented by the set of ordered pairs $(F, A) = \{ (e, F(e)) : e \in A, F(e) \in I^U \}$. In other words, the fuzzy soft set is a parameterized family of fuzzy subsets of the set U.

Definition 2.5. [7, 8] A fuzzy soft set (F, A) over U is called a *null* fuzzy soft set, denoted by $\tilde{0}_E$, if $F(e) = 0_U$ for all $e \in A \subseteq E$.

Remark 2.1. According to the definition of fuzzy soft set $F(e) \neq 0_U$ if $e \in A \subseteq E$, 0_U does not belong to the co-domain of F. Therefore, the concept of null fuzzy soft set can be defined as follows.

Definition 2.6. A fuzzy soft set (F, A) over U is called a *null* fuzzy soft set or an *empty* fuzzy soft set, whenever $A = \varphi$.

Definition 2.7. A fuzzy soft set (F, A) over U is said to be an *A–universal* fuzzy soft set if $F(e) = 1_U$ if $e \in A$, where $1_U(u) = 1$ for all $u \in U$. An *A–universal* fuzzy soft set is denoted by $\tilde{1}_A$.

Definition 2.8. [36] A fuzzy soft set (F, A) over U is said to be an *absolute* fuzzy soft set or a *universal* fuzzy soft set if $A = E$ and $F(e) = 1_U$ for all $e \in E$. An *absolute* fuzzy soft set is denoted by $\tilde{1}_E$.

Definition 2.9. [28] A fuzzy soft set (F, A) is said to be a fuzzy soft subset of a fuzzy soft set (G, B) over a common universe U if $A \subseteq B$ and $F(e) \leq G(e)$, for all $e \in A$.

We redefine fuzzy soft subset as follows.

Definition 2.10. A fuzzy soft set (F, A) is said to be a fuzzy soft subset of a fuzzy soft set (G, B) over a common universe U if either $F(e) = 0_U$ for all $e \in A$ or $A \subseteq B$ and $F(e) \leq G(e)$, for all $e \in A$.

If a fuzzy soft set (F, A) is fuzzy soft subset of a fuzzy soft set (G, B) we write $(F, A) \tilde{\subseteq} (G, B)$. (F, A) is is said to be a fuzzy soft superset of a fuzzy soft set (G, B) if (G, B) is fuzzy soft subset of (F, A) and we write $(F, A) \tilde{\supseteq} (G, B)$.

Definition 2.11.[36] Two fuzzy soft sets (F, A) and (G, B) over a common universe is said to be equal, denoted by $(F, A) = (G, B)$ if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$. That is, if $F(e) \le G(e)$ and $G(e) \le F(e)$ for all $e \in E$.

Definition 2.12.[2] The intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cap B$ and $H(e) = F(e) \wedge G(e)$ for all $e \in C$ and we write $(H, C) = (F, A) \tilde{\cap} (G, B)$.

In particular if $A \cap B = \varphi$ or $F(e) \wedge G(e) = 0_U$ for every $e \in A \cap B$ then $H(e) = 0_U$.

Definition 2.13.[28] The union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A–B$, $H(e) = G(e)$ if $e \in B–A$, $H(e) = F(e) \vee G(e)$ if $e \in A \cap B$ and in this case we write $(H, C) = (F, A) \tilde{\cup} (G, B)$.

Definition 2.14.[28] The complement of a fuzzy soft sets (F, A), denoted by $(F, A)^C$, and is defined as $(F, A)^C = (F^C, \neg A)$, where $F^C : \neg A \rightarrow I^U$ is a mapping given by $F^C(e) = (F(\neg e))^C$ for all $e \in \neg A$.

Alternatively, the complement of a fuzzy soft sets can be defined as follows.

Definition 2.15. [39] The fuzzy soft complement of a fuzzy soft sets (F, A), denoted by $(F, A)^C$, and is defined as $(F, A)^C = (F^C, A)$, where $F^C(e) = 1 - F(e)$, for every $e \in A$. Clearly $((F, A)^C)^C = (F, A)$ and $(\tilde{1}_E)^C = \tilde{0}_E$ and $(\tilde{0}_E)^C = \tilde{1}_E$.

Proposition 2.1. Let (F, A) be a fuzzy soft sets over (U, E). Then

(1) $(F, A) \tilde{\cup} (F, A) = (F, A)$, $(F, A) \tilde{\cap} (F, A) = (F, A)$

(2) $(F, A) \tilde{\cup} \tilde{0}_E = (F, A)$, $(F, A) \tilde{\cap} \tilde{0}_E = (F, A)$

(3) $(F, A) \tilde{\cup} \tilde{1}_E = \tilde{1}_E$, $(F, A) \tilde{\cap} \tilde{1}_E = (F, A)$

(4) $(F, A) \tilde{\cup} (F, A)^C = \tilde{1}_E$, $(F, A) \tilde{\cap} (F, A)^C = \tilde{0}_E$

Proposition 2.2. Let (F, A), $(G B)$, (H, C) be fuzzy soft sets over (U, E). Then

(1) $(F, A) \tilde{\cup} (G B) = (G, B) \tilde{\cup} (F, A)$, $(F, A) \tilde{\cap} (G, B) = (G, B) \tilde{\cap} (F, A)$

(2) $((F, A) \tilde{\cup} (G, B))^C = (G, B)^C \tilde{\cap} (F, A)^C$, $((F, A) \tilde{\cap} (G, B))^C = (G, B)^C \tilde{\cup} (F, A)^C$

(3) $((F, A)\tilde{\cup}(G, B))\tilde{\cup}(H, C) = (F, A)\tilde{\cup}((G, B)\tilde{\cup}(H, C))$, $((F, A)\tilde{\cap}(G, B))\tilde{\cap}(H, C) = (F, A)\tilde{\cap}((G, B)\tilde{\cap}(H, C))$

(4) $(F, A)\tilde{\cup}((G, B)\tilde{\cap}(H, C)) = ((F, A)\tilde{\cup}(G\ B))\tilde{\cap}((F, A)\tilde{\cup}(H, C))$, $(F, A)\tilde{\cap}((G, B))\tilde{\cup}(H, C)) = ((F, A)\tilde{\cap}(G, B))\tilde{\cup}((F, A)\tilde{\cap}(H, C))$

Definition 2.16.[36, 39] A fuzzy soft topology τ on (U, E) is a family of fuzzy soft sets over (U, E), satisfying the following properties.

