Intern. J. Fuzzy Mathematical Archive
Vol. 9, No. 2, 2015, 179-187
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 8 October 2015
www.researchmathsci.org

International Journal of
Fuzzy Mathematical
Archive

Solution Procedures for Multi-Objective Intuitionistic Fuzzy Linear and Fractional Linear Programming Problems using Weighting Factor

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Received 9 September 2015; accepted 1 October 2015

Abstract. The paper focuses its attention on solving Multi-objective intuitionistic fuzzy linear programming problem and Multi-objective intuitionistic fuzzy linear fractional programming problem by using weighting factor in which the constraints and the cost coefficients are intuitionistic fuzzy numbers. Weighting factor is used to convert problems that are multi-objective into single objective and then it is solved by simplex method for different weights. Finally, a numerical example is provided to check the feasibility of the proposed method.

Keywords: Multi-Objective Intuitionistic Fuzzy Linear Programming, Multi-objective intuitionistic fuzzy linear fractional programming problem, Weighting factor, Triangular intuitionistic fuzzy number

AMS Mathematics Subject Classification (2010): 90B50

1. Introduction

Zimmermann (1978) first discussed the concept of fuzzy Multi-objective mathematical programming problems. Optimization in fuzzy environment was further studied and was applied in various areas by many researchers. Attanassov (1986) introduced the intuitionistic fuzzy sets as a powerful extension of fuzzy set by adding an additional non-membership degree, which may express more abundant and flexible information as compared with the fuzzy set. Recently, the research on intuitionistic fuzzy numbers has received a little attention and several definition of intuitionistic fuzzy numbers and ranking methods have been proposed. The concept of optimization in intuitionistic fuzzy environment was given by Angelov (1997). Dubey *et al.*, (2012) studied linear programming problem in intuitionistic fuzzy environment using intuitionistic fuzzy number and interval uncertainty in fuzzy numbers.

Linear Fractional Programming (LFP) problems are a special type of non-linear programming problems in which the objective function is a ratio of linear functions and the constraints are linear functions. In real life situations, linear fractional models arise in decision making such as construction planning, economic and commercial planning, health care and hospital planning. Several methods (Bajalinov,2003;Stancu-

Minasian,1997,2006) have been recommended to solve LFP Problems. Isbell and Marlow (1956) first identified an example of LFP Problems and solved it by a sequence of linear programming problems. Charnes and Cooper (1962) considered variable transformation method to solve LFP and the updated objective function method were developed for solving the LFP problem by Bitran and Novaes (1973).

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. In many real world problems, there are situations where multiple objectives may be more appropriate rather than considering single objective. However, in such cases emphasis is on efficient solutions, which are optimal in a certain multi-objective sense.

Thakre *et al.,* [6] solved Multi-objective fuzzy linear programming problem (MOFLPP) and I.M. Stancu-Minasian *et al.,* [11] solved Multi-objective fuzzy fractional linear programming problem (MOFFLPP) using weighting factor under constraints with fuzzy coefficients. Based on these papers, we propose to solve Multi-objective intuitionistic fuzzy linear programming problem (MOIFLPP) and Multi-objective intuitionistic fuzzy linear fractional programming problem (MOIFLFPP) with intuitionistic fuzzy coefficients using weighting factor.

The paper is organized as follows : Section 2 briefly describes some basic concepts on intuitionistic fuzzy set, weighting factor and linear fractional programming. Algorithms for solving the multi-objective Intuitionistic fuzzy linear programming problem and Multi-objective intuitionistic fuzzy linear fractional programming problem are developed in section 3. Numerical examples are solved in section 4 to demonstrate the efficiency of proposed method. Finally the paper is concluded in section 5.

2. Preliminaries

2.1. Basic definitions

Definition 2.1.1. An **Intuitionistic fuzzy sets** (IFS) $\bar{A}$assigns to each element x of the universe X to a membership degree $\mu_{\bar{a}}(x)\epsilon[0,1]$ and a non-membership $\nu_{\bar{a}}(x)\epsilon[0,1]$ such that$\mu_{\bar{a}}(x)+\nu_{\bar{a}}(x)\leq 1$. An IFS $\bar{A}$ is mathematically, represented as$\{<x,\mu_{\bar{a}}(x),\nu_{\bar{a}}(x)>/x\in X\}$. The value $\pi_{\bar{a}}(x)=(1-\mu_{\bar{a}}(x))-\nu_{\bar{a}}(x))$ is called the degree of **hesistancy**or the **intuitionistic index** of x to $\bar{a}$.

