

Study on Fuzzy Soft Hausdorff Spaces

Thangaraj Beaula¹ and R.Raja²

¹PG & Research Department of Mathematics, TBML College
Porayar - 609307, TamilNadu, India
Email: edwinbeaula@yahoo.co.in

²PG & Research Department of Mathematics,
TBML College, Porayar - 609307, TamilNadu, India
Email: itmraja@gmail.com

Received 12 September 2015; accepted 3 October 2015

Abstract. The aim of this paper is to study some properties of fuzzy soft hausdorff space for this we have introduced notions like diagonal sets, injective, surjective, continuous functions. Also fuzzy soft compact is defined.

Keywords: Fuzzy soft diagonal set, fuzzy soft continuous function, fuzzy soft open, homeomorphism and fuzzy soft compact

AMS Mathematics Subject Classification (2010): 37F35, 03B52

1. Introduction

Several set theories can be considered as tools for dealing with uncertainties, say theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [4], theory of Vague sets, theory of interval mathematics [2,5] and theory of rough sets [2], but all these theories have their own difficulties. The reason for these difficulties is possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [9].

Fuzzy soft set which is a combination of fuzzy and soft sets were first introduced by Maji et.al. [8] in 2001. Many researchers improved this study and gave new results ([1],[3]). Aygunoglu and Aygun [6] applied fuzzy soft sets on group theory. Tanay and Kandemir [13] defined fuzzy soft topology on a fuzzy soft set over an initial universe. They introduced new concepts like fuzzy soft base, fuzzy soft neighborhood system, fuzzy soft subspace topology and they presented basic properties. Roy and Samanta [11] defined fuzzy soft topology over the initial universe and they introduced base and subbase for this space also they gave some characterizations.

In this paper, notion like fuzzy soft diagonal set, fuzzy soft continuity, fuzzy soft homeomorphism are introduced. The concept of fuzzy soft hausdorff is coined and some properties of this space is established.

2. Preliminaries

In this section we present some basic definitions of fuzzy soft set. Throughout our discussion, U refers to an initial universe, E the set of all parameters for U and $P(\tilde{U})$ the set of all fuzzy sets of U . (U, E) means the universal set U and the parameter set E .

Definition 2.1. [6] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U .

In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε elements of the soft set (F, E) , or as the set of ε -approximate elements of the soft set.

Definition 2.2. [8] A pair (F, A) is called a fuzzy soft set over U where $F : A \rightarrow P(\tilde{U})$ is a mapping from A into $P(\tilde{U})$.

Definition 2.3 [8] For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E) , we say that (F, A) is a fuzzy soft subset of (G, B) , if

(i) $A \subseteq B$

(ii) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \subseteq (G, B)$.

Definition 2.4. [8] Union of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \cup (G, B) = (H, C)$.

Definition 2.5. [8] Intersection of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where

$C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A) \cap (G, B) = (H, C)$.

Definition 2.6. [11] Let $A \subseteq E$ then the mapping $F_A : E \rightarrow \tilde{P}(U)$, defined by $F_A(e) = \mu^e F_A$ (a fuzzy subset of U), is called soft set over (U, E) , where $\mu^e F_A = \tilde{0}$ if $e \in E - A$ and $\mu^e F_A \neq \tilde{0}$ if $e \in A$. The set of all fuzzy soft set over (U, E) is denoted by $FS(U, E)$.

Definition 2.7. [11] The fuzzy soft set $F_\emptyset \in FS(U, E)$ is called null fuzzy soft set and it is denoted by $\tilde{\Phi}$. Here $F_\emptyset(e) = \tilde{0}$ for every $e \in E$.

Definition 2.8. [11] Let $F_E \in FS(U, E)$ and $F_E(e) = \tilde{1}$ for all $e \in E$. Then F_E is called absolute fuzzy soft set. It is denoted by \tilde{E} .

Definition 2.9. [11] Let $F_A, G_B \in FS(U, E)$. If $F_A(e) \subseteq G_B(e)$ for all $e \in E$, i.e., if $\mu^e F_A \subseteq \mu^e G_B$ for all $e \in E$, i.e., if

Fuzzy Soft Hausdorff Spaces

$\mu^e F_A(x) \leq \mu^e G_B(x)$ for all $x \in U$ and for all $e \in E$, then F_A is said to be fuzzy soft subset of G_B , denoted by $F_A \tilde{\subseteq} G_B$.

