Intern. J. Fuzzy Mathematical Archive
Vol. 9, No. 2, 2015, 197-202
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 8 October 2015
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

The Solution Procedure "PASEB" for Solving Fuzzy Multi Objective Linear Programing Problem

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Received 1 September 2015; accepted 5 October 2015

Abstract. This paper presents a new method 'PASEB' fuzzy multi objective linear programming problem(FMOLPP).An Algorithm is developed here the optimal solution for FMOLPP, which may be the Best Compromise Solution (BCS). Finally, a numerical example is provided to demonstrate the effectiveness of the proposed method.

Keywords: 'PASEB' assumptions, new fuzzy arithmetic operations, triangular fuzzy numbers.

AMS Mathematics Subject Classification (2010): 65K05, 90C90, 90C70, 90C29

1. Introduction

Most of the real life problems exhibit the properties of multiobjectivity and fuzziness in nature..Some solution methodologies for solving linear programming problem were given in references[2-4]. Some examples by Belenson and Kapur [1] application is in practice and a practicable solution method for Multi Objective Linear Programing problems has yet to be found.

PASEB is an iterative and interactive solution procedure developed with special attention to its practicability. The basis of the method is the pattern search procedure proposed by Hooke and Jeeves for unconstrained optimization. PASEB can be used to determine a Best Compromise Solution to a Multiobjective linear programming problem, provided certain conditions referred to later are satisfied, this is time consuming in practice. In this paper has been developed a solution procedure based on PASEB to solve a Fuzzy Multi-objective linear programming problem .

The paper is organized as follows: Section 2 introduces preliminaries it contains Fuzzy set, Fuzzy number, Triangular Fuzzy Number, The Fuzzy Arithmetic Operations under Function Principle, Graded Mean Integration Method, New Operation on Triangular Fuzzy Number. Section 3 deals with PASEB Algorithm, Properties. The PASEB Algorithm for FMOLPP. In Section 4, Proposes a method for solving fuzzy multi-objective linear programming problems**.** Finally, section 5 contains some concluding remarks**.**

2. Preliminaries

This section introduces the preliminary notations of the area of intuitionistic fuzzy set theory.

Fuzzy set: A fuzzy set Ã is defined by Ã={ (x,μ_A(x)): x [0.1]}.In the pair (x, μ_A(x)) the first element x belong to the classical set A , the second element , μ_A(x), belong to the interval [0,1] called membership function.

Fuzzy number: A fuzzy set Ã on R must posses at least the following properties to quality as a fuzzy number.
i) Ã must be a normal fuzzy set.
ii) Ã must be closed interval for every α∈[0.1]

Triangular fuzzy number

A triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A}=\left(a_1,a_2,a_3\right)$ and is defined by the membership function as

$$\mu_{\tilde{A}}\left(x\right)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, \text{ if } a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, \text{ if } a_2\le x\le a_3\\ 0, \text{ otherwise}\end{cases}$$

The fuzzy arithmetic operations under function principle

Suppose $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then the fuzzy arithmetic operations under function principle are furnished below.

Addition:

$\tilde{A}$ + $\tilde{B}$ = ($a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$), where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

Subtraction:

$\tilde{A}$ - $\tilde{B}$ = ($a_1 - b_3$, $a_2 - b_2$, $a_3 - b_1$), where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

Scalar Multiplication:

Let $\lambda \in$ R, then $\lambda\tilde{A}$ = (λa_1, λa_2, λa_3), $\lambda \ge 0$ and
$\lambda\tilde{A}$ = (λa_3, λa_2, λa_1), $\lambda < 0$.

Multiplication:

$\tilde{A}\,.\,\tilde{B} = (c_1, c_2, c_3)$, where

$$T=\left\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\right\}, T_1=a_2b_2, c_1=\min T, c_2=T_1, c_3=\max T.$$

If a_1, a_2, a_3, b_1, b_2, b_3 are all nonzero positive real numbers, then
$\tilde{A}\,.\,\tilde{B}=(a_1b_1, a_2b_2, a_3b_3)$.

Division:

$\tilde{A}/\tilde{B}=(a_1/b_3, a_2/b_2, a_3/b_1)$, if all b_i's are non-zero positive real numbers.

