Intern. J. Fuzzy Mathematical Archive
Vol. 9, No. 2, 2015, 189-196
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 8 October 2015
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

On Solving a Multi-Objective Intuitionistic Fuzzy Linear Fractional Programming Problem

***R.Sophia Porchelvi*[1] and *S.Rukmani*[2]**

[1]Department of Mathematics, A.D.M. College for women, Nagappattinam, Tamilnadu, email-sophiaporchelvi@gmail.com
[2]Department of Mathematics, S.T.E.T. Women's College, Mannargudi, Tamilnadu

Received 9 September 2015; accepted 1 October 2015

Abstract. In this paper, a multi-objective intuitionistic fuzzy linear fractional programm - ing problem is discussed with two different methods namely Kanti Swarup's Fractional algorithm and weighting factor method in which the constraints are triangular intuitionistic fuzzy number. The optimal solutions are verified for the proposed methods. Numerical examples are provided to check the feasibility of the above methods.

Keywords: Linear fractional programming problem, multi-objective intuitionistic fuzzy set, triangular intuitionistic fuzzy number, weighting factor.

AMS Mathematics Subject Classification (2010): 90B50

1. Introduction

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. In many real world problems, there are situations where multiple objectives may be more appropriate rather than considering single objective. However, in such cases emphasis is on efficient solutions, which are optimal in a certain multi-objective sense.

Zimmermann (1978) first discussed the concept of fuzzy Multi-objective mathematical programming problems. Optimization in fuzzy environment was further studied and was applied in various areas by many researchers. Attanassov (1986) introduced the intuitionistic fuzzy sets as a powerful extension of fuzzy set by adding an additional non-membership degree, which may express more abundant and flexible information as compared with the fuzzy set. Recently, the research on intuitionistic fuzzy numbers has received a little attention and several definition of intuitionistic fuzzy numbers and ranking methods have been proposed.

Sophia Porchelvi and Rukmani [7] solved Multi-objective intuitionistic fuzzy linear programming problem (MOIFLPP) using a ranking procedure. This paper is used to transform the MOIFLP problem into a Multi-objective linear programming problem (MOLPP) and can be solved accordingly.

Linear Fractional Programming (LFP) problems are a special type of non-linear programming problems in which the objective function is a ratio of linear functions and the constraints are linear functions. In real life situations, linear fractional models arise in decision making such as construction planning, economic and commercial planning, health care and hospital planning. Several methods (Bajalinov 2003; Stancu-Minasian, 1997, 2006) have been recommended to solve LFP Problems. Isbell and Marlow (1956) first identified an example of LFP Problems and solved it by a sequence of linear programming problems. Charnes and Cooper (1962) considered variable transformation method to solve LFP and the updated objective function method were developed for solving the LFP problem by Bitran and Novaes (1973).

Charnes and Cooper (1962) solved LFP by resolving it into two linear programming problems. Later, KantiSwarup (1986) gave an algorithm for the solution of programming problems with linear fractional functional without reducing it to linear programming problems. Here, we used this procedure to solve the problem.

Using weighting factor, P.A. Thakre *et al.,* [5] solved Multi-objective fuzzy linear programming problem (MOFLPP) and I.M. Stancu-Minasian *et al.,* [9] solved multi-objective fuzzy linear fractional programming under constraints with fuzzy coefficients. Based on these papers, here to solve multi-objective intuitionistic fuzzy linear fractional programming problem (MOIFLFPP) with intuitionistic fuzzy coefficients.

This Paper is organized as follows : The paper begins with the Preliminaries section which provides some basic definitions and the concept of MOIFLFPP. The two different procedures for solving MOIFLFPP are given in section 3. A numerical example is solved in section 4, to show the efficiency of the proposed algorithm. The paper is concluded at the end of the section.

