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Neighborhood and Efficient Triple Connected Domination Number of a Fuzzy Graph

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Abstract. A subset S of V of a nontrivial fuzzy graph G is said to be a Neighborhood triple Connected dominatingset, if S is a dominating set and the induced sub graph < N(S) > is a triple Connected. The minimum cardinality taken over all Neighborhood triple Connected dominating sets is called the Neighborhood triple Connected domination number. In this paper we introduce the concept of Neighborhood and Efficienttriple Connected domination number of a fuzzy graph and obtain some interesting results for the new parameter in fuzzy graph.

Keywords: Connected dominating set, fuzzy graphs, triple connected dominating set, neighborhood triple connected domination number, efficient triple connected domination number

AMS Mathematics Subject Classification (2010): 05C72

1.Introduction

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths cycles and connectedness are introduced by Rosenfeld [12]. Bhattacharya [3] has established some connectivity regarding fuzzy cut node and fuzzy bridges. The concept of domination in fuzzy graphs are introduced by Somasudaram and Somasundaram [15] in 1998. In 2012, Bounds on connected domination in square graph of graph is introduced by Muddabihal and Srinivasa. Triple connected domination number of a graph introduced by Mahadevan, Selvam. In this paper, we analyze bounds on Neighborhood and Efficient triple connected dominating set of fuzzy graph and proves some results based on triple connected domination number of a fuzzy graph

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2. Preliminaries

Definition 2.1. A fuzzy subset of a nonempty set V is mapping σ: V $\rightarrow$ [0, 1] and A fuzzy relation on V is fuzzy subset of V x V. A fuzzy graph is a pair G:(σ,μ) where σ is a fuzzy subset of a set V and μ is a fuzzy relation on σ , where μ (u,v) $\leq \sigma$ (u) $\Lambda\sigma$ (v) $\forall$ u , v $\in$ V

Definition 2.2. Let G:(σ,μ) be a fuzzy graph. Then D $\subseteq$V is said to be a fuzzy dominating set of G if for every*v*$\in$ V-D, There exists u in D such that

$\mu\ (u,v) = \sigma\ (u)\Lambda\sigma\ (v)$. The minimum scalar cardinality of D is called the fuzzy domination number and is denoted by γ (G). Note that scalar cardinality of a fuzzy sub set D of V is $|D| = \sum_{v\in V} \sigma\ (v)$

Definition 2.3. A dominating set D of a fuzzy graph G:(σ, μ) is connected dominating set if the induced fuzzy subgraph <D> is connected. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by γ_c (G)

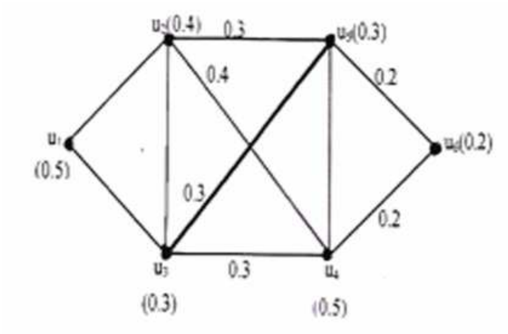

Fig.1 D= { u_3,u_5}, γ_c (G)=0.6

Figure 1:

Definition 2.4. Let G be a fuzzy graph, the neighborhood of a vertex *v* in V is defined by N(v) ={u∈V:$\mu\ (u,v) = \sigma\ (u)\Lambda\sigma\ (v)$}. The scalar cardinality of N(V) is the neighborhood degree of V, Which is denoted by $d_N(v)$ and the effective degree of V is the sum of the weights of the edges incident on V denoted by $d_E(v)$

Definition 2.5. Let u and v be any two vertices of a fuzzy graph G. Then u strongly dominates v(v weakly dominates u) if (i) $\mu\ (u,v) = \sigma\ (u)\Lambda\sigma\ (v)$.and
(ii) $d_N(u) > d_N(v)$

Definition 2.6. A fuzzy graph G is said to be triple connected if any three vertices lie on a path in G.All paths, cycles, complete graphs are some standard examples of triple connected fuzzy graphs.

