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An Application of Fuzzy Matrices in Medical Diagnosis

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Abstract. Fuzzy set theory plays a vital role in medical fields. There are varieties of models involving fuzzy matrices to deal with different complicated aspects of medical diagnosis. Sanchez formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between the symptoms and diseases. In this paper the above concept is applied to Hypotension and Anaemia.

Keywords: Medical diagnosis, fuzzy matrix, membership function, triangular fuzzy number

AMS Mathematics Subject Classification (2010): 15B15

1. Introduction

In recent years, computational intelligence has been used to solve many complex problems by developing intelligent systems. And fuzzy logic has proved to be a powerful tool for decision-making systems, such as expert systems and pattern classification systems. Fuzzy set theory has already been used in some medical expert systems. Sanchez [10] formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases. Esogbue and Elder utilized fuzzy cluster analysis to model medical diagnostic. Meenakshi and Kaliraja [7] have extended Sanchez's approach for medical diagnosis using the representation of a interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it. Baruah [1,2] used the definition of complement of a fuzzy soft set proposed by Neog and Sut [8] and put forward a matrix representation of fuzzy soft set and extended Sanchez's approach for medical diagnosis. Edward Samuel and Balamurugan [4] studied Sanchez's approach for medical diagnosis using Intuitionist fuzzy set. Fuzzy set theory also plays a vital role in the field of Decision Making. Decision Making is a most important scientific, social and economic endeavour. In classical crisp decision making theories, decisions are made under conditions of certainty but in real life situations this is not possible which gives rise to fuzzy decision making theories. For decision making in fuzzy environment one may refer Bellman and Zadeh [3]. Most probably the fuzzy decision model in which overall ranking or ordering of different fuzzy sets are determined by using comparison matrix, introduced and developed by Shimura [11].

In this paper, we would like to discuss how fuzzy set theory and fuzzy logic can be used for developing knowledge based systems in medicine. Some basic definitions of fuzzy set theory has been discussed in section 2. In Section 3, a novel approach is presented for medical diagnosis which is also an extension of Sanchez's approach with modified procedure using triangular fuzzy number matrices and its new membership function. Some conclusions are given in section 4.

2. Preliminaries

Definition 2.1. [13] (Triangular fuzzy number matrix)

Triangular fuzzy number matrix of order mxn is defined as A = $(a_{ij})_{mxn}$ where a_{ij} = $(a_{ijL,}a_{ijM},\ a_{ijU}\ )$ is the ij^{th} element of A. a_{ijL}, a_{ijU} are the left and right spreads of a_{ij}respectively and a_{ijM} is the mean value.

Definition 2.2. (Addition and subtraction operation on triangular fuzzy number matrix)

Let A = $(a_{ij})_{nxn}$and B = $(b_{ij})_{nxn}$ be two triangular fuzzy number matrices of same order.

Then (i) Addition Operation

A(+)B = $(a_{ij}+ b_{ij})_{nxn}$ where

$a_{ij} + b_{ij} = (a_{ijL} + b_{ijL}, a_{ijM} + b_{ijM}, a_{ijU} + b_{ijU})$ is the ijth element of A(+)B

(ii) Subtraction Operation

A(-)B = $(a_{ij} -b_{ij})_{nxn}$ where $a_{ij} - b_{ij} = (a_{ijL} - b_{ijU}, a_{ijM} -b_{ijM}, a_{ijU} - b_{ijL})$ is the ijth element of A(-)B

The same condition holds for triangular fuzzy membership number.

Definition 2.3. (Multiplication operation on triangular fuzzy number matrix)

Let A = $(a_{ij})_{mxp}$ and B = $(b_{ij}\ )_{pxn}$ be two triangular fuzzy number matrices. Then the Multiplication Operation: A(.)B = $(c_{ij})_{mxn}$, where $(c_{ij}) = \sum_{k=1}^{p} a_{ik}\, b_{kj}$ for i = 1, 2,.....m and j = 1,2,.....n

Definition 2.4. (Max-min composition on fuzzy membership value matrices)

Let F_{mn} denote the set of all mxn matrices over F. Elements of F_{mn} are called as fuzzy membership value matrices.