(1) $\tilde{0}_E, \tilde{1}_E \in \tau$

(2) If $(F, A), (G\ B) \in \tau$ then $(F, A)\tilde{\cap}(G, B) \in \tau$

(3) If $(F, A)_\alpha \in \tau$, $\forall \alpha \in \Lambda$ then $\tilde{\bigcup}_{\alpha\in\Lambda} (F, A)_\alpha \in \tau$

Definition 2.17.[36] If τ is a fuzzy soft topology on (U, E), the triple (U, E, τ), is said to be fuzzy soft topological space. Each member of τ is called fuzzy soft open set in (U, E, τ).

Definition 2.18.[33] Let (U, E, τ) be a fuzzy soft topological space. A fuzzy soft set is called fuzzy soft closed if its complement is a member of τ.

Proposition 2.3.[33] Let (U, E, τ) be a fuzzy soft topological space) and let τ' be the collection of all fuzzy soft closed sets. Then

(1) $\tilde{0}_E, \tilde{1}_E \in \tau'$

(2) If $(F, A), (G\ B) \in \tau'$ then $(F, A)\tilde{\cup}(G, B) \in \tau'$

(3) If $(F, A)_\alpha \in \tau'$, $\forall \alpha \in \Lambda$ then $\tilde{\bigcap}_{\alpha\in\Lambda} (F, A)_\alpha \in \tau'$

3. Fuzzy Soft Closure, Fuzzy Soft Interior, Fuzzy Soft Exterior, Fuzzy Soft Boundary

In this section fuzzy soft closure, fuzzy soft interior of a fuzzy soft set are studied and some properties related to these structures are investigated. Also the concept of fuzzy soft exterior and fuzzy soft boundary are introduced and some properties are established.

Definition 3.1.[33, 39] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft closure of (F, A), denoted by $\overline{(F, A)}$, is defined as the intersection of all fuzzy soft closed sets which contain (F, A). That is, $\overline{(F, A)} = \tilde{\cap}\{ (G, B) : (G, B)$ is fuzzy soft closed and $(F, A)\tilde{\subseteq} (G, B)\}$. Clearly, $\overline{(F, A)}$ is the smallest fuzzy soft closed set over (U, E) which contain (F, A). It is also clear that $\overline{(F, A)}$ is fuzzy soft closed and $(F, A)\tilde{\subseteq} \overline{(F, A)}$.

Example 3.1. Let $U = \{x, y, z\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2,\} \subseteq E$ Let us consider the following fuzzy soft sets over (U, E).
$(F, A) = \{F(e_1) = \{ x/0.6, y/0.1, z/0\}, F(e_2) = \{ x/0.7, y/0.9, z/0.5\}, F(e_3) = \{ x/0, y/0, z/0\}, F(e_4) = \{ x/0.5, y/0.3, z/0.9\}\}$,
$(G, B) = \{G(e_1) = \{ x/0.4, y/0.1, z/0\}, G(e_2) = \{ x/0.6, y/0.5, z/0.2\}, G(e_3) = \{ x/0, y/0, z/0\}, G(e_4) = \{ x/0, y/0, z/0\}\}$

Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (F, A), (G, B)\}$.

Now $(F, A)^C = (F^C, A) = \{ F^C(e_1) = \{ x/0.4, y/0.9, z/1\}, F^C(e_2) = \{ x/0.3, y/0.1, z/0.5\}, F^C(e_3) = \{ x/1, y/1, z/1\}, F^C(e_4) = \{ x/0.5, y/0.7, z/0.1\}\}$
and $(G, B)^C = (G^C, B) = \{G^C(e_1) = \{ x/0.6, y/0.9, z/1\}, G^C(e_2) = \{ x/0.4, y/0.5, z/0.8\}, G^C(e_3) = \{ x/1, y/1, z/1\}, G^C(e_4) = \{ x/1, y/1, z/1\}\}$.
Clearly, $(F, A)^C$ and $(G, B)^C$ are fuzzy soft closed.
Let us consider the following fuzzy soft sets over (U, E). $(M, C) = \{M(e_1) = \{ x/0.5, y/0.7, z/0.8\}, M(e_2) = \{ x/0.3, y/0.5, z/0.6\}, M(e_3) = \{x/0, y/0, z/0\}, M(e_4) = \{ x/0, y/0, z/0\}\}$
Then the fuzzy soft closure of (M, C), denoted by $\overline{(M, C)}$, is the intersection of all fuzzy soft closed sets containing (M, C). That is, $\overline{(M, C)} = (G, B)^C \tilde{\cap}\, \tilde{1}_E = (G, B)^C$ $= \{G^C(e_1) = \{ x/0.6, y/0.9, z/1\}, G^C(e_2) = \{ x/0.4, y/0.5, z/0.8\}, G^C(e_3) = \{ x/1, y/1, z/1\}, G^C(e_4) = \{ x/1, y/1, z/1\}\}$.