Definition 2.7. A subset D of V of a nontrivial connected fuzzy graph G is said to be triple connected dominating set. If D is the dominating set and the induced fuzzy subgraph <D> is triple connected .The minimum cardinality taken over all triple connected dominating set of G is called the triple connected domination number of G and is denoted by γ_{tc}(G).

Examples 2.8. For the fuzzy graph Fig.2, D = {v_1,v_2,v_5 } forms a γ_{tc} set of G

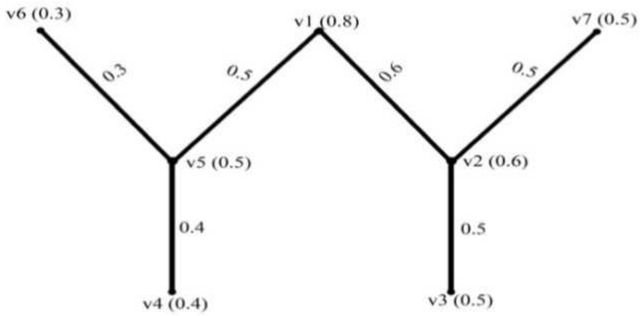

Figure 2: $\gamma_{tc}(G)$ =0.8+0.5+0.6 =1.9

3. Neighborhood triple connected domination number of a fuzzy graph

A subset S of V of a nontrivial fuzzy graph G is said to be a Neighborhood triple Connected dominating set, if S is a dominating set and the induced sub graph < N(S) > is triple Connected. The minimum cardinality taken overall Neighborhood triple Connected dominating sets is called the Neighborhood triple Connected domination number. In this section we present few elementary bounds on Neighborhood triple connected domination number of a fuzzy graph and the correspondingsome results.

Example 3.1. For the fuzzy graph, in Fig.3.1,S ={v_1, v_2} forms a γ_{ntc}-set { v_3, v_4 ,v_5} of G. Hence $\gamma_{ntc}(G)$ = 1.1.

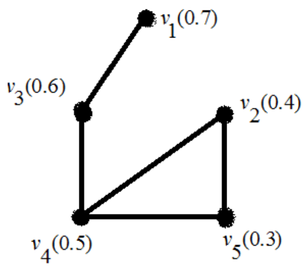

Figure 3.1: Fuzzy graph with $\gamma_{ntc}(G)$ = 1.1.

Observation 3.2. Neighborhood triple connected dominating set (γ_{ntc} -set or ntcd set) does not exist for all fuzzy graphs.

Remark 3.3. Throughout this section we consider only connected fuzzygraphs for which neighborhood triple connected dominating set exists.

Observation 3.4. The complement of a neighborhood triple connected dominating set S need not be a neighborhood triple connected dominating set.

Example 3.5. For the fuzzy graph in the Fig.3.2, S = {v_1, v_5, v_6} is a neighborhood triple connected dominating set. But the complement V – S = {v_2, v_3, v_4} is not a neighborhood triple connected dominating set.

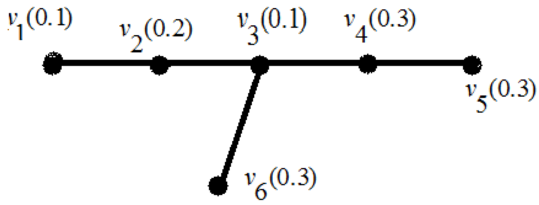

Figure 3.2:

Observation 3.6. Every neighborhood triple connected dominating set is a dominating set but not conversely.

Example 3.7. For the fuzzy graph in Fig.3.3, S = {v_1, v_2} is a neighborhood triple connected dominating set as well as a dominating set. For the fuzzy graph in Fig.3.4, S = {v_3, v_5} is a dominating set but not a neighborhood triple connected dominating set.