Definition 2.10. [11] Let $F_A, G_B \in FS(U, E)$

Then the union of F_A and G_B is also a fuzzy soft set H_C , defined by $H_C(e) = \mu^e H_C = \mu^e F_A \cup \mu^e G_B$ for all $e \in E$ where $C = A \cup B$. Here we write $H_C = F_A \tilde{\cup} G_B$.

Definition 2.11. [11] Let $F_A, G_B \in FS(U, E)$.

Then the intersection of F_A and G_B is also a fuzzy softset

H_C defined by $H_C(e) = \mu^e H_C = \mu^e F_A \cap \mu^e G_B$ for all $e \in E$ where $C = A \cap B$. Here we write $H_C = F_A \tilde{\cap} G_B$.

Definition 2.12. Let $F_A \in FS(U, E)$. The complement of F_A is denoted by F_A^C and is defined by $F_A^C : E \rightarrow \tilde{P}(U)$ is a mapping given by $F_A^C(\varepsilon) = [F(\varepsilon)]^C, \forall \varepsilon \in E$.

3. Fuzzy soft Hausdorff spaces

Definition 3.1. Let $FS(U, E)$ be the set of all fuzzy soft sets over U . Let $F_A \tilde{\in} FS(U, E)$ $a \tilde{\in} U$ and $A \tilde{\subseteq} E$. Then $(F_A)_\Delta$ is a fuzzy soft set over $I^{U \times U}$ for which

$$(F_A)_\Delta : E \rightarrow I^{U \times U} \text{ and } (F_A)_\Delta(e) = \Delta = \left\{ \begin{array}{l} \mu_{F_e^{(a \times a)}}(s) \text{ if } a = e \\ 0 \text{ if } a \neq e \end{array} \right\}. \text{ Then } (F_A)_\Delta \text{ is the fuzzy}$$

soft diagonal set.

Theorem 3.2. (U, E, \mathfrak{S}) be a fuzzy soft hausdorff space if and only if the fuzzy soft diagonal set $(F_A)_\Delta$ is fuzzy soft closed.

Proof: Let (U, E, \mathfrak{S}) be a fuzzy soft hausdorff space. We must show that $(F_A)_\Delta^C$ is fuzzy soft open. Suppose that $\mu_{F_e^{(a_1 \times a_2)}} \tilde{\in} (F_A)_\Delta^C$ then $\mu_{F_e^{(a_1 \times a_2)}} \not\tilde{\in} (F_A)_\Delta$ and for some $S \tilde{\in} E$ $\mu_{F_e^{(a_1 \times a_2)}}(s) \not\tilde{\in} (F_A)_\Delta(s)(e)$. Denote $\mu_{F_e^{(a_1 \times a_2)}}(s) = (\mu_{F_e^{a_1}} \times \mu_{F_e^{a_2}})(s)$. Thus $\mu_{F_e^{a_1}} \neq \mu_{F_e^{a_2}}$ and are two fuzzy soft points say F_e and F_e in the fuzzy soft topological space. Since (U, E, \mathfrak{S}) is fuzzy soft hausdorff there exists $G_A, H_A \tilde{\in} \mathfrak{S}$ such that $F_e \tilde{\in} G_A, F_e \tilde{\in} H_A$ with $G_A \tilde{\cap} H_A = \tilde{\emptyset}$. Hence for each $e \tilde{\in} F$, $\mu_{F_e^{(a_1 \times a_2)}}(e) \tilde{\in} G_A(e) \times H_A(e)$ and $(G_A(e) \times H_A(e)) \tilde{\cap} (F_A)_\Delta = \tilde{\Phi}$. Hence $(F_A)_\Delta$ is fuzzy soft closed.