Definition 2.1.2. An Intuitionistic Fuzzy Set $\bar{A}=\{(x\,,\mu_A(x)\,,\nu_A(x)\mid x\in X)\}$of the real line is called an **Intuitionistic Fuzzy Number** (IFN) if

a) A is IF-normal,
b) A is IF-convex
c) μ_A is upper semicontinuous and ν_A is lower semicontinuous,
d) A = $\{x\in X\,|\nu_A(x)<1\}$ is bounded.

From the definition given above we get at once that for any IFN A there exists eight numbers $a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4\in\mathbb{R}$ such that $b_1\leq a_1\leq b_2\leq a_2\leq a_3\leq b_3\leq a_4\leq b_4$ and four functions $f_A,g_A,h_A,k_A:\mathbb{R}\to[0,1]$, called the sides of a intuitionistic fuzzy number, where f_A and k_A are nondecreasing and g_A and h_A are nonincreasing, such that we can describe a **membership** fuction μ_A in form

$$\mu_A(x)=\begin{cases} 0 & if \quad x<a_1, \\ f_A(x) & if \quad a_1\le x\le a_2, \\ 1 & if \quad a_2\le x\le a_3, \\ g_A(x) & if \quad a_4<x \end{cases}$$

while a **nonmembership** function ν_A has a following form

$$\mu_A(x)=\begin{cases} 0 & if \quad x<b_1, \\ h_A(x) & if \quad b_1\le x\le b_2, \\ 1 & if \quad b_2\le x\le b_3, \\ k_A(x) & if \quad b_4<x \end{cases}$$

Definition 2.1.3. A is **trapezoidal intuitionistic fuzzy number** with parameters $b_1\le a_1\le b_2\le a_2\le a_3\le b_3\le a_4\le b_4$ is denoted by A = (b_1, a_1, b_2 , a_2 ,a_3 ,b_3 ,a_4 ,b_4)
In this case , the membership and non-membership functions are

$$\mu_A(x)=\begin{cases} 0 & if \quad x<a_1, \\ L\left(\dfrac{x-a_1}{a_2-a_1}\right) & if \quad a_1\le x\le a_2, \\ 1 & if \quad a_2\le x\le a_3, \\ R\left(\dfrac{x-a_4}{a_3-a_4}\right) & if \quad a_3\le x\le a_4, \\ 0 & if \quad a_4<x \end{cases}$$

$$\nu_A(x)=\begin{cases} 0 & if \quad x<b_1, \\ L\left(\dfrac{x-b_1}{b_2-b_1}\right) & if \quad b_1\le x\le b_2, \\ 1 & if \quad b_2\le x\le b_3, \\ R\left(\dfrac{x-b_4}{b_3-b_4}\right) & if \quad b_3\le x\le b_4, \\ 0 & if \quad b_4<x \end{cases}$$

If $b_2=b_3$ in a trapezoidal intuitionistic fuzzy number A , then it gives a **triangular intuitionistic fuzzy number** (TrIFN) with parameters $b_1\le a_1\le b_2$ ($a_2 = a_3= b_3$) $\le a_4\le b_4$, and denoted by A = (b_1, a_1, b_2 ,a_2 ,b_3)

Definition 2.1.4. An **ordered weighted averaging (OWA)** operators of dimension n is a mapping $f: R^n \to R$ that has an associated n vector W

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

such that (1).$w_i \in [0,1]$ (2).$\sum_i w_i = 1$

Definition 2.1.5. A **Fractional linear programming problem** is defined as

$$\text{Max } Z = \frac{cx + p}{dx + q}$$

Subject to $AX \le B$, $X \ge 0$

where $c = (c_1, c_2, \ldots c_n)$, $d = (d_1, d_2, \ldots d_n)$, $B = (b_1, b_2, \ldots b_m)^T$,

$X \in R^n$, $x \in X$, p and q are scalar and $A = [a_{ij}]_{nxm}$

Definition 2.1.6. A **Multi objective fractional linear programming problem** is defined as

$$\text{Max } Z_i = \frac{f_i(x)}{g_i(x)} \qquad i = 1,2\ldots..k$$

Subject to $AX \le B$, $X \ge 0$

where $B = (b_1, b_2, \ldots b_m)^T$, $X \in R^n$, $x \in X$, and $A = [a_{ij}]_{nxm}$