Graded mean integration method

Suppose $\tilde{A}$ = (a_1, a_2, a_3) is a given triangular fuzzy number. Then the defuzzification of the fuzzy number by graded mean integration method is $p\,(\tilde{A}) = \dfrac{(a_1 + 4a_2 + a_3)}{6}$

3. PASEB algorithm

Thanassoulies [1] developed “PASEB”, a solution procedure to solve Multi-objective linear programming problem. This section provides Properties and PASEB Algorithm for solving fuzzy multi objective linear programming problem.

Properties

$U(C(x^1)) > U(C(x^2)) \rightarrow C(x^1)$ is provided to c(x)

$U(C(x^1)) = U(C(x^2)) \rightarrow C(x^1)\ C(x^2)$ are equally desired where x^i i=1,2,……. U(C(x)) is referred to as the DM’S value function.

The PASEB algorithm for FMOLPP

Step 1. Determine $\tilde{x}^i$,i=1,2,…….,N, and a starting feasible solution, to be first current solution.

Step 2. If the current solution is the s^{th} consecutive “similar” solution goto next process. Otherwise carry out local search.

Step 3. If the current solution is efficient gotoStep:4 4.Else determine an efficient solution dominating the current solution and goto **Step** 2.

Step 4. Let x^k be the optimal solution to the efficiency testing model. If x^k is a basic solution to p1 otherwise determine the DM’S preferred solution in the direction of trade offs between the objectives offered by some non-basic variable at x^k. if they are attractive and goto 2. Other wise repeat the forgoing for some non basic variable at x^k. and so on. If no non basic variable at x^k offers desirable trade off between the objectives goto **Step** 6.

Step 5. If some nonbasic variable at x^k does not generate feasible non basic solution to P1 when alter marginally discard it. Oterwise determine the DM’S preferred in the direction of trade offs between the objective offered by the variable at x^k offers desirable trade offs between the objectives goto **Step** 6.

Step 6. The DM to specify one or more pairs of sets of objectives I_p and I_q wish to see improved and worse respectively, at the current solution likely offer objective values changed compatible with those expressed in some pair I_p,I_q

Step 7. Determine the DMS preferred solution in the direction of the pattern of improving solution established and goto **Step** 2.

Step 8. Determine, if it exists, an efficient solution dominating the current, if the solution is the same or similar to the current goto **Step** 6 otherwise goto **Step** 2.

4. Numerical example

Consider the following Fuzzy multi objective linear programming problem

Maximize $\tilde{z}_1$=(2,3,4)$\tilde{x}_1$+(1,1,1) $\tilde{x}_2$+(1,2,3) $\tilde{x}_3$+(0.5,1,1.5) $\tilde{x}_4$

Maximize $\tilde{z}_2$=(1,1,1) $\tilde{x}_1$+(0.5,1,1.5) $\tilde{x}_2$+(1,2,3) $\tilde{x}_3$+(3,4,5) $\tilde{x}_4$

Maximize $\tilde{z}_3$=(0.5,1,1.5) $\tilde{x}_1$+(4,5,6) $\tilde{x}_2$+(1,1,1) $\tilde{x}_3$+(1,2,3) $\tilde{x}_4$

Subject to the constraints

$(1,2,3)\ \tilde{x}_1+(0.5,1,1.5)\ \tilde{x}_2+(3,4,5)\ \tilde{x}_3+(2,3,4)\ \tilde{x}_4 \leq (50,60,70)$
$(1,2,3)\ \tilde{x}_1+(3,4,5)\ \tilde{x}_2+(1,1,1)\ \tilde{x}_3+(1,2,3)\ \tilde{x}_4 \leq (40,60,80)$
$\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4 \geq 0$ (a)
The set of feasible solution of the above problem is denoted by $\tilde{x}$.
The DM'S value function for the above problem is taken as follows.
U{C(X)}= On the basis of the DM'S value function , the BCS of(i)
[$\tilde{x}_1,\tilde{x}_4$ basic variable , $\tilde{x}_2,\tilde{x}_3$ are non basic variables $x_2,x_3=0$] (b)