Thangaraj Beaula and R.Raja

Conversly, let $(F_A)_\Delta$ is fuzzy soft closed. Let $F_e, F_{e'} \in (U, E, \mathfrak{S})$ such that $F_e \neq F_{e'}$. Then $(F_e, F_{e'}) \not\subseteq (F_A)_\Delta$ and so $(F_e, F_{e'}) \subseteq (F_A)_\Delta^C$ such that there exists fuzzy soft open sets H_A and G_A such that $(F_e, F_{e'}) \subseteq H_A \times G_A \subseteq (F_A)_\Delta^C$. Hence $F_e \subseteq H_A$ and $F_{e'} \subseteq G_A$ again $H_A \cap G_A = \tilde{\Phi}$.

Definition 3.3. Let U, U' be universe sets E, E' be the corresponding parameter sets. The map $h_{up} : FS(U_E) \rightarrow FS(U'_{E'})$ is a fuzzy soft map from U to U' which maps the fuzzy soft subset F_A of U to fuzzy soft subset $hup(F_A)$ of U' where $u : U \rightarrow U'$

and $p : E \rightarrow E'$ is defined as

$$[h_{up}(F_A)]_{e'}(S') = \begin{cases} \sup_{s \in U^{-1}(S')} \left[\sup_{e \in p^{-1}(e')} F_A(e) \right](S) & \text{if } p^{-1}(e') \neq \tilde{\Phi} \text{ and } u^{-1}(S') \neq \tilde{\Phi} \\ 0 & \text{otherwise} \end{cases}$$

Also the universe $[h_{up}^{-1}(F_{A'})]_e$ is defined as

$$[h_{up}^{-1}(F_{A'})]_e(S) = \begin{cases} F_{A'}(p(e)u(S)), \text{ for } p(e) \in E' \\ 0 & \text{otherwise} \end{cases}$$

Definition 3.4. Let (U, E, \mathfrak{S}_1) and (U, E, \mathfrak{S}_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $h_{up} : FS(U_E) \rightarrow FS(U'_{E'})$ is said to be fuzzy soft continuous if $h_{up}^{-1}(F_{A'}) \subseteq \mathfrak{S}_1, \forall F_{A'} \subseteq \mathfrak{S}_2$.

Theorem 3.5. If (U, E, \mathfrak{S}) is fuzzy soft hausdorff and $h_{up} : FS(U_E) \rightarrow FS(U'_{E'})$ is a fuzzy soft map which is injective, surjective and fuzzy soft open then (U', E', \mathfrak{S}') is fuzzy soft hausdorff.

Proof: Let F_{e_1}, F_{e_2} be fuzzy soft sets in $FS(U'_{E'})$ such that $F_{e_1} \neq F_{e_2}$. Since hup is surjective there exists F_{e_1}, F_{e_2} in $FS(U_E)$ such that $h_{up}(F_{e_1}) = F_{e_1}$ and $h_{up}(F_{e_2}) = F_{e_2}$. As (U, E, \mathfrak{S}) is fuzzy soft hausdorff there exists $F_A, G_A \subseteq \mathfrak{S}$ such that $F_{e_1} \subseteq F_A$ and $F_{e_2} \subseteq G_A$ and $F_A \cap G_A = \tilde{\Phi}$.

Fuzzy Soft Hausdorff Spaces

Hence for each $e \in E$, $F_{e_1} \in F_A(e)$ and $F_{e_2} \in G_A(e)$ and $F_A(e) \cap G_A(e) = \emptyset$. Thus $h_{up}(F_{e_1}) = F_{e_1} \in h_{up}(F_A(e))$ and $h_{up}(F_{e_2}) = F_{e_2} \in h_{up}(G_A(e))$. Since h_{up} is fuzzy soft open $h_{up}(F_A)$ and $h_{up}(G_A)$ belongs to \mathcal{S}' . since h_{up} is injective, $h_{up}(F_A) \cap h_{up}(G_A) = h_{up}(F_A \cap G_A) = \emptyset$.

Thus (U', E', \mathcal{S}') is fuzzy soft hausdorff.

Definition 3.6. Fuzzy soft mapping $h_{up} : FS(U_{E_1}) \rightarrow FS(U'_{E_2})$ is called fuzzy soft open if $h_{up}(F_A) \in \mathcal{S}_2$, for every $F_A \in \mathcal{S}_1$.

Definition 3.7. $h_{up} : FS(U_E) \rightarrow FS(U'_{E'})$ is said to be injective if u and p are injective.

It is said to be surjective if u and p are surjective.