$f_i(x) = cx + p$ and $g_i(x) = dx + q$, $c = (c_1, c_2, \ldots c_n)$, $d = (d_1, d_2, \ldots d_n)$, p and q are scalar

2.2. Intuitionistic fuzzy linear programming problem

Consider the multi-objective intuitionistic fuzzy linear programming problem(MOIFLPP) with cost of decision variables are in intuitionistic fuzzy number and coefficient matrix of constraints are in triangular intuitionistic fuzzy number.

$$Max \sum_{j=1}^{n} \widetilde{c_j} x_j$$

Subject to (2.2.1)

$$\sum_{x_{ij} \ge 0} \{(a_{ij}, b_{ij}, c_{ij})(a_{ij}{}', b_{ij}{}', c_{ij}{}')\} \le \{(t_i, u_i, v_i)(t_i{}', u_i{}', v_i{}')\}$$

$$0 \le i \le m \,; 0 \le j \le n.$$

Moreover, we have used one theorem given by Thakre *e at.* [6] which is stated as

For any two triangular intuitionistic fuzzy numbers $A = (a_1, b_1, c_1)$&$B = (a_2, b_2, c_2), A \le B$iff $a_1 \le a_2, b_1 - a_1 \le b_2 - a_2, a_1 + c_1 \le a_2 + c_2$.

It can be extended to intuitionistic fuzzy set as

For any two triangular intuitionistic fuzzy numbers

$A = \{(a_1, b_1, c_1)(a_1', b_1', c_1')\}$&$B = \{(a_2, b_2, c_2)(a_2', b_2', c_2')\}$iff $a_1 \le a_2, b_1 - a_1 \le b_2 - a_2, a_1 + c_1 \le a_2 + c_2, a_1' \le a_2', b_1' - a_1' \le b_2' - a_2', a_1' + c_1' \le a_2' + c_2'$.

Therefore, the constraints (2.2.1) can be rewritten for membership as,

$$\left.\begin{aligned}&\sum_{j=1}^{n} a_{ij}x_j \le t_i\\ &\sum_{j=1}^{n}\left(b_{ij}-a_{ij}\right)x_j \le u_i - t_i\\ &\sum_{j=1}^{n}\left(a_{ij}+c_{ij}\right)x_j \le t_i + v_i, x_i \ge 0\end{aligned}\right\} \quad (2.2.2)$$

And for non-membership as,

$$\left.\begin{aligned}&\sum_{j=1}^{n} a_{ij}^{'}x_j \le t_i^{'}\\ &\sum_{j=1}^{n}\left(b_{ij}^{'}-a_{ij}^{'}\right)x_j \le u_i^{'} - t_i^{'} \quad \sum_{j=1}^{n}\left(a_{ij}^{'}+c_{ij}^{'}\right)x_j \le t_i^{'} + v_i^{'}, x_i \ge 0\end{aligned}\right\}$$
(2.2.3)

2.3 Multi-objective linear programming with intuitionistic fuzzy coefficients

The multi-objective intuitionistic fuzzy linear programming problem with intuitionistic fuzzy coefficients can be formulated as

$$\underset{x\in X}{\text{Max}}\{\tilde{f}_1(x), \tilde{f}_2(x), \dots \dots, \tilde{f}_k(x)\}$$

subject to (2.2.1)

where $f_i: R^n \to R^i$

where R be the set of all real numbers and R^n be an n-dimensional Euclidean space.

By considering the weighting factor, the multi-objective intuitionistic fuzzy linear programming problem is defined as

$$\underset{x\in X}{\text{Max}}\{w_1\tilde{f}_1(x), w_2\tilde{f}_2(x), \dots \dots, w_k\tilde{f}_k(x)\}$$

i.e. $\text{Max}_{x\in X}\sum_{m=1}^{k} w_m\tilde{f}_m(x)$

subject to (2.2.1)

where $w_m \in [0,1]$ and $\sum_{m=1}^{k} w_m = 1$

2.4. Multi-objective linear fractional programming with intuitionistic fuzzy coefficients

The multi-objective intuitionistic fuzzy linear programming problem with intuitionistic fuzzy coefficients can be formulated as

$$\underset{x\in X}{\text{Max}}\{\frac{\tilde{f}_1(x)}{\tilde{g}_1(x)}, \frac{\tilde{f}_2(x)}{\tilde{g}_2(x)}, \dots \dots, \frac{\tilde{f}_k(x)}{\tilde{g}_k(x)}\}$$

subject to (2.2.1)

where $f_i: R^n \to R^i$ and $g_i: R^n \to R^i$

where R be the set of all real numbers and R^n be an n-dimensional Euclidean space.