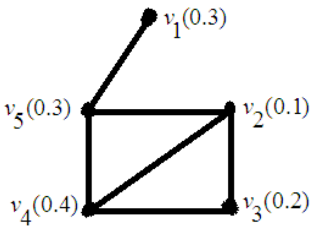

Figure 3.3:

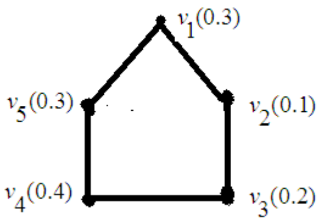

Figure 3.4:

Exact value for some special graphs

1) The diamond fuzzy graph with 4 vertices and 5 edges as shown in Fig.3.5

For any diamond fuzzygraph G of order 4, $\gamma_{ntc}(G) = 0.1$

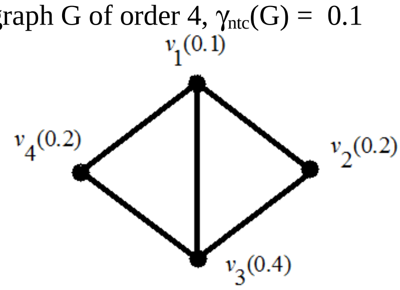

Figure 3.5:

In Fig: 3.5, S = {v_1} is a neighborhood triple connected dominating set.

2) The Moser spindle (also called the Mosers' spindle or Moser graph) with seven vertices and eleven edges as shown in Fig.3.6.

For the Moser spindle fuzzy graph G, $\gamma_{ntc}(G) = 0.7$

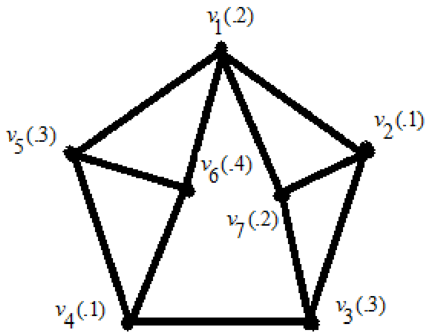

Figure 3.6:

In Fig 3.6, S = {v_3, v_6} is a neighborhood triple connected dominating set.

3) For any Fan graph of order $n \geq 4$,in Fig:3.7,$\gamma_{ntc}(G)$ = 0.2

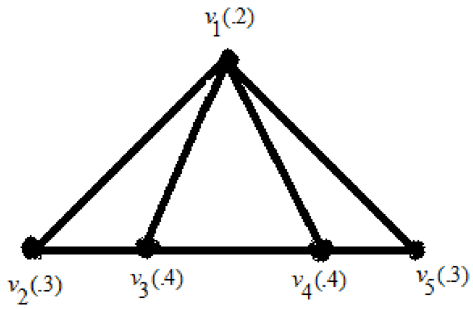

Figure 3.7:

In Fig: 3.7,S = {v_1} is a neighborhood triple connected dominating set.

Theorem 3.8. For any connected fuzzy graph G with $p \geq 3$, we have $[p/[1 + \Delta s(G)] \leq \gamma_{ntc}(G) \leq p - 1$ and the bounds are sharp.

Proof. Let S be a dominating set of any neighborhood triple connected dominating set, the each node of S dominating atmost $\Delta s(G)$ other nodes of G, Thus
$p \leq \gamma(G)\, [1 + \Delta s(G)] \leq \gamma_{ntc}(G)$ Also for a connected fuzzy graph ,clearly
$\gamma_{ntc}(G) \leq p - 1$.

Theorem 3.9. For any connected fuzzy graph G with $p \geq 3$ vertices, Neighborhood triple connected dominating vertices is p - 1 if and only if G is isomorphic to P3, C3.

Proof. Suppose G is isomorphic to any one of the fuzzy graphs as stated in the theorem, then clearly, Neighborhood triple connected dominating vertices is p – 1. Conversely, assume that G is a connected fuzzy graph with $p \geq 3$ vertices and Neighborhood triple connected dominating vertices is p – 1. Let S = {v_1, v_2, …,v_{p-1}} be a $\gamma_{ntc}(G)$ -set. Let <V - S> = {v_p}. Since S is the neighborhood triple connected dominating set, there existsv$_i$ in S such that v_i is adjacent to v_p. Also <N(S)> is triple connected, we have v_i is adjacent to v_j for $i \neq j$ in S.