Definition 3.8. Let (U, E, \mathcal{S}) and (U', E', \mathcal{S}') be two fuzzy soft topological spaces. A fuzzy soft function $h_{up} : FS(U_E) \rightarrow FS(U'_{E'})$ is called homeomorphism if h_{up} is one to one, onto, continuous and open.

Theorem 3.9. In fuzzy soft hausdorff space, a sequence converges to a unique point.

Proof: Suppose that $\{F_{e_n}\}$ is a sequence in (U, E, \mathcal{S}) converging to F_e and let $F_{e_n} \neq F_e$. Since (U, E, \mathcal{S}) is fuzzy soft hausdorff space there exist fuzzy soft open sets $F_A, G_A \in \mathcal{S}$ such that $F_{e_1} \in F_A, F_{e_2} \in G_A$ and $F_A \cap G_A = \emptyset$. This implies that for all $e \in E$, $F_{e_1} \in F_A(e), F_{e_2} \in G_A(e)$ and $F_A(e) \cap G_A(e) = \emptyset$. As $\{F_{e_n}\}$ converges to F_e and F_A is fuzzy soft open set containing F_e . there exist $n_1 \in \mathbb{N}$ such that $F_{e_n} \in F_A$ for all $n \geq n_1$. Since F_{e_n} converges to F_e and G_A is fuzzy soft neighbourhood of F_e then there exists $n_2 \in \mathbb{N}$ such that $F_{e_n} \in G_A$ for all $n \geq n_2$. Let $n_0 = \max(n_1, n_2)$ then for all $n \geq n_0, F_{e_n} \in F_A$ and $F_{e_n} \in G_A$. This implies that $F_{e_n} \in F_A(e)$ and $F_{e_n} \in G_A(e)$ for all $e \in E$. Then $F_A(e) \cap G_A(e) \neq \emptyset$. Hence $F_A \cap G_A \neq \emptyset$. This is contradiction.

Definition 3.10. Let $F_A \in (U, E, \mathcal{S})$ and $G_B \in (U', E', \mathcal{S}')$. The Cartesian product $F_A \times G_B$ is defined by $(F \times G)_{(A \times B)}$ as

$$(F \times G)_{(A \times B)}(e) = \mu_{F_A}^e \times \mu_{G_B}^{e'} \text{ where } \mu_{F_A}^e \text{ and } \mu_{G_B}^{e'} \text{ are fuzzy subset of } U$$

and U' where $\mu_{F_A}^e = \emptyset$ if $e \in E - A$ and $\mu_{F_A}^e \neq \emptyset$ if $e \in A$ also $\mu_{G_B}^{e'} = \emptyset$ if $e' \in E' - B$ and $\mu_{G_B}^{e'} \neq \emptyset$ if $e' \in B$.

Definition 3.11. Let $p:U \times U' \rightarrow U$, $p':U \times U' \rightarrow U'$ and $q:E \times E' \rightarrow E$, $q':E \times E' \rightarrow E'$ be the projections on first and second factors. Then (p, q) and (p', q') are homeomorphisms defined from $U \times U'$ to U and $U \times U'$ to U' as follows.

$$(p, q)(F_A \times G_B) = \mu_{p(F_A \times G_B)}^{q(e \times e')} \text{ where } e \in A, e' \in B = \mu_{F_A}^e$$

$$(p', q')(F_A \times G_B) = \mu_{p'(F_A \times G_B)}^{q'(e \times e')} \text{ where } e \in A, e' \in B = \mu_{G_B}^{e'}$$

Lemma 3.12. Let (U, E, \mathfrak{S}) and (U', E', \mathfrak{S}') be two fuzzy soft topological spaces. Then U, U' are homeomorphic to the subspace of $U \times U'$.

Proof: Let $(F_e, F_{e'}) \in U \times U'$ and $(e, e') \in E \times E'$. Our aim is to show that $h_{up}:U \rightarrow U \times \{F_e\} \cong U \times U'$ is a homeomorphism where $u:U \rightarrow U \times \{F_e\}$ and $p:E \rightarrow E \times \{e'\}$. u and p are one to one and onto, so h_{up} is one to one and onto.