Theorem 3.1. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and (G, B) are fuzzy soft sets over (U, E). Then

(1) $\overline{\tilde{0}_E} = \tilde{0}_E$, $\overline{\tilde{1}_E} = \tilde{1}_E$.

(2) $(F, A) \tilde{\subseteq} \overline{(F, A)}$.

(3) (F, A) is fuzzy soft closed if and only if $(F, A) = \overline{(F, A)}$.

(4) $\overline{(\overline{(F, A)})} = \overline{(F, A)}$.

(5) $(F, A) \tilde{\subseteq} (G, B)$ implies $\overline{(F, A)} \tilde{\subseteq} \overline{(G, B)}$.

(6) $\overline{(F, A) \tilde{\cup} (G, B)} = \overline{(F, A)} \tilde{\cup} \overline{(G, B)}$.

(7) $\overline{(F, A) \tilde{\cap} (G, B)} \tilde{\subseteq} \overline{(F, A)} \tilde{\cap} \overline{(G, B)}$

Proof. (1) and (2) are obvious.
(3) If (F, A) is fuzzy soft closed set over (U, E) then (F, A) is itself a fuzzy soft closed set over (U, E) which contains (F, A). So (F, A) is smallest fuzzy soft closed set over (U, E) containing (F, A) and $(F, A) = \overline{(F, A)}$.

Conversely, let $(F, A) = \overline{(F, A)}$. Since $\overline{(F, A)}$ is fuzzy soft closed set, so (F, A) is a fuzzy soft closed set over (U, E).
(4) Since $\overline{(F, A)}$ is fuzzy soft closed set therefore, by (3) we have $\overline{(\overline{(F, A)})} = \overline{(F, A)}$.
(5) Let $(F, A) \tilde{\subseteq} (G, B)$. Then every fuzzy soft closed superset of (G, B) will also contain (F, A). That is, every fuzzy soft closed superset of (G, B) is also a fuzzy soft closed superset of (F, A). Hence the intersection of fuzzy soft closed superset of

(F, A) is contained in the fuzzy soft intersection of fuzzy soft closed superset of (G, B). Thus $\overline{(F,A)} \mathrel{\tilde{\subseteq}} \overline{(G,B)}$.

(6) Since $(F, A) \mathrel{\tilde{\subseteq}} (F, A) \mathrel{\tilde{\cup}} (G, B)$ and $(G, B) \mathrel{\tilde{\subseteq}} (F, A) \mathrel{\tilde{\cup}} (G, B)$, so by (5), $\overline{(F,A)} \mathrel{\tilde{\subseteq}} \overline{(F,A) \tilde{\cup} (G,B)}$ and $\overline{(G,B)} \mathrel{\tilde{\subseteq}} \overline{(F,A) \tilde{\cup} (G,B)}$.

Thus $\overline{(F,A)} \mathrel{\tilde{\cup}} \overline{(G,B)} \mathrel{\tilde{\subseteq}} \overline{(F,A) \tilde{\cup} (G,B)}$.

Conversely, as $(F, A) \mathrel{\tilde{\subseteq}} \overline{(F,A)}$ and $(G, B) \mathrel{\tilde{\subseteq}} \overline{(G,B)}$. So $(F, A) \mathrel{\tilde{\cup}} (G, B) \mathrel{\tilde{\subseteq}} \overline{(F,A)} \mathrel{\tilde{\cup}} \overline{(G,B)}$. Then by (5), $\overline{(F,A)} \mathrel{\tilde{\cup}} \overline{(G,B)}$ is a fuzzy soft closed set over (U, E) being the union of two fuzzy soft closed sets. Thus $\overline{(F,A) \tilde{\cup} (G,B)} \mathrel{\tilde{\subseteq}} \overline{(F,A)} \mathrel{\tilde{\cup}} \overline{(G,B)}$. Thus $\overline{(F,A) \tilde{\cup} (G,B)} = \overline{(F,A)} \mathrel{\tilde{\cup}} \overline{(G,B)}$.

(7) Since $(F, A) \mathrel{\tilde{\cap}} (G, B) \mathrel{\tilde{\subseteq}} (F, A)$ and $(F, A) \mathrel{\tilde{\cap}} (G, B) \mathrel{\tilde{\subseteq}} (G, B)$, so by (5) $\overline{(F,A) \tilde{\cap} (G,B)} \mathrel{\tilde{\subseteq}} \overline{(F,A)}$ and $\overline{(F,A) \tilde{\cap} (G,B)} \mathrel{\tilde{\subseteq}} \overline{(G,B)}$.

Thus $\overline{(F,A) \tilde{\cap} (G,B)} \mathrel{\tilde{\subseteq}} \overline{(F,A)} \mathrel{\tilde{\cap}} \overline{(G,B)}$.

Definition 3.2. [33, 39] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft interior of (F, A), denoted by $(F, A)^o$, is defined as the union of all fuzzy soft open sets contained in (F, A). That is, $(F, A)^o = \tilde{\bigcup}\{ (G, B) : (G, B)$ is fuzzy soft open and $(G, B) \mathrel{\tilde{\subseteq}} (F, A) \}$. Clearly, $(F, A)^o$ is the largest fuzzy soft open set over (U, E) contained in (F, A). It is also clear that $(F, A)^o$ is fuzzy soft open and $(F, A)^o \mathrel{\tilde{\subseteq}} (F, A)$.

Example 3.2. Let $U = \{x, y, z\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2,\} \subseteq E$. Let us consider the following fuzzy soft sets over (U, E). $(F, A) = \{F(e_1) = \{ x/0.6, y/0.1, z/0\}, F(e_2) = \{ x/0.7, y/0.9, z/0.5\}, F(e_3) = \{ x/0, y/0, z/0\}, F(e_4) = \{ x/0.5, y/0.3, z/0.9\}\}$.