The constraints will become,
$(1,2,3)\ \tilde{x}_1+(2,3,4)\ \tilde{x}_4 = (50,60,70)$ (c)
$(2,3,4)\ \tilde{x}_1+(1,2,3)\ \tilde{x}_4=(40,60,80)$ (d)

(c) X 3 → $(3,6,9)\ \tilde{x}_1+(6,9,12)\ \tilde{x}_4= (150,189,210)$
(d) X 2 → $(4,6,8)\ \tilde{x}_1+(2,4,6)\ \tilde{x}_4 = (80,120,160)$
$(-1,0,1)\ \tilde{x}_1+(4,5,6)\ \tilde{x}_4 = (70,60,50)$
$(4,5,6)\ \tilde{x}_4= (70,60,50)$
$\tilde{x}_4 =$ 70,60,50/4,5,6
= (11.6,12,12.5)
Substitute the value in the above equation (d) ,we get
$(2,3,4)\ \tilde{x}_1+ (1,2,3)\ \tilde{x}_4 = (40,60,80)$
$(2,3,4)\ \tilde{x}_1+(11.6,24,37.5) = (40,60,80)$
$(2,3,4)\tilde{x}_1= (40,60,80)-(11.6,24,37.5)$
=(2.5,36,68.4)
Hence x^*=[(.625,12,34.2),(0,0,0),(0,0,0)(1.6,12,12.5)] (e)

$\tilde{c}_1.\tilde{x}$ =3(.625,12,34.2)+(0,0,0)+(0,0,0)+(11.6,12,12.5)
=(1.875,36,102.6)+(11.6,12,12.5)
=(13.475,48,115.1)
$c_2.\tilde{x}$ =(.625,12,34.2)+(0,0,0)+(0,0,0)+4(11.6,12,12.5)
=(.625,12,34.2)+(46.4,48,50)
=(47.025,60,84.2)
$c_3.\tilde{x}$ =-(.625,12,34.2)+(0,0,0)+(0,0,0)+2(11.6,12,12.5)
=-(.625,12,34.2)+(23.2,24,25)
=(23.2,24,25)- (.625,12,34.2)
=(-11,12,24.375)

X_3= (3.9,5.8,7.8)
X_1=(.96,18.3,25.4)
Max $\tilde{z}_2=(1,1,1)\ \tilde{x}_1-(.5,1,1.5)\ \tilde{x}_2+(1,2,3)\ \tilde{x}_3+(3,4,5)\ \tilde{x}_4$

Such that
$(1,2,3)\ \tilde{x}_1+(.5,1,1.5)\ \tilde{x}_2+(3,4,5)\ \tilde{x}_3+(2,3,4)\ \tilde{x}_4 = (50,60,70)$
$(2,3,4)\ \tilde{x}_1+(3,4,5)\ \tilde{x}_2+(1,1,1)\ \tilde{x}_3+(1,2,3)\ \tilde{x}_4= (40,60,80)$
$(2,3,4)\ \tilde{x}_1+(1,1,1)\ \tilde{x}_2+(1,2,3)\ \tilde{x}_3+(.5,1,1.5)\ \tilde{x}_4 = (65,66,67)$
$\tilde{x}$ = (16.6,20,23.3)

maximize $z = (3,4,5)\ (16.6,20,23.3)$
$= (69.9,80,83)$
$= 80$
Maximize $z = (-1,-1,-1)\tilde{x}_1+(4,5,6)\ \tilde{x}_2+(1,1,1)\ \tilde{x}_3+(1,2,3)\tilde{x}_4$
Subject to the constraints
$(1,2,3)\ \tilde{x}_1+(.5,.1,1.5)\ \tilde{x}_2+(3,4,5)\ \tilde{x}_3+(2,3,4)\ \tilde{x}_4= (50,60,70)$
$(2,3,4)\ \tilde{x}_1+(3,4,5)\ \tilde{x}_2+(1,1,1)\ \tilde{x}_3+ (1,2,3)\ \tilde{x}_4 = (40,60,80)$
$(2,3,4)\ \tilde{x}_1+(1,1,1)\ \tilde{x}_2+(1,2,3)\tilde{x}_3+(.5,.1,1.5)\ \tilde{x}_4 = (65,66,67)$
$(1,1,1)\tilde{x}_1+(.5,.1,1.5)\tilde{x}_2+(1,2,3)\tilde{x}_3+(3,4,5)\ \tilde{x}_4 = (70,80,90)$