2. Preliminaries

2.1. Basic definitions

Definition 2.1.1. An **Intuitionistic fuzzy sets** (IFS) $\bar{A}$assigns to each element x of the universe X to a membership degree $\mu_{\bar{a}}(x)\epsilon[0,1]$ and a non-membership $\nu_{\bar{a}}(x)\epsilon[0,1]$ such that $\mu_{\bar{a}}(x)+\nu_{\bar{a}}(x)\leq 1$. An IFS $\bar{A}$ is mathematically, represented as $\{<x,\mu_{\bar{a}}(x),\nu_{\bar{a}}(x)>/x\in X\}$. The value $\pi_{\bar{a}}(x)=(1-\mu_{\bar{a}}(x))-\nu_{\bar{a}}(x))$ is called the degree of **hesistancy** or the **intuitionistic index** of x to $\bar{a}$.

Definition 2.1.2. A is **trapezoidal intuitionistic fuzzy number** with parameters $b_1\leq a_1\leq b_2\leq a_2\leq a_3\leq b_3\leq a_4\leq b_4$ is denoted by A = (b_1, a_1, b_2 , a_2 ,a_3 ,b_3 ,a_4 ,b_4)

In this case, the membership and non-membership functions are

$$\mu_A(x)=\begin{cases} 0 & if \quad x<a_1, \\ L\left(\dfrac{x-a_1}{a_2-a_1}\right) & if \quad a_1\leq x\leq a_2, \\ 1 & if \quad a_2\leq x\leq a_3, \\ R\left(\dfrac{x-a_4}{a_3-a_4}\right) & if \quad a_3\leq x\leq a_4, \\ 0 & if \quad a_4<x \end{cases}$$

$$\nu_A(x)=\begin{cases} 0 & if \quad x<b_1, \\ L\left(\dfrac{x-b_1}{b_2-b_1}\right) & if \quad b_1\le x\le b_2, \\ 1 & if \quad b_2\le x\le b_3, \\ R\left(\dfrac{x-b_4}{b_3-b_4}\right) & if \quad b_3\le x\le b_4, \\ 0 & if \quad b_4<x \end{cases}$$

If $b_2=b_3$ in a trapezoidal intuitionistic fuzzy number A, then it gives a **triangular intuitionistic fuzzy number** (TrIFN) with parameters

$b_1\le a_1\le b_2$ ($a_2 = a_3= b_3$) $\le a_4\le b_4$, and denoted by A = (b1, a_1, b_2 ,a_2 ,b_3)

Definition 2.1.3. An **ordered weighted averaging (OWA)** operators of dimension n is a mapping $f : R^n \rightarrow R$ that has an associated n vector W

$$w=\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

such that (1).$w_i \in [0,1]$ (2).$\sum_i w_i = 1$

Definition 2.1.4. A **Linear fractional programming problem** is defined as follows:

$$MaximizeorMinimizeR(x)=\frac{(c^T x+\alpha)}{(d^T x+\beta)}$$

subject to the constraints : $Ax = b, x \geq 0$, where

(i) x,c and d are n×1 vectors
(ii) A is an m×n matrix, $A = (a_{ij}), i = 1, \dots . m; j = 1, \dots . . n$
(iii) b is an m×1 vector,
(iv) α, β are scalars.

Definition 2.1.5. A **Multi objective linear fractional programming problem** is defined as

$$MaxZ_i=\frac{f_i(x)}{g_i(x)}, i = 1, \dots \dots k$$

where$f_i(x) = c^T x + \alpha, g_i(x) = d^T x + \beta$
subject to the constraints : $Ax = b, x \geq 0$, where

(i) x,c and d are n×1 vectors
(ii) A is an m×n matrix, $A = (a_{ij}), i = 1, \dots . m; j = 1, \dots . . n$
(iii) b is an m×1 vector,
(iv) α, β are scalars.