By considering the weighting factor used in Minasian *et al.* [11], the multi-objective intuitionistic fuzzy linear fractional programming problem is defined as

$\text{Max}_{x\in X}\{(w_1\tilde{f}_1(x), w_2\tilde{f}_2(x), \dots \dots, w_k\tilde{f}_k(x) +$
$(w_1{'}\tilde{g}_1(x), w_2{'}\tilde{g}_2(x), \dots \dots, w_k{'}\tilde{g}_k(x)\}$i.e.$\text{Max}_{x\in X}\sum_{m=1}^{k}\{w_m\tilde{f}_m(x) + w_m{'}\tilde{g}_m(x)\}$

subject to (2.2.1)

where $w_m \in [0,1]$ and $\sum_{m=1}^{k}(w_m + w_m{'}) = 1$

3. Proposed algorithms

3.1. Algorithm for solving multi-objective intuitionistic fuzzy linear programming problem (MOIFLPP)

1. Consider the MOIFLPP in which the constraints are triangular intuitionistic fuzzy numbers.
2. Convert the constraints into the form (2.2.2) and (2.2.3) for membership and non-membership.
3. Using weighting factor, the multi-objective function $\text{Max}_{x\in X}\{\tilde{f}_1(x), \tilde{f}_2(x), \dots\dots, \tilde{f}_k(x)\}$ is defined as $\text{Max}_{x\in X}\sum_{m=1}^{k} w_m \tilde{f}_m(x)$.
4. Solve the Linear programming problem

$$\underset{x\in X}{\text{Max}} \sum_{m=1}^{k} w_m \tilde{f}_m(x)$$

Subject to (2.2.2) and (2.2.3)
where $w_m \in [0,1]$ and $\sum_{m=1}^{k} w_m = 1$ by simplex method for different weights.

3.2. Algorithm for solving multi-objective intuitionistic fuzzy linear fractional programming problem (MOIFLFPP)

1. Consider the MOIFLFPP in which the constraints are triangular intuitionistic fuzzy numbers.
2. Convert the constraints into the form (2.2.2) and (2.2.3) for membership and non-membership.
3. Using weighting factor, the multi-objective function is defined as

   $\text{Max}_{x\in X}\sum_{m=1}^{k}\{w_m \tilde{f}_m(x) + w_m{}' \tilde{g}_m(x)\}$.
4. Solve the Linear programming problem

$$\underset{x\in X}{\text{Max}} \sum_{m=1}^{k} \{w_m \tilde{f}_m(x) + w_m{}' \tilde{g}_m(x)\}$$

subject to (2.2.2) and (2.2.3).
where$w_m \in [0,1]$ and $\sum_{m=1}^{k}(w_m + w_m{}') = 1$ by simplex method for different weights.

4. Numerical examples

4.1. Numerical example for MOIFLPP

Consider the MOIFLP problem

$$Max\tilde{5}x_1 + \tilde{3}x_2$$
$$Max\tilde{4}x_1 + \tilde{7}x_2$$
$$Max\tilde{6}x_1 + \tilde{5}x_2$$

Subject to $\{(2,3,5)(1,3,5.5)\}x_1 + \{(3,4,4.5)(2,4,6)x_2\} \leq \{(2,7,10)(1,7,12)\}$
$\{(3,5,6)(2,5,7)\}x_1 + \{(1.5,2,4)(1,2,5)x_2\} \leq \{(1,8,10)(1,8,12)\}$

Using (2.2.2) and (2.2.3) the membership constraints becomes,

$$\left.\begin{aligned} 2x_1 + 3x_2 &\leq 2 \\ x_1 + x_2 &\leq 5 \\ 7x_1 + 7.5x_2 &\leq 12 \\ 3x_1 + 1.5x_2 &\leq 1 \end{aligned}\right\} \qquad (4.1.1)$$

$$2x_1 + 0.5x_2 \le 7$$
$$9x_1 + 5.5x_2 \le 11$$

and the non-membership constraints becomes,

$$\left.\begin{aligned} x_1 + 2x_2 &\le 1 \\ 2x_1 + 2x_2 &\le 6 \\ 6.5x_1 + 8x_2 &\le 13 \\ 2x_1 + x_2 &\le 1 \\ 3x_1 + x_2 &\le 7 \\ 9x_1 + 6x_2 &\le 13 \end{aligned}\right\} \quad (4.1.2)$$