For A = $(a_{ij}) \in F_{mxp}$ and B = $(b_{ij}) \in F_{pxn}$ the max-min product A(.)B = $(\sup_k[\{\inf\{a_{ik}; b_{kj}\}\}]) \in F_{mxn}$.

Definition 2.5. (Maximum operation on triangular fuzzy number)

Let A = $(a_{ij})_{nxn}$ where $a_{ij} = (a_{ijL},\ a_{ijM},\ a_{ijU})$ and B = $(b_{ij})_{nxn}$ where $b_{ij} =(b_{ijL}, b_{ijM}, b_{ijU})$ be two triangular fuzzy number matrices of same order. Then the maximum operation on it is given by L_{max} = max(A, B) = $(\sup\{a_{ij} ; b_{ij}\})$ where $\sup\{a_{ij} , b_{ij}\} = (\sup(a_{ijL}; b_{ijL}), \sup(a_{ijM}; b_{ijM}), \sup(a_{ijU}; b_{ijU}))$ is the ij^{th} element of max(A, B).

Definition 2.6 (Arithmetic mean (AM) for triangular fuzzy number)

Let A = $(a_1, a_2\ , a_3)$ be a triangular fuzzy number then AM (A) $=\frac{a_1+a_2+ a_3}{3}$. The same condition holds for triangular fuzzy membership number.

3. Medical diagnosis under fuzzy environment

Let S be the set of symptoms of certain diseases, D is a set of diseases and P is a set of patients. The elements of triangular fuzzy number matrix are defined as $A = (a_{ij})_{mxl}$ where $(a_{ij}) = (a_{ijL}, a_{ijM}, a_{ijU})$ is the ijth element of A, $0 \le a_{ijL} \le a_{ijM} \le a_{ijU} \le 10$. Here a_{ijL} is the lower bound, a_{ijM} is the moderate value and a_{ijU} is the upper bound.

Procedure 3.1.

Step 1: Construct a triangular fuzzy number matrix (F, D) over S, where F is a mapping given by $F : D \to \check{F}(S)$, $\check{F}(S)$ is a set of all triangular fuzzy sets of S. This matrix is denoted by R_0 which is the fuzzy occurrence matrix or symptom-disease triangular fuzzy number matrix.

Step 2: Construct another triangular fuzzy number matrix (F_1, S) over P, where F_1 is a mapping given by $F_1: S \to F(P)$. This matrix is denoted by R_S which is the patient-symptom triangular fuzzy number matrix.

Step 3: Convert the elements of triangular fuzzy number matrix into its membership function as follows: Membership function of $(a_{ij}) = (a_{ijL}, a_{ijM}, a_{ijU})$ is defined as
$\mu_{aij} = \left(\frac{a_{ijL}}{10} \le \frac{a_{ijM}}{10} \le \frac{a_{ijU}}{10}\right)$, if $0 \le a_{ijL} \le a_{ijM} \le a_{ijU} \le 10$, where $0 \le \frac{a_{ijL}}{10} \le \frac{a_{ijM}}{10} \le \frac{a_{ijU}}{10} \le 1$.

Now the matrix R_0 and R_S are converted into triangular fuzzy membership matrices namely $(R_0)_{mem}$ and $(R_S)_{mem}$.

Step 4: Compute the following relation matrices.
$R_1 = (R_S)_{mem}(.)(R_0)_{mem}$ it is calculated using Definition 2.5.
$R_2 = (R_S)_{mem}(.)(J(-)(R_0)_{mem})$, where J is the triangular fuzzy membership matrix in which all entries are (1, 1, 1). $(J(-)(R_0)_{mem})$ is the complement of $(R_0)_{mem}$ and it is called as non symptom-disease triangular fuzzy membership matrix.
$R_3 = (J(-)(R_S)_{mem})(.)(R_0)_{mem}$, where $(J(-)(R_S)_{mem})$ is the complement of
R_S and it is called as non patient-symptom triangular fuzzy membership matrix.
R_2 and R_3 are calculated using subtraction operation and Definition 2.5.
$R_4 = \max\{R_2;R_3\}$. It is calculated using Definition 2.6.
The elements of R_1,R_2,R_3,R_4 is of the form $Y_{ij} = (y_{ijL}, y_{ijM}, y_{ijU})$ where $0 \le y_{ijL} \le y_{ijM} \le y_{ijU} \le 1$.
R_5 = R1(-)R4. It is calculated using subtraction operation. The elements of R_5 is of the form $Z_{ij} = (z_{ijL}, z_{ijM}, z_{ijU}) \in [-1, 1]$ where $z_{ijL} \le z_{ijM} \le z_{ijU}$.