$(G, B) = \{G(e_1) = \{ x/0.4, y/0.1, z/0\}, G(e_2) = \{ x/0.6, y/0.5, z/0.2\}, G(e_3) = \{ x/0, y/0, z/0\}, G(e_4) = \{ x/0, y/0, z/0\}\}$

Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (F, A), (G, B)\}$. Let us consider the following fuzzy soft sets over (U, E). $(N, D) = \{N(e_1) = x/0.6, y/0.3, z/0.2\}, N(e_2) = \{ x/0.7, y/0.5, z/0.4\}, N(e_3) = \{ x/0.2, y/0.8, z/0.6\}, N(e_4) = \{ x/0, y/0, z/0\}\}$. Then the fuzzy soft interior of (N, D), denoted by $(N, D)^o$, is the union of all fuzzy soft open sets contained in (N, D). That is, $(N, D)^o = (G, B) \mathrel{\tilde{\cup}} \tilde{0}_E = (G, B) = \{G(e_1) = \{ x/0.4, y/0.1, z/0\}, G(e_2) = \{ x/0.6, y/0.5, z/0.2\}, G(e_3) = \{ x/0, y/0, z/0\}, G(e_4) = \{ x/0, y/0, z/0\}\}$.

Theorem 3.2. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and (G, B) are fuzzy soft sets over (U, E). Then

(1) $(\tilde{0}_E)^o = \tilde{0}_E$ and $(\tilde{1}_E)^o = \tilde{1}_E$.

(2) $(F, A)^o \tilde{\subseteq} (F, A)$.
(3) $((F, A)^o)^o = (F, A)^o$.
(4) (F, A) is a fuzzy soft open set if and only if $(F, A)^o = (F, A)$.
(5) $(F, A) \tilde{\subseteq} (G, B)$ implies $(F, A)^o \tilde{\subseteq} (G, B)^o$.
(6) $(F, A)^o \tilde{\cap} (G, B)^o = ((F, A) \tilde{\cap} (G, B))^o$.
(7) $(F, A)^o \tilde{\cup} (G, B)^o \tilde{\subseteq} ((F, A) \tilde{\cup} (G, B))^o$.

Proof. (1) and (2) are obvious.

(3) Since $(F, A)^o$ is fuzzy soft open and $((F, A)^o)^o$ is the union of all fuzzy soft open subsets over (U, E) contained in $(F, A)^o$, $(F, A)^o \tilde{\subseteq} ((F, A)^o)^o$. But by (2) $((F, A)^o)^o \tilde{\subseteq} (F, A)^o$. Hence $((F, A)^o)^o = (F, A)^o$.

(4) Let (F, A) is a fuzzy soft open set over (U, E). Then (F, A) is itself a fuzzy soft open set over (U, E) which contains (F, A). So $(F, A)^o$ is the largest fuzzy soft open set contained in (F, A) and $(F, A)^o = (F, A)$. Conversely, let $(F, A) = (F, A)^o$. Since $(F, A)^o$ is a fuzzy soft open set, (F, A) is a fuzzy soft open set over (U, E).

(5) Let $(F, A) \tilde{\subseteq} (G, B)$. Since $(F, A)^o \tilde{\subseteq} (F, A) \tilde{\subseteq} (G, B)$, $(F, A)^o$ is a fuzzy soft open subset of (G, B), so by definition of $(F, A)^o \tilde{\subseteq} (G, B)^o$.

(6) $((F, A) \tilde{\cap} (G, B)) \tilde{\subseteq} (F, A)$, $((F, A) \tilde{\cap} (G, B)) \tilde{\subseteq} (G, B)$. Therefore, by (5) $((F, A) \tilde{\cap} (G, B))^o \tilde{\subseteq} (F, A)^o$, $((F, A) \tilde{\cap} (G, B))^o \tilde{\subseteq} (G, B)^o$. So $((F, A) \tilde{\cap} (G, B))^o \tilde{\subseteq} ((F, A)^o \tilde{\cap} (G, B)^o)$. Also, since $(F, A)^o \tilde{\subseteq} (F, A)$ and $(G, B)^o \tilde{\subseteq} (G, B)$ implies $((F, A)^o \tilde{\cap} (G, B)^o) \tilde{\subseteq} (F, A) \tilde{\cap} (G, B)$, so $((F, A) \tilde{\cap} (G, B))^o$ is the biggest fuzzy soft open subset of $((F, A) \tilde{\cap} (G, B))$. Hence $((F, A)^o \tilde{\cap} (G, B)^o) \tilde{\subseteq} ((F, A) \tilde{\cap} (G, B))^o$. Thus $(F, A)^o \tilde{\cap} (G, B)^o = ((F, A) \tilde{\cap} (G, B))^o$.

(7) $(F, A)^o \tilde{\subseteq} (F, A)$ and $(G, B)^o \tilde{\subseteq} (G, B)$. Then $(F, A)^o \tilde{\cup} (G, B)^o \tilde{\subseteq} (F, A) \tilde{\cup} (G, B)$. Therefore, $((F, A) \tilde{\cup} (G, B))^o$ is the biggest fuzzy soft open set contained by $((F, A) \tilde{\cup} (G, B))$. Hence $(F, A)^o \tilde{\cup} (G, B)^o \tilde{\subseteq} ((F, A) \tilde{\cup} (G, B))^o$.

Corollary 3.1. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then $(\overline{(F, A)})^C = ((F, A)^C)^o$.