Definition 2.1.6. If M = sup(s(A),s(B)), where s(A), s(B) is the support of$\tilde{A}$ and $\tilde{B}$ respectively. The **distance from intuitionistic fuzzy number $\widetilde{A}$ to M** is,

$$D(\tilde{A},M)=\int_0^1\int_{-1/2}^{1/2}\left\{-\left[\left[\frac{\mu(a_l(\lambda))+\mu(a_R(\lambda))}{2}+x\left(\mu(a_R(\lambda))-\mu(a_l(\lambda))\right)\right]+\left[\frac{\nu(a_l(\lambda))+\nu(a_R(\lambda))}{2}+x\left(\nu(a_R(\lambda))-\nu(a_l(\lambda))\right)\right]\right]\right\}dxd\lambda$$

2.2. Multi-objective intuitionistic fuzzy linear fractional programming (MOIFLFP) using ranking algorithm

Consider multi-objective intuitionistic fuzzy linear fractional programming problem

$$\left.\begin{array}{l} Max\widetilde{Z_i}=\frac{f_i(x)}{g_i(x)}=\frac{\tilde{c}^Tx+\alpha}{\tilde{d}^Tx+\beta}\ \text{, i = 1,2,3,….} \\ \text{subject to } \tilde{a}_{i1}x_1+\tilde{a}_{i2}x_2+\cdots\cdots+\tilde{a}_{in}x_n\le\tilde{b}_i, \\ x_1,x_2,\cdots\cdots,x_n\ge 0, i=1,2,\cdots\cdots m. \end{array}\right\} \quad (2.2.1)$$

where $\tilde{a}_{ij}$ and $\tilde{b}_i$ are triangular intuitionistic fuzzy numbers,

$$\tilde{a}_{i1}=\{(a_{i11},a_{i12},a_{i13})(a_{i11'},a_{i12'},a_{i13'})\};$$

$$\tilde{a}_{i2}=(a_{i21},a_{i22},a_{i23})(a_{i21'},a_{i22'},a_{i23'})$$

,……………………….,

$\tilde{a}_{in}=\{(a_{in1},a_{in2},a_{in3})(a_{in1'},a_{in2'},a_{in3'})\}$and $\tilde{b}_i=\{(b_{i1},b_{i2},b_{i3})(b_{i1'},b_{i2'},b_{i3'})\}$.

By the ranking algorithm used in Sophia Porchelvi [7], the above MOIFLFPP is transformed into a MOLFPP as follows:

$$\left.\begin{array}{l} Max\widetilde{Z_i}=\frac{f_i(x)}{g_i(x)}=\frac{\tilde{c}^Tx+\alpha}{\tilde{d}^Tx+\beta}\ , i = 1,2,3,.... \\ \text{subject to} \\ (a_{i11}x_1+a_{i21}x_2+\cdots\cdots+a_{in1}x_n)+(a_{i11'}x_1+a_{i21'}x_2+\cdots\cdots+a_{in1'}x_n)+ \\ 4(a_{i12}x_1+a_{i22}x_2+\cdots\cdots+a_{in2}x_n)+(a_{i13}x_1+a_{i23}x_2+\cdots\cdots+a_{in3}x_n)+ \\ (a_{i13'}x_1+a_{i23'}x_2+\cdots\cdots+a_{in3'}x_n)\le b_{i1}+b_{i1}'+4b_{i2}+b_{i3}+b_{i3}' \quad (2.2.2) \\ x_1,x_2,\cdots\cdots x_n\ge 0, i=1,2,\cdots\cdots m. \end{array}\right\}$$

2.3. Multi-Objective linear fractional programming problem using weighting factor

The multi-objective intuitionistic fuzzy linear programming problem with intuitionistic fuzzy coefficients can be formulated as

$$\underset{x\in X}{\text{Max}}\left\{\frac{\tilde{f}_1(x)}{\tilde{g}_1(x)},\frac{\tilde{f}_2(x)}{\tilde{g}_2(x)},\dots\dots,\frac{\tilde{f}_k(x)}{\tilde{g}_k(x)}\right\}$$

subject to (2.2.1) where $f_i: R^n\to R^i$ and $g_i: R^n\to R^i$, where R be the set of all real numbers and R^n be an n-dimensional Euclidean space.