Step 5: Calculate $R_6 = AM(z_{ij})$ and Row'_i= Maximum of i^{th} row which helps the decision maker to strongly confirm the disease for the patient.

Illustrative Example 3.1.

Suppose there are three patient's P_1, P_2 and P_3 in a hospital with symptoms fatigue, dizziness, headache and cramps in legs. Let the possible diseases relating to the above symptoms be hypotension and anaemia.

Step 1: We consider the set $S = \{S_1, S_2, S_3, S_4\}$ as universal set where S_1, S_2, S_3 and S_4 represent the symptoms fatigue, dizziness, headache and cramps in legs respectively and the set $D = \{d_1, d_2\}$ where d_1 and d_2 represent the parameters Hypotension and Anaemia respectively. Suppose that

$F(d_1) = [< e_1, (7, 8.5, 10) >,< e_2, (2, 3.5, 5) >,< e_3, (6, 6.5, 7) >,< e_4, (3, 4, 5) >]$

$F(d_2) = [< e_1, (6, 7.5,9) >,< e_2, (5,6,7) >,< e_3, (4, 5.5,7) >, < e_4, (8, 9, 10) >]$

The triangular fuzzy number matrix (F,D) is a parameterized family ($F(d_1)$, $F(d_2)$) of all triangular fuzzy number matrix over the set S and are determined from expert medical documentation. Thus the triangular fuzzy number matrix (F,D) represents a relation matrix R_0 and it gives an approximate description of the triangular fuzzy number matrix medical knowledge of the two diseases and their symptoms given by

$R_O =$

	d_1	d_2
S_1	(7, 8.5, 10)	(6, 7.5,9)
S_2	(2, 3.5, 5)	(5, 6, 7)
S_3	(6,6.5,7)	(4, 5.5,7)
S_4	(3, 4, 5)	(8, 9, 10)

Step 2: Again we take $P = \{p_1, p_2, p_3\}$ as the universal set where p_1, p_2 and p_3 represent patients respectively and $S = \{s_1, s_2, s_3, s_4\}$ as the set of parameters. Suppose that,

$F_1(s_1) = [<P_1, (6, 7.5, 9) >;<P_2, (3, 4, 5) >;<P_3(6, 7, 8) >]$

$F1(s2) = [<P_1, (4, 5, 6) >;<P_2, (4, 6, 8) >;<P_3(3, 5, 7) >]$

$F1(s3) = [<P_1, (8, 9, 10) >;<P_2, (3, 4, 5) >;<P_3, (6, 7, 8) >]$

$F1(s4) = [<P_1, (7, 8.5, 10) >;<P_2, (4, 5, 6) >;<P_3, (3, 4.5.6) >]$

The triangular fuzzy number matrix (F_1, S) is another parameterized family of triangular fuzzy number matrix and gives a collection of approximate description of the patient-symptoms in the hospital. Thus the triangular fuzzy number matrix (F_1, S) represents a relation matrix R_S called patient-symptom matrix given by

$R_S =$

	S_1	S_2	S_3	S_4
P_1	(6,7.5, 9)	(4, 5, 6)	(8, 9, 10)	(7, 8.5, 10)
P_2	(3, 4, 5)	(4, 6, 8)	(3, 4, 5)	(4, 5, 6)
P_3	(6, 7, 8)	(3, 5, 7)	(6, 7, 8)	(3, 4.5, 6)

Step 3:

$(R_0)_{mem} =$

	d_1	d_2
S_1	(0.7, 0.85, 1)	(0.6, 0.75,0.9)
S_2	(0.2, 0.35, 0.5)	(0.5, 0.6, 0.7)
S_3	(0.6, 0.65, 0.7)	(0.4, 0.55,0.7)
S_4	(0.3, 0.4, 0.5)	(0.8, 0.9, 1)