Definition 3.3. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft exterior of (F, A), denoted by $(F, A)_o$, is defined as $(F, A)_o = ((F, A)^C)^o$.

Example 3.3. Let $U = \{x, y, z\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2,\} \subseteq E$. Let us consider the following fuzzy soft sets over (U, E). $(F, A) = \{F(e_1) = \{x/0.6, y/0.1, z/0\}, F(e_2) = \{x/0.7, y/0.9, z/0.5\}, F(e_3) = \{x/0, y/0, z/0\}, F(e_4) = \{x/0.5, y/0.3, z/0.9\}\}$

$(G, B) = \{G(e_1) = \{ x/0.4, y/0.1, z/0\}, G(e_2) = \{ x/0.6, y/0.5, z/0.2\}, G(e_3) = \{ x/0, y/0, z/0\}, G(e_4) = \{ x/0, y/0, z/0\}\}$.

Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (F, A), (G, B)\}$. Let us consider the following fuzzy soft over (U, E).

$(H, O) = \{H(e_1) =\{ x/0.4, y/0.7, z/0.8\}, H(e_2) = \{ x/0.3, y/0.5, z/0.6\}, H(e_3) =\{ x/0.8, y/0.2, z/0.4\}, H(e_4) = \{ x/1, y/1, z/1\}$

Then $(H, O)^C = (H^C, O) =\{H^C(e_1) =\{ x/0.6, y/0.3, z/0.2\}, H^C(e_2) = \{ x/0.7, y/0.5, z/0.4\}, H^C(e_3) = \{ x/0.2, y/0.8, z/0.6\}, H^C(e_4) = \{ x/0, y/0, z/0\}\}$

Then $((H, O)^C)^o$ is the union of all fuzzy soft open sets contained in $(H, O)^C$. That is, $((H, O)^C)^o = (G, B) \tilde{\cup} \tilde{0}_E = (G, B)=\{G(e_1) = \{ x/0.4, y/0.1, z/0\}, G(e_2) = \{ x/0.6, y/0.5, z/0.2\}, G(e_3) =\{x/0, y/0, z/0\}, G(e_4) = \{x/0, y/0, z/0\}\}$ which is the fuzzy soft exterior of (H, O).

Theorem 3.3. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and $(G, B)$are fuzzy soft sets over (U, E). Then

(1) $(F, A)_o =((F, A)^C)^o$.

(2) $((F, A) \tilde{\cup} (G, B))_o = (F, A)_o \tilde{\cap} (G, B)_o$.

(3) $(F, A)_o \tilde{\cup} (G, B)_o \tilde{\subseteq} ((F, A) \tilde{\cap} (G, B))_o$.

Proof. (1) Proof follows from the definition.

(2) $((F, A) \tilde{\cup} (G, B))_o = (((F, A) \tilde{\cup} (G, B))^C)^o = ((F, A)^C \tilde{\cap} (G, B)^C)^o = ((F, A)^C)^o \tilde{\cap} ((G, B)^C)^o$(by theorem 3.2(6)) $= (F, A)_o \tilde{\cap} (G, B)_o$.

(3) $(F, A)_o \tilde{\cup} (G, B)_o = ((F, A)^C)^o \tilde{\cup} ((G, B)^C)^o \tilde{\subseteq} ((F, A)^C \tilde{\cup} (G, B)^C)^o$ (by theorem 3.2(7)) $= (((F, A) \tilde{\cap} ((G, B))^C)^o = ((F, A) \tilde{\cap} (G, B))_o$.

We now define "fuzzy soft boundary" of a fuzzy soft set (F, A) in the following way.

Definition 3.4. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft boundary of (F, A), denoted by $(F, A)^b$, is defined as $(F, A)^b = \overline{(F, A)} \tilde{\cap} \overline{(F, A)^C}$.

Example 3.4. Let $U = \{x, y, z\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A =\{e_1, e_2,\} \subseteq E$, $B = \{e_1, e_2, e_4\} \subseteq E$. Let us consider the following fuzzy soft sets over (U, E).

$(F, A) = \{F(e_1) = \{ x/0.4, y/0.1, z/0\}, F(e_2) = \{ x/0.6, y/0.5, z/0.2\}, F(e_3) = \{ x/0, y/0, z/0\}, F(e_4) = \{ x/0, y/0, z/0\}\}$

$(G, B) = \{G(e_1) = \{ x/0.5, y/0.6, z/0.7\}, G(e_2) = \{ x/0.1, y/0.5, z/0.5\}, G(e_3) = \{ x/0, y/0, z/0\}, G(e_4) = \{ x/0.1, y/0.2, z/0.3\}\}$.

Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (F, A), (G, B)\}$. Now $(F, A)^C = (F^C, A) = \{ F^C(e_1) = \{ x/0.6, y/0.9, z/1\}, F^C(e_2) = \{ x/0.4, y/0.5, z/0.8\}, F^C(e_3) = \{ x/1, y/1, z/1\}, F^C(e_4) = \{ x/1, y/1, z/1\}\}$

and $(G, B)^C = (G^C, B) = \{G^C(e_1) = \{x/0.5, y/0.4, z/0.3\}, G^C(e_2) = \{x/0.9, y/0.5, z/0.5\}, G^C(e_3) = \{x/1, y/1, z/1\}, G^C(e_4) = \{x/0.9, y/0.8, z/0.7\}\}$. Clearly, $(F, A)^C$ and $(G, B)^C$ are fuzzy soft closed.
Let us also consider the following fuzzy soft sets over (U, E).
$(H, C) = \{H(e_1) = \{x/0.5, y/0.7, z/0.8\}, H(e_2) = \{x/0.3, y/0.5, z/0.6\}, H(e_3) = \{x/0, y/0, z/0\}, H(e_4) = \{x/0.2, y/0.3, z/0.4\}\}$.