By considering the weighting factor used in Minasian *et al.* [2], the multi-objective intuitionistic fuzzy linear fractional programming problem is defined as

$\text{Max}_{x\in X}\{(w_1\tilde{f}_1(x),w_2\tilde{f}_2(x),\dots\dots,w_k\tilde{f}_k(x)+$
$(w_1'\tilde{g}_1(x),w_2'\tilde{g}_2(x),\dots\dots,w_k'\tilde{g}_k(x)\}$i.e.$\text{Max}_{x\in X}\sum_{m=1}^{k}\{w_m\tilde{f}_m(x)+w_m'\tilde{g}_m(x)\}$
subject to (2.2.2)
where$w_m\in[0,1]$&$\sum_{m=1}^{k}(w_m+w_m')=1$

3. Proposed algorithms

3.1. Procedure for solving MOIFLPP using KantiSwarup’s fractional algorithm

Step 1: Consider the MOIFLPP in which the constraints are triangular intuitionistic fuzzy numbers.
Step 2: Convert the constraints into the form (2.2.2)
Step 3: Consider the most important objective subject to the intuitionistic fuzzy constraints (2.2.2).
Step 4: Solve the problem using KantiSwarup's Fractional algorithm.
Step 5: Consider the next important objective subject to (2.2.2) with an additional constraint, obtained from step 3 which is nothing but the preemptive optimization .
Step 6: Solve it again by using KantiSwarup's Fractional algorithm to get the optimal solution.

3.2. Procedure for solving MOIFLPP using weighting factor
Step 1: Consider the MOIFLPP in which the constraints are triangular intuitionistic fuzzy numbers.
Step 2: Convert the constraints into the form (2.2.2)
Step 3: Using weighting factor, the multi-objective function is defined as $\text{Max}_{x \in X} \sum_{m=1}^{k} \{w_m \tilde{f}_m(x) + w_m{}' \tilde{g}_m(x)\}$.
Step 4: Solve the Linear programming problem

$$\underset{x \in X}{\text{Max}} \sum_{m=1}^{k} \{w_m \tilde{f}_m(x) + w_m{}' \tilde{g}_m(x)\}$$

subject to (2.2.2)
where $w_m \in [0,1] \& \sum_{m=1}^{k} (w_m + w_m{}') = 1$ by simplex method for different weights.

4. Numerical example
Consider the MOIFLPP

$$MaxZ_1 = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}, \qquad MaxZ_2 = \frac{5x_1 + 2x_2}{x_1 + 8x_2 + 1}$$

subject to $3x_1 + 5x_2 \le 15,\ 5x_1 + 2x_2 \le 10,\ x_1, x_2 \ge 0$
$\tilde{3}$ = {(2.5,3,3.8)(2.4,3,4.2)}
$\tilde{5}$ = {(4.5,5,6.5)(4,5,7)}
$\widetilde{15}$ = {(14,15,15.5)(13,15,16)}
$\tilde{5}$ = {(4,5,6.5)(3,5,7.5)}
$\tilde{2}$ = {(1.5,2,3)(1,2,3.5)}
$\widetilde{10}$ = {(9,10,11.5)(8.5,10,12)}

By [2.2.2], the constraints becomes

$$\begin{gathered} 24.9x_1 + 42x_2 \le 118.5 \\ 41x_1 + 19.5x_2 \le 81 \\ x_1, x_2 \ge 0 \end{gathered} \tag{4.1}$$

4.1. Numerical example for MOIFLPP using KantiSwarup's fractional algorithm
Consider the first objective

$$MaxZ_1 = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$$ subject to (4.1)

Now, solve this using KantiSwarup's Fractional algorithm.