Maximize $z =(4,5,6)(10,15,20)$
$=(40,75,120)$
$C(\tilde{x}_1) = (60,30,-12)$
$C(\tilde{x}_2) = (20,80,40)$
$C(\tilde{x}_3) = (15,-15,75)$
In view of the linearity of U(c(x)) and the fact that $c(\tilde{x}')$is preferred to $c(x^{\tilde{}'})$.The DM'S preferred feasible solutions is $c(x'')$ in the direction $c(x')$, $c(\tilde{x}')$is
$c(x'')= c(\tilde{x}')= (66,30,-12)$
Maximize $z^{\square\square\square\square\square} = \tilde{e}_1+\tilde{e}_2+\tilde{e}_3$
such that
$c_{1.}\tilde{\tilde{x}}-\tilde{e}_{\ 1}=(65,66,67)$
$c_{2.}\tilde{x}-\tilde{e}_2=(29,30,31)$
$c_{3.}\tilde{x}-\tilde{e}_3=(-12,-12,-12)$
At the optimal solution $x^{\square 2}$ to $z^{\square} =0.x^{\square 2}$ is an efficient basic solution.
At $\tilde{x}^2$ in the above problem e is non basic while e ,e and x are basic at zero level.
Now the trade offs between the objectives offered by e and diseriable and they are as follows,
$(c(a^j)) = -a_{ij}$,i=1,2,…….N
$a_{11} = -1$
$(13.48,48,115.1) = (65,66,67)$
$-\tilde{e}_1 = (65,66,67) -(13.48,48,115.1)$
$= (-50.1,18,51.52)$
$(47.03,60,84.2)- \tilde{e}_2 = (29,30,31)$
$-\tilde{e}_2 = (29,30,31)- (47.03,60,84.2)$
$= (-55.2,-30,-16.03)$
$(-11,12,24.36)-\tilde{e}_3 = (-12,-12,-12)$
$-\tilde{e}_3 = (-12,-12,-12) -(-11,12,24.36)$
$=(-48,-24,1)$
$a_{21}= \tilde{e}_2/\tilde{e}_1= (55.2,30,16.03)/(-50.1,18,51.52)=1.66$
$a_{31}= \tilde{e}_3/\tilde{e}_1= (-48,-24,1)/(-50.1,18,51.52)=1.33$
$c(a^{\square})=(-1,1.66,1.33)$
The feasible solution as along $c(a^{\square})$ at $c(x^{\square})$ as follows,
$C(x)= [\tilde{x}^2+ \lambda\ [c(\tilde{a}')] \quad 0\leq \lambda \leq \lambda^*$
where λ^* is the maximal value of λ in the following
$C(x) = (66,30,-13)+ \lambda(-1,1.66,1.33)$
Max λ such that

$C_1.x+ \lambda = (65,66,67)$
$C_2.x-1.66 \lambda = (29,30,31)$
$C_3.x-1.33 \lambda = (-12,-12,-12)$
$x \in X, \lambda \geq 0$
Solving this equation
$(13.48,48,115.1) + \lambda = (65,66,67)$
$\lambda = (65,66,67)- (13.48,48,115.1)$
$=(-50.1,18,51.50)$
$(47.025,60,84.2) -1.66 \lambda= (29,30,31)$
$-1.66 \lambda = (29,30,31) - (47.025,60,84.2)$
$=(-18.025,-30,-53.2)$
$\lambda =(10.8,18,31.9)$
$(-11,12,24.35) - 1.33 \lambda = (-12,-12,-12)$
$-1.33 \lambda = (-12,-12,-12) - (-11,12,24.35)$
$= (-1,-24,-36.35)$
$\lambda = (0.75,18,27.26)$
Hence $\lambda = \lambda^* = 18$
The DM's preferred solution is as follows,
$Cx^3 = (48,60,12)$

5. Conclusion

PASEB is a relatively simple procedure for obtaining a good solution for MOLP Problem. In this paper, this method is extended to solve Fuzzy multi objective linear programming problem.

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