Then the fuzzy soft closure of (H, C), denoted by $\overline{(H,C)}$, is the intersection of all fuzzy soft closed sets containing (H, C). That is, $\overline{(H,C)} = (F, A)^C \tilde{\cap} \tilde{1}_E = (F, A)^C = \{F^C(e_1) = \{x/0.6, y/0.9, z/1\}, F^C(e_2) = \{x/0.4, y/0.5, z/0.8\}, F^C(e_3) = \{x/1, y/1, z/1\}, F^C(e_4) = \{x/1, y/1, z/1\}\}$. Complement of (H, C), denoted by $(H, C)^C$, is given by $(H, C)^C = (H^C, C) = \{H^C(e_1) = \{x/0.5, y/0.3, z/0.2\}, H^C(e_2) = \{x/0.7, y/0.5, z/0.4\}, H^C(e_3) = \{x/1, y/1, z/1\}, H^C(e_4) = \{x/0.8, y/0.7, z/0.6\}\}$. Then the fuzzy soft closure of $(H, C)^C$, denoted by $\overline{(H,C)^C}$, is the intersection of all fuzzy soft closed sets containing $(H, C)^C$. That is, $\overline{(H,C)^C} = (G, B)^C \tilde{\cap} \tilde{1}_E = (G, B)^C = \{G^C(e_1) = \{x/0.5, y/0.4, z/0.3\}, G^C(e_2) = \{x/0.9, y/0.5, z/0.5\}, G^C(e_3) = \{x/1, y/1, z/1\}, G^C(e_4) = \{x/0.9, y/0.8, z/0.7\}\}$. Then the fuzzy soft boundary $(H, C)^b$ of the fuzzy soft set (H, C) over (U, E) is given by $(H, C)^b = \overline{(H,C)} \tilde{\cap} \overline{(H,C)^C} = \{H^b(e_1) = \{x/0.5, y/0.4, z/0.3\}, H^b(e_2) = \{x/0.4, y/0.5, z/0.5\}, H^b(e_3) = \{x/1, y/1, z/1\}, H^b(e_4) = \{x/0.9, y/0.8, z/0.7\}\}$.

Theorem 3.4. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be fuzzy soft set over (U, E). Then

(1) $(F, A)^b \tilde{\subseteq} \overline{(F,A)}$.

(2) $(F, A)^b = \overline{(F,A)} \tilde{\setminus} (\mathrm{F, A})^{\mathrm{o}}$.

Proof. (1) Proof follows from the definition of fuzzy soft boundary.

(2) $\overline{(F,A)} \tilde{\setminus} (F, A)^o = \overline{(F,A)} \tilde{\cap} ((F, A)^o)^C = \overline{(F,A)} \tilde{\cap} (\tilde{\cup} (\mathrm{F, B})_\alpha)^C$ (where $(F, B)_\alpha \tilde{\subseteq} (F, A)$, $(F, B)_\alpha \in \tau$))

$$= \overline{(F,A)} \tilde{\cap} (\tilde{\cap} ((F, B)_\alpha)^C) = \overline{(F,A)} \tilde{\cap} \overline{(F,A)^C} = (F, A)^b.$$

Theorem 3.5. Let (U, E, τ) be a fuzzy soft topological space. Let (F, B) be fuzzy soft set over (U, E). Then

(1) $((F, B)^b)^C = (F, B)^o \tilde{\cup} ((F, B)^C)^o = (F, B)^o \tilde{\cup} (F, B)_o$.

(2) $\overline{(F,B)} = (F, B)^o \tilde{\cup} (F, B)^b$.

(3) $(F, B)^o = (F, B) \tilde{\setminus} (F, B)^b$.

(4) $(F, B)^b = \overline{(F,B)} \tilde{\cap} \overline{(F,B)^C} = \overline{(F,B)} \tilde{\setminus} (F, B)^{\mathrm{o}}$.

Proof. (1) $(F, B)^o \tilde{\cup} ((F, B)^C)^o = (((F, B)^o)^C)^C \tilde{\cup} ((((F, B)^C)^o)^C)^C$
$= [((F, B)^o)^C \tilde{\cap} (((F, B)^C)^o)^C]^C = [\overline{(F,B)^C} \tilde{\cap} \overline{(F,B)}]^{\mathrm{C}} = ((F, B)^b)^C$.

(2) $(F, B)^o \tilde{\cup} (F, B)^b = (F, B)^o \tilde{\cup} (\overline{(F,B)} \tilde{\cap} \overline{(F,B)^C})$

$= [(F, B)^o \tilde{\cup} \overline{(F,B)}] \tilde{\cap} [(F, B)^o \tilde{\cup} \overline{(F,B)^C}]$

$= \overline{(F,B)} \tilde{\cap} [(F, B)^o \tilde{\cup} \overline{(F,B)^C}]$

$= \overline{(F,B)} \tilde{\cap} \tilde{1}_E = \overline{(F,B)}$.

(3) $(F, B) \tilde{\setminus} (F, B)^b = (F, B) \tilde{\cap} ((F, B)^b)^C$

$= (F, B) \tilde{\cap} ((F, B)^o \tilde{\cup} ((F, B)^C)^o)$ (by (a))

$= [(F, B) \tilde{\cap} ((F, B)^o] \tilde{\cup} [(F, B) \tilde{\cap} ((F, B)^C)^o]$

$= (F, B)^o \tilde{\cup} \tilde{0}_E = (F, B)^o$.