$(R_S)_{mem} =$

	S_1	S_2	S_3	S_4
P_1	(0.6,0.75, 0.9)	(0.4, 0.5, 0.6)	(0.8, 0.9, 1)	(0.7, 0.85, 1)
P_2	(0.3, 0.4, 0.5)	(0.4, 0.6, 0.8)	(0.3, 0.4, 0.5)	(0.4, 0.5, 0.6)
P_3	(0.6, 0.7, 0.8)	(0.3, 0.5, 0.7)	(0.6, 0.7, 0.8)	(0.3, 0.4, 0.6)

Step 4: Computing the following relation matrices

$R_1 = (R_s)_{mem}(.)\,(R_0)_{mem} =$

	d_1	d_2
P_1	(0.6, 0.75, 0.9)	(0.6, 0.85,1)
P_2	(0.3, 0.4, 0.5)	(0.4, 0.6, 0.7)
P_3	(0.6, 0.7, 0.8)	(0.6, 0.7,0.8)

$$R_2 = (R_s)_{mem}(.)(J(-)(R_0)_{mem}) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{pmatrix} d_1 & d_2 \\ (0.7, 0.6, 0.6) & (0.6, 0.5, 0.6) \\ (0.4, 0.6, 0.5) & (0.4, 0.4, 0.3) \\ (0.4, 0.45, 0.5) & (0.6, 0.45, 0.3) \end{pmatrix}$$

$$R_3 = (J(-)(R_s)_{mem})(.)(R_0)_{mem} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{pmatrix} d_1 & d_2 \\ (0.4, 0.35, 0.9) & (0.6, 0.55, 0.9) \\ (0.7, 0.6, 0.5) & (0.6, 0.6, 0.5) \\ (0.4, 0.4, 0.4) & (0.7, 0.55, 0.4) \end{pmatrix}$$

$$R_4 = \max\{R_2, R_3\} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{pmatrix} d_1 & d_2 \\ (0.7, 0.6, 0.9) & (0.6, 0.55, 0.9) \\ (0.7, 0.6, 0.5) & (0.6, 0.6, 0.5) \\ (0.4, 0.45, 0.5) & (0.7, 0.55, 0.4) \end{pmatrix}$$

$$R_5 = R_1(-)R_4 = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{pmatrix} d_1 & d_2 \\ (-0.3, 0.15, 0.3) & (-0.3, -0.3, 0.4) \\ (-0.4, -0.2, 0) & (-0.2, 0, 0.2) \\ (-0.1, 0.25, 0.4) & (-0.1, 0.15, 0.4) \end{pmatrix}$$

Step 5:

$$R_6 = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{pmatrix} d_1 & d_2 & \text{Row'i = maximum of ith row} \\ 0.05 & 0.13 & 0.13 \\ -0.2 & 0 & 0.00 \\ 0.18 & 0.15 & 0.18 \end{pmatrix}$$

This can be represented in the form of a graph namely network as follows:

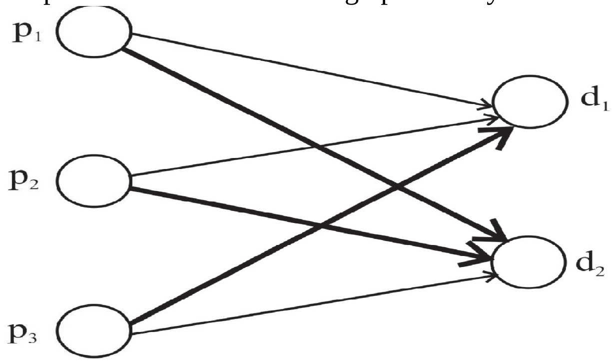

Figure 1: Fuzzy medical diagnosis network

In the above network, nodes or vertices denote the patients and diseases, lengths or edges denote the assumption of diseases to the patients. The darken edges denotes the strong confirmation of disease to the patient.

4. Conclusion

Medicine is one of the field in which the applicability of fuzzy set theory was recognized quite early. The physician generally gathers knowledge about the patient from the past history, laboratory test result and other investigative procedures such as x-rays and ultra sonic rays etc. The knowledge provided by each of these sources carries with it varying degrees of uncertainty. Thus the best and most useful descriptions of disease entities often use linguistic terms that are vague. Hence in this paper, Fuzzy set framework has been utilized in several different approaches to model the medical diagnostic process and decision making process. From the above analysis it is obvious that, the patient P_1 and P_2 suffer from Anaemia whereas the patient P_3 faces Hypotension.

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