Initial Iteration: S_1 and S_2 are basic variables

		c_j	5	3	0	0	
		d_j	5	2	0	0	
d_B	C_B	X_B	x_1	x_2	s_1	s_2	θ
0	0	S_1=118.5	24.9	42	1	0	4.75
0	0	S_2=81	(41)	19.5	0	1	1.9*
$Z^2 = 1$	$Z^1 = 0$						
		Z_j^1-c_j	-5	-3	0	0	
		Z_j^2-d_j	-5	-2	0	0	
		Δ_j	(-5)	-3	0	0	

First Iteration : S_2 leaves and x_1 enters into the basis

		c_j	5	3	0	0	
		d_j	5	2	0	0	
d_B	C_B	X_B	x_1	x_2	s_1	s_2	θ
0	0	S_1=69.2	0	(30)	1	-0.5	2.3*
5	5	x_1=1.98	1	0.48	0	0.02	4.13
$Z^2 = 11$	$Z^1 = 10$						
		Z_j^1-c_j	0	-0.6	0	0.1	
		Z_j^2-d_j	0	0.4	0	0.1	
		Δ_j	0	(-10.6)	0	0.1	

Second Iteration : S_1 leaves and x_2 enters into the basis

		c_j	5	3	0	0
		d_j	5	2	0	0
d_B	C_B	X_B	x_1	x_2	s_1	s_2
2	3	x_2=2.3	0	1	0.03	0.17
5	5	x_1=0.88	1	0	-0.14	-0.06
$Z^2 = 10$	$Z^1 = 11.3$					
		Z_j^1-c_j	0	0	-0.61	0.21
		Z_j^2-d_j	0	0	-0.64	0.04
		Δ_j	0	0	1.1	1.64

Here, all $\Delta_j \geq 0$, the solution is optimal.

The optimum basic feasible solution is $\boldsymbol{x_1 = 0.88, x_2 = 2.3, Z = \frac{Z^1}{Z^2} = 1.13}$

Next, proceed to solve the problem with another objective function and an additional constraint, $\boldsymbol{-0.65x_1 + 0.74x_2 \geq 1.13}$, obtained from the previous step.

$$MaxZ_2 = \frac{5x_1 + 2x_2}{x_1 + 8x_2 + 1}$$

subject to $24.9x_1 + 42x_2 \leq 118.5$

$$41x_1 + 19.5x_2 \leq 81$$

$-0.65x_1 + 0.74x_2 \geq 1.13$ and $x_1, x_2 \geq 0$

Again solve it by using KantiSwarup's fractional algorithm,

Initial Iteration : S_1, S_2 and R_1 are basic variables

		c_j	5	2	0	0	0	-M	
		d_j	1	8	0	0	0	-M	
d_B	C_B	X_B	x_1	x_2	s_1	s_2	s_3	R_1	θ
0	0	s_1=118.5	24.9	42	1	0	0	0	2.82
0	0	s_2=81	41	19.5	0	1	0	0	4.15
-M	-M	R_1=1.13	-0.65	(0.74)	0	0	-1	1	1.5*
Z^2 = 1	Z^1 = 0								
		Z_j^1-c_j	0.65M-5	-0.74M-2	0	0	M	0	
		Z_j^2-d_j	0.65M-1	-0.74M-8	0	0	M	0	
		Δ_j	0.65M-5	-0.74M-2	0	0	M	0	

First Iteration : R_1 leaves and x_2 enters into the basis

		c_j	5	2	0	0	0	
		d_j	1	8	0	0	0	
d_B	C_B	X_B	x_1	x_2	s_1	s_2	s_3	θ
0	0	s_1=54.24	(61.86)	0	1	0	56.7	0.876*
0	0	s_2=51.16	58.16	0	0	1	26.33	0.879
8	2	x_2=1.53	-0.88	1	0	0	-1.35	-
Z^2 = 13.24	Z^1 = 3.06							
		Z_j^1-c_j	-6.76	0	0	0	-2.7	
		Z_j^2-d_j	-8.04	0	0	0	-10.8	
		Δ_j	-64.9	0	0	0	-2.71	

Second Iteration : S_1 leaves and x_1 enters into the basis

		c_j	5	2	0	0	0
		d_j	1	8	0	0	0
d_B	C_B	X_B	x_1	x_2	s_1	s_2	s_3
1	5	x_1=0.88	1	0	0.016	0	0.92
0	0	s_2=-0.02	0	0	-0.93	1	-27.17
8	2	x_2=2.3	0	1	0.014	0	-0.54
Z^2 = 20.28	Z^1 = 9						
		Z_j^1-c_j	0	0	0.108	0	3.52
		Z_j^2-d_j	0	0	0.128	0	-3.4
		Δ_j	0	0	1.04	0	101.99

Here, all $\Delta_j \geq 0$, the solution is optimal.