(4) $(F, B)^b = \overline{(F,B)} \tilde{\setminus} (F, B)^o = \overline{(F,B)} \tilde{\cap} ((F, B)^o)^C = \overline{(F,B)} \tilde{\cap} \overline{(F,B)^C}$.

Theorem 3.6. Let (U, E, τ) be a fuzzy soft topological space. Let (F, B) be fuzzy soft set over (U, E). Then

(1) $(F, B)^b \tilde{\cap} (F, B)^o = \tilde{0}_E$.

(2) $(F, B)^b \tilde{\cap} (F, B)_o = \tilde{0}_E$.

Proof. (1) $(F, B)^b \tilde{\cap} (F, B)^o = (F, B)^o \tilde{\cap} (F, B)^b$

$= (F, B)^o \tilde{\cap} (\overline{(F,B)} \tilde{\cap} \overline{(F,B)^C}) = (F, B)^o \tilde{\cap} \overline{(F,B)} \tilde{\cap} ((F, B)^o)^C = \tilde{0}_E$.

(2) $(F, B)^b \tilde{\cap} (F, B)_o = ((F, B)^C)^o \tilde{\cap} (\overline{(F,B)} \tilde{\cap} \overline{(F,B)^C})$

$= ((F, B)^C)^o \tilde{\cap} \overline{(F,B)} \tilde{\cap} \overline{(F,B)^C} = (\overline{(F,B)})^C \tilde{\cap} \overline{(F,B)} \tilde{\cap} \overline{(F,B)^C} = \tilde{0}_E$.

Theorem 3.7. Let (U, E, τ) be a fuzzy soft topological space. Let (F, B) be fuzzy soft set over (U, E). Then

(1) (F, B) is a fuzzy soft open set over (U, E) if and only if $(F, B) \tilde{\cap} (F, B)^b = \tilde{0}_E$.

(2) (F, B) is a fuzzy soft closed set over (U, E) if and only if $(F, B)^b \tilde{\subseteq} (F, B)$.

Proof. (1) Let (F, B) be a fuzzy soft open set over (U, E). Then $(F, B)^o = (F, B) \Longrightarrow (F, B) \tilde{\cap} (F, B)^b = (F, B)^o \tilde{\cap} (F, B)^b = \tilde{0}_E$ (by theorem 3.6(1)).

Conversely, let $(F, B) \tilde{\cap} (F, B)^b = \tilde{0}_E$. Then $(F, B) \tilde{\cap} \overline{(F,B)} \tilde{\cap} \overline{(F,B)^C} = \tilde{0}_E$. That is, $(F, B) \tilde{\cap} \overline{(F,B)^C} = \tilde{0}_E$. So $\overline{(F,B)^C} \tilde{\subseteq} (F, B)^C$ which shows that $(F, B)^C$ is a fuzzy soft closed set. Hence (F, B) is fuzzy soft open.

(2) Let (F, B) be fuzzy soft closed set over (U, E). Then $\overline{(F,B)} = (F, B)$. Now $(F, B)^b = \overline{(F,B)} \tilde{\cap} \overline{(F,B)^C} \tilde{\subseteq} \overline{(F,B)} = (F, B)$. That is, $(F, B)^b \tilde{\subseteq} (F, B)$. Conversely, let $(F, B)^b \tilde{\subseteq} (F, B)$. Then $((F, B)^b \tilde{\cap} (F, B)^C = \tilde{0}_E$. Since $(F, B)^b =$

$((F, B)^C)^b = \tilde{0}_E$, we have $((F, B)^C)^b \tilde{\cap} (F, B)^C = \tilde{0}_E$. Then by (1) $(F, B)^C$ is fuzzy soft open and hence (F, B) is a fuzzy soft closed set over (U, E).

Theorem 3.8. Let (U, E, τ) be a fuzzy soft topological space. Let (F, B) and (G, B) are fuzzy soft sets over (U, E). Then

(1) $((F, A) \tilde{\cup} (G, B))^b \tilde{\subseteq} [(F, A) \tilde{\cap} (G, B)^C]^b \tilde{\cup} [(G, B)^b \tilde{\cap} \overline{(F, A)^C}]$.

(2) $((F, A) \tilde{\cap} (G, B))^b \tilde{\subseteq} [(F, A)^b \tilde{\cap} \overline{(G, B)}] \tilde{\cup} [(G, B)^b \tilde{\cap} \overline{(F, A)}]$.

Proof. (1) $((F, A) \tilde{\cup} (G, B))^b = (\overline{(F, A) \tilde{\cup} (G, B)}) \tilde{\cap} \overline{(((F, A) \tilde{\cup} (G, B))^C)}$

$= (\overline{(F, A)} \tilde{\cup} \overline{(G, B)}) \tilde{\cap} \overline{(((F, A)^C \tilde{\cap} (G, B))^C)}$

$\tilde{\subseteq} (\overline{(F, A)} \tilde{\cup} \overline{(G, B)}) \tilde{\cap} [\overline{(F, A)^C} \tilde{\cap} \overline{(G, B)^C}]$

$= [\overline{(F, A)} \tilde{\cap} \overline{(F, A)^C}) \tilde{\cap} (\overline{(G, B)^C}] \tilde{\cup} [\overline{(G, B)}) \tilde{\cap} [\overline{(F, A)^C} \tilde{\cap} \overline{(G, B)^C}]$

$= [(\overline{(F, A)} \tilde{\cap} \overline{(F, A)^C}) \tilde{\cap} \overline{(G, B)^C}] \tilde{\cup} [(\overline{(G, B)} \tilde{\cap} \overline{(G, B)^C}) \tilde{\cap} \overline{(F, A)^C}]$