Using weighting factor the multi-objective function becomes,

$$MaxZ = w_1(5x_1 + 3x_2) + w_2(4x_1 + 7x_2) + w_3(6x_1 + 5x_2)$$

Subject to (4.1.1) & (4.1.2)

Using TORA software we obtain the solution for different weights.
For example, $w_1=0$, $w_2=0$, $w_3=1$

$$MaxZ = 6x_1 + 5x_2$$

Subject to (4.1.1) and (4.1.2)

The solution is Z = 2.89 , x_1 = 0.11, x_2 = 0.44

Following table lists the solution for the above problem for various weights and it also shows that the solutions are independent of weights.

S.No	**W_1**	**W_2**	**W_3**	**(x_1,x_2)**
1.	0	0	1	(0.11,0.44)
2.	0	1	0	(0.11,0.44)
3.	1	0	0	(0.11,0.44)
4.	0.5	0.2	0.3	(0.11,0.44)
5.	0.3	0.5	0.2	(0.11,0.44)
6.	0.6	0.2	0.2	(0.11,0.44)
7.	0.4	0.3	0.3	(0.11,0.44)
8.	0.6	0.3	0.1	(0.11,0.44)
9.	0.2	0.7	0.1	(0.11,0.44)
10.	0.6	0.4	0	(0.11,0.44)

4.2. Numerical example for MOIFLFPP

Consider the MOIFLFPP

$$MaxZ_1 = \frac{5x_1+3x_2}{5x_1+2x_2+1}, \quad MaxZ_2 = \frac{5x_1+2x_2}{x_1+8x_2+1}$$

Subject to $\{(2,3,5)(1,3,5.5)\}x_1 + \{(3,4,4.5)(2,4,6)x_2\} \le \{(2,7,10)(1,7,12)\}$
$\{(3,5,6)(2,5,7)\}x_1 + \{(1.5,2,4)(1,2,5)x_2\} \le \{(1,8,10)(1,8,12)\}$

Using (2.1.2) and (2.1.3) the constraints becomes,
For membership constraints,

$$\left.\begin{aligned} 2x_1 + 3x_2 &\le 2 \\ x_1 + x_2 &\le 5 \\ 7x_1 + 7.5x_2 &\le 12 \\ 3x_1 + 1.5x_2 &\le 1 \\ 2x_1 + 0.5x_2 &\le 7 \end{aligned}\right\} \quad (4.2.1)$$

$$9x_1 + 5.5x_2 \leq 11$$

For non-membership constraints,

$$\left.\begin{aligned} x_1 + 2x_2 &\leq 1 \\ 2x_1 + 2x_2 &\leq 6 \\ 6.5x_1 + 8x_2 &\leq 13 \\ 2x_1 + x_2 &\leq 1 \\ 3x_1 + x_2 &\leq 7 \\ 9x_1 + 6x_2 &\leq 13 \end{aligned}\right\} \quad (4.2.2)$$

Using weighting factor the multi-objective function becomes,

$$MaxZ = w_1(5x_1 + 3x_2) + w_2(5x_1 + 2x_2) + w_1{}'(5x_1 + 2x_2 + 1) + w_2{}'(x_1 + 8x_2 + 1)$$

Using TORA software we obtain the solution for different weights. Following table lists the solution for the above problem for various weights and it also shows that the solutions are independent of weights.

S.No	**W_1**	**W_2**	**$W_1{}^1$**	**$W_2{}^1$**	**(x_1,x_2)**
1.	0.235	0.475	0.175	0.115	(0.11,0.44)
2.	0.005	0.685	0.225	0.085	(0.11,0.44)
3.	0.125	0.375	0.425	0.075	(0.11,0.44)
4.	0.175	0.225	0.385	0.215	(0.11,0.44)
5.	0.115	0.415	0.235	0.235	(0.11,0.44)
6.	0.715	0.085	0.135	0.065	(0.11,0.44)
7.	0.1	0.6	0.2	0.1	(0.11,0.44)
8.	0.225	0.575	0.125	0.075	(0.11,0.44)
9.	0.991	0.007	0.001	0.001	(0.11,0.44)
10.	0.035	0.465	0.475	0.025	(0.11,0.44)

5. Conclusion

In this paper, a multi-objective intuitionistic fuzzy linear programming problem and a multi-objective intuitionistic fuzzy linear fractional programming problem are discussed using weighting factor and it is proved that the solutions obtained are independent of weights.

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