Case (i) < N(S)> has three vertices
Then the induced subgraph <N(S)> has the following possibilities. <N(S)> = P3 or C3. Hence G is isomorphic to P3, C3

Case (ii) <N(S)> has more than three vertices
Then there exists at least one v_k for $i \neq j \neq k$ in S which is adjacent to either v_i or v_j.
If v_k is adjacent to v_i, then the induced subgraph <N(S)> is not triple connected. If we increase the degrees of the vertices in S we can find a neighborhood triple connected dominating set with fewer elements than S. Hence no graph exists in this case.
If v_k is adjacent to v_j, then we can remove v_k from S and find a neighborhood triple connected dominating set of G with less than p – 1 vertices, which is a contradiction. Hence no graph exist in this case.

Theorem 3.10. For any connected fuzzy graph G with $p \geq 5$ vertices, $\gamma_{tc}(G) + \gamma_{ntc}(G) < 2p - 3$.

Proof. Let G be a connected graph with $p \geq 5$ vertices. We know that$\gamma_{tc}(G) \leq p - 2$ for $p \geq 5$ vertices and by theorem 3.8, $\gamma_{ntc}(G) \leq p - 1$ for $p \geq 3$ vertices. Hence $\gamma_{tc}(G) + \gamma_{ntc}(G) \leq 2p - 3$ for $p \geq 5$ vertices. Also by theorem 3.9, the bound is not sharp.

4. Efficient triple connected domination number of a fuzzy graph

Definition 4.1. A subset S of V of a nontrivial fuzzy graph G is said to be an efficient dominating set, if every vertex is dominated exactlyonce.The minimum cardinality taken over all efficient dominating sets is called the efficient domination number and is denoted by γ_e.

Definition 4.2. A subset S of V of a nontrivial fuzzy graph G is said to be a efficient triple connected dominating set, if S is a efficient dominating set and the induced subgraph<S> is triple connected. The minimum cardinality taken over all efficient triple connected dominating sets is called the efficient triple connected domination number of G and is denoted by $\gamma_{etc}(G)$. Any efficient triple connected dominating set with γ_{etc} vertices is called a γ_{etc}-set of G. In this section, we introduce new domination parameter efficient triple connected domination number of a fuzzy graph.

Example 4.3. For the fuzzy graph G, in Fig.4.1, S = {v_1, v_2, v_6} forms aγ_{etc} -set of G. Hence $\gamma_{etc}(G) = 0.7$.

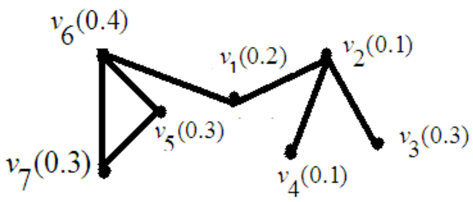

Figure 4.1:

Observation 4.4. Efficient Triple connected dominating set (γ_{etc} -set or etcd set) does not exists for all graphs.

Observation 4.5. The complement of the efficient triple connected dominating set need not be a efficient triple Connected dominating set.

Example 4.6. For the fuzzy graph, in Fig.4.2, = {v_2, v_5, v_6} forms a efficient triple connected dominating set of G, But the complement V – S = {v_1, v_3, v_4, v_7, v_8} is not a efficient triple connected dominating set.

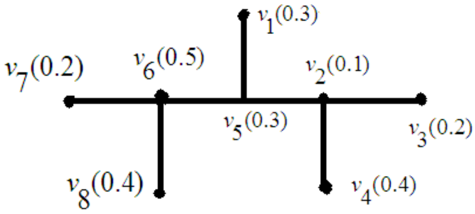

Figure 4.2: Fuzzy Graph in which V – S is not an etcd set

Observation 4.7. Every efficient triple connected dominating set is a dominating set but not conversely.

5.Conclusion

The Neighborhood and Efficient triple connected domination number of fuzzy graph is defined. Theorems related to this concept are derived and the relation between triple connected dominating number of fuzzy graphs and Neighborhood, Efficient triple connected dominating number of fuzzy graphs are established.

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