$= ((F, A)^b \tilde{\cap} \overline{(G, B)^C}) \tilde{\cup} ((G, B)^b \tilde{\cap} \overline{(F, A)^C}) \tilde{\subseteq} (F, A)^b \tilde{\cup} (G, B)^b$

(2) $((F, A) \tilde{\cap} (G, B))^b = (\overline{(F, A) \tilde{\cap} (G, B)}) \tilde{\cap} \overline{(((F, A) \tilde{\cap} (G, B))^C)}$

$\tilde{\subseteq} [\overline{(F, A)} \tilde{\cap} \overline{(G, B)}] \tilde{\cap} [\overline{((F, A)^C \tilde{\cup} (G, B)^C)}]$

$= [\overline{(F, A)} \tilde{\cap} \overline{(G, B)}] \tilde{\cap} [\overline{(F, A)^C} \tilde{\cup} \overline{(G, B)^C}]$

$= [(\overline{(F, A)} \tilde{\cap} \overline{(G, B)}) \tilde{\cap} \overline{(F, A)^C}] \tilde{\cup} [(\overline{(F, A)} \tilde{\cap} \overline{(G, B)}) \tilde{\cap} \overline{(G, B)^C}]$

$= [(F, A)^b \tilde{\cap} \overline{(G, B)}] \tilde{\cup} [(G, B)^b \tilde{\cap} \overline{(F, A)}]$.

Theorem 3.9. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be fuzzy soft set over (U, E). Then $(((F, A)^b)^b)^b = ((F, A)^b)^b$.

Proof: $(((F, A)^b)^b)^b = \overline{(((F, A)^b)^b)} \tilde{\cap} \overline{((((F, A)^b)^b)^C)}$

$= (((F, A)^b)^b) \tilde{\cap} \overline{((((F, A)^b)^b)^C)}$ (1)

Now let us consider

$((((F, A)^b)^b)^C) = [\overline{(F, A)^b} \tilde{\cap} \overline{(((F, A)^b)^C)}]^C = [(F, A)^b \tilde{\cap} \overline{(((F, A)^b)^C)}]^C$

$= ((F, A)^b)^C \tilde{\cup} (\overline{((F, A)^b)^C})^C$.

Therefore, $\overline{((((F, A)^b)^b)^C)} = \overline{((F, A)^b)^C \tilde{\cup} (\overline{((F, A)^b)^C})^C}$

$= \overline{(((F, A)^b)^C)} \tilde{\cup} \overline{(\overline{((F, A)^b)^C})^C}$

$= (G, B) \tilde{\cup} \overline{(G, B)^C} = \tilde{1}_E$, where $(G, B) = \overline{(((F, A)^b)^C)}$. (2)

From (1) and (2) we have, $(((F, A)^b)^b)^b = (((F, A)^b)^b) \tilde{\cap} \tilde{1}_E = (((F, A)^b)^b)$.

Theorem 3.10. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and (F, B) are fuzzy soft sets over (U, E). Then

(1) $((F, A)\backslash (G, B))^o \,\tilde{\subseteq}\, (F, A)^o \backslash (G, B)^o$.

(2) $((F, A)^o)^b \,\tilde{\subseteq}\, (F, A)^b$.

Proof. (1) $((F, A)\backslash (G, B))^o = ((F, A) \tilde{\cap} (G, B)^C)^o = (F, A)^o \tilde{\cap} ((G, B)^C)^o$
$= (F, A)^o \tilde{\cap} (\overline{(G,B)})^C$ (by Corollary 3.1) $= (F, A)^o \backslash \overline{(G,B)} \,\tilde{\subseteq}\, (F, A)^o \backslash (G, B)$

(2) $((F, A)^o)^b$
$= \overline{(F,A)^o} \,\tilde{\cap}\, \overline{(((F,A)^o)^C)} \,\tilde{\subseteq}\, \overline{(F,A)^o} \,\tilde{\cap}\, \overline{((F,A)^C)} \,\tilde{\subseteq}\, \overline{(F,A)} \,\tilde{\cap}\, \overline{((F,A)^C)} = (F, A)^b$.

Theorem 3.11. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be fuzzy soft set over (U, E). Then $(F, A)^b = \tilde{0}_E$ if and only if (F, A) is a fuzzy soft closed set and a fuzzy soft open set.

Proof. Let $(F, A)^b = \tilde{0}_E \Rightarrow \overline{(F,A)} \,\tilde{\cap}\, \overline{(F,A)^C} = \tilde{0}_E$
$\Rightarrow \overline{(F,A)} \,\tilde{\subseteq}\, (\overline{(F,A)^C})^C = (F, A)^o \Rightarrow \overline{(F,A)} \,\tilde{\subseteq}\, (F, A) \Rightarrow \overline{(F,A)} = (F, A)$ and this implies that (F, A) is a fuzzy soft closed set. Again let $(F, A)^b = \tilde{0}_E \Rightarrow \overline{(F,A)} \,\tilde{\cap}\, \overline{(F,A)^C} = \tilde{0}_E$. That is, $(F, A) \tilde{\cap} ((F, A)^o)^C = \tilde{0}_E \Rightarrow$ $(F, A) \,\tilde{\subseteq}\, (F, A)^o \Rightarrow (F, A)^o = (F, A)$ and this implies that (F, A) is a fuzzy soft open set.

Conversely, let (F, A) is a fuzzy soft open and a fuzzy soft closed set. Then $(F, A)^b = \overline{(F,A)} \,\tilde{\cap}\, \overline{(F,A)^C} = \overline{(F,A)} \,\tilde{\cap}\, ((F, A)^o)^C = (F, A) \tilde{\cap} (F, A)^C = \tilde{0}_E$.

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