The optimum basic feasible solution is $\boldsymbol{x_1 = 0.88, x_2 = 2.3, Z = \frac{Z^1}{Z^2} = 0.44}$
i.e. $\boldsymbol{x_1 = 0.88, x_2 = 2.3, Max\ Z_1 = 1.13, Max\ Z_2 = 0.44}$.

4.2. Numerical example for MOIFLPP using weighting factor

Consider the problem as in 4.1.

Using weighting factor the multi-objective function becomes,

$MaxZ = w_1(5x_1 + 3x_2) + w_2(5x_1 + 2x_2) + w_1'(5x_1 + 2x_2 + 1) + w_2'(x_1 + 8x_2 + 1)$

subject to $\quad 24.9x_1 + 42x_2 \leq 118.5$

$$41x_1 + 19.5x_2 \leq 81$$

And $x_1, x_2 \geq 0$

Using TORA software we obtain the solution for different weights.

Following table lists the solution for the above problem for various weights and it also shows that the solutions are independent of weights.

S.No	$\mathbf{W_1}$	$\mathbf{W_2}$	$\mathbf{W_1^1}$	$\mathbf{W_2^1}$	$(\mathbf{x_1},\mathbf{x_2})$
1.	0.991	0.007	0.001	0.001	(0.88,2.3)
2.	0.225	0.575	0.125	0.075	(0.88,2.3)
3.	0.1	0.6	0.2	0.1	(0.88,2.3)
4.	0.715	0.085	0.135	0.065	(0.88,2.3)
5.	0.115	0.415	0.235	0.235	(0.88,2.3)
6.	0.175	0.225	0.385	0.215	(0.88,2.3)
7.	0.125	0.375	0.425	0.075	(0.88,2.3)
8.	0.005	0.685	0.225	0.085	(0.88,2.3)
9.	0.235	0.475	0.175	0.115	(0.88,2.3)
10.	0.435	0.225	0.335	0.005	(0.88,2.3)

We get same result in both methods. In comparison of both methods we conclude that weighting factor method is the best method because the solution procedure is very simple and the optimal solution can be obtained using software package.

5. Conclusion

In this paper, we discussed two different approaches for solving MOIFLPP such as KantiSwarup's method and weighting factor method and conclude that the optimal solutions are same in both methods. The results are verified by means of a numerical example.

REFERENCES

1. H.J.Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, 1 (1978) 45-55.
2. L.A.Zadeh, Fuzzy Sets, *Information and Control*, 8 (1965) 338-353.
3. K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.
4. P.P.Angelov, Optimization in intuitionistic fuzzy environment, *Fuzzy Sets and Systems*, 86 (1997) 299-306.
5. P.A.Thakre, D.S.Sholar and S.P.Thakre, Solving Fuzzy Linear Programming Problem as Multi-Objective Linear Programming Problem, *Proceedings of the World Congress on Engineering, Vol II,* (2009).
6. R.Sophia Porchelvi, An algorithmic approach to multi-objective fuzzy linear programming problem, *International Journal of Algorithms, Computing and Mathematics, Eashwar Publications*, 3(4) (2010) 61-66.
7. R.Sophia Porchelvi and S.Rukmani, On solving multi-objective intuitionistic fuzzy linear programming problem, *International journal of Applied Engineering Research*, 10 (51) (2015) 1046-1050.
8. Kanti Swarup, P.K.Gupta and Man Mohan, Operations Research, Sultand Chand & Sons Pvt Ltd., New Delhi (2014).
9. I.M.Stancu-Minasian, B.Pop, On a fuzzy set approach to solving multiple objective linear fractional programming problem, *Fuzzy Sets and Systems*, 134 (2003) 397-405.