Now, let us show that h_{up} is continuous. Let F_A be the fuzzy soft set in the subspace $U \times \{F_e\}$. Then there exists open set $G_B \times H_C$ in the subspace $S(U \times U', E \times E')$ such that $F_A = (G_B \times H_C) \tilde{\cap} \tilde{E}_{U \times \{F_e\}}$ for $p(e) = (e, e')$.

$$\begin{aligned} [(h_{up}^{-1})(F_A)]_{(e, e')}(S) &= (h_{up}^{-1})((G_B \times H_C) \tilde{\cap} \tilde{E}_{U \times \{F_e\}})_{(e, e')}(S) \\ &= h_{up}^{-1}((G_B \times H_C) \tilde{\cap} \tilde{E}_{U \times \{F_e\}})_{p(e)}^{(S)} \\ &= h_{up}^{-1}((G_B(e) \times H_C(e')) \tilde{\cap} U \times \{F_e\})_{p(e)}^{(S)} \\ &= \begin{cases} h_{up}^{-1}(G_B(e) \times \{F_e\})_{p(e)}^{(S)} & \text{if } F_e \in H_C(e') \\ \tilde{\Phi} & \text{otherwise} \end{cases} \\ &= \begin{cases} G_B(p(e)) \cap u(S) & \text{for } F_e \in H_C(e') \\ \tilde{\Phi} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Then } (h_{up})^{-1}(F_A) = \begin{cases} G_B & \text{if } F_e \in H_C(e') \\ \tilde{\Phi} & \text{otherwise} \end{cases}$$

Fuzzy Soft Hausdorff Spaces

Hence $h_{up}^{-1}F_A$ is fuzzy soft open. So h_{up} is fuzzy soft continuous. Now we show that h_{up} is open. Let F_A be a fuzzy soft open set on U . For $e' \in E$.

$$\begin{aligned} [(h_{up})(F_A)_{e'}](s) &= \begin{cases} \sup_{s \in u^{-1}(s')} [\sup_{e \in p^{-1}(e')} F_A(e)](s) & \text{if } p^{-1}(e') \neq \tilde{\Phi} \text{ and } u^{-1}(s') \neq \tilde{\Phi} \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} F_A(e) \times \{F_{e'}\}, p^{-1}(e') \tilde{\cap} A \neq \tilde{\Phi} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Hence $(h_{up})F_A = F_A \times \{F_{e'}\}$. Which is open and hence the mapping h_{up} is open.

Theorem 3.13. U and U' are fuzzy soft hausdorff spaces then $U \times U'$ is fuzzy soft hausdorff space.

Proof: Let U and U' be fuzzy soft hausdorff spaces. Let

$\mu_{F_e}(a_1 \times b_1), \mu_{F_e}(a_2 \times b_2) \tilde{\in} U \times U'$ and $\mu_{F_e}(a_1 \times b_1) \neq \mu_{F_e}(a_2 \times b_2)$. So we have $\mu_{F_e} a_1 \neq \mu_{F_e} a_2$ or $\mu_{F_e} b_1 \neq \mu_{F_e} b_2$. Assume that $\mu_{F_e} a_1 \neq \mu_{F_e} a_2$ since U is fuzzy soft hausdorff space there exist fuzzy soft open set F_A and G_B such that $\mu_{F_e} a_1 \tilde{\in} F_A$, $\mu_{F_e} a_2 \tilde{\in} G_B$ and $F_A \tilde{\cap} G_B = \tilde{\Phi}$. Then $F_A \times \tilde{E}'$ and $G_B \times \tilde{E}'$ are fuzzy soft open set on $U \times U'$. Hence $(\mu_{F_e}(a_1 \times b_1)) \tilde{\in} F_A \times \tilde{E}'$, $(\mu_{F_e}(a_2 \times b_2)) \tilde{\in} G_B \times \tilde{E}'$ and $(F_A \times \tilde{E}') \tilde{\cap} (G_B \times \tilde{E}') = \tilde{\Phi}$.

Definition 3.14. Let (U, E, \mathfrak{S}) be a fuzzy soft topological spaces. A collection of fuzzy soft open subsets of $\{F_{A_\alpha} / \alpha \in \mathfrak{S}\}$, (U, E, \mathfrak{S}) is said to form an open cover if

$\tilde{E} = \tilde{\cup}_{\alpha \in \mathfrak{S}} F_{A_\alpha}$. If the finite subcollection of $\{F_{A_\alpha} / \alpha \in \mathfrak{S}\}$ covers \tilde{E} then \tilde{E} is said to be

fuzzy soft compact ie, $\tilde{E} = \tilde{\cup}_{i=1}^n F_{A_{\alpha_i}}$.

Theorem 3.15. Let (U, E, \mathfrak{S}) be a fuzzy soft hausdorff space. If F_A is fuzzy soft compact on U , then F_A is fuzzy soft closed.

Proof: We must show that F_A^C is fuzzy soft open. Let $F_e \tilde{\in} F_A^C$. Then $F_e \not\tilde{\in} F_A$ choose $F_{e'} \tilde{\in} F_A$ clearly $F_e \neq F_{e'}$ for $e, e' \tilde{\in} \tilde{E}$. As \tilde{E} is fuzzy soft hausdorff there exist disjoint fuzzy soft open subsets G_B and $G_{C'}$ for F_e and $F_{e'}$ respectively such that $G_B \tilde{\cap} G_{C'} = \tilde{\Phi}$, (ie) $F_e \tilde{\in} (G_B)_{F_e}$ and $F_{e'} \tilde{\in} (G_{C'})_{F_{e'}}$. Then $F_A(e) \tilde{\subseteq} (G_{C'})_{F_{e'}}(e)$. The collection

Thangaraj Beaula and R.Raja

$E = \{(G_c)_{F_e}(e) : F_e \tilde{\in} F_A\}$ is a fuzzy soft open covering of F_A . As F_A is fuzzy soft

compact there exist has a finite sub cover, so $F_A \tilde{\subseteq} \bigcup_{i=1}^n (G_c)_{F_{e_i}}(e)$. It is clear that

$\bigcup_{i=1}^n (G_c)_{F_{e_i}}(e)$ and $\bigcap_{i=1}^n (G_B)_{F_{e_i}}(e)$ are disjoint. Then $F_e \tilde{\in} (G_B)_{F_{e_i}} \tilde{\subseteq} (\bigcup_{i=1}^n (G_c)_{F_{e_i}})^C \tilde{\subseteq} F_A^C$.

Hence F_A^C is fuzzy soft open.

4. Conclusion

Fuzzy soft set are very popular subject researchers. This hybrid model which is more general than fuzzy and fuzzy soft sets can be applied several directions easily. In this paper we construct fuzzy soft hausdorff spaces and proved some theorems.

REFERENCES

1. B.Ahamad and A. Kharal, On fuzzy soft sets, *Adv. Fuzzy Syst.* (2009) doi: 10.1155/2009/586507.
2. M.I.Ali, A note on soft sets, rough soft sets and fuzzy soft sets, *Fuzzy Sets and Systems* 64 (1994) 159-174.
3. K.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.
4. K.Atanassov, Operators over interval values intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 64 (1994) 159-174.
5. A. Aygunoglu and H. Aygun, Introduction to fuzzy soft groups, *Comput. Math. Appl.* 58 (2009) 1279-1286.
6. C.Chang, Fuzzy Topological Spaces, *J. Math. Anal. Appl.*, 24 (1968) 182-190.
7. M.B. Gorzalzany, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo*, 19(2) (1970). 89-96.
8. P.K.Maji, A. R. Roy and R Biswas, Fuzzy soft sets, *J. Fuzzy Math.* 9(3) (2001) 589-602.
9. P.A.Mohodstov, Soft set theory-first result, *Computers and Mathematics with Application*, 37(1999) 19-31.
10. Z.Pawlak, Rough sets, *Int. J. Comput Sci.*, 11 (1982) 341-356.
11. S.Roy and T.K.Samanta, A note on fuzzy soft topological spaces, *Ann. Fuzzy Math. Inform.*, 3 (2012) 305-311.
12. B.Tanay and M.B.Kandemir, Topological structure of fuzzy soft sets, *Comput. Math, Appl.*, 61 (2011) 2592 – 2957.
13. L.A.Zadeh, Fuzzy sets, *Inf. Control*, 8 (1965) 338-353.