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A Method for Ranking of Fuzzy Numbers Using new Area Method

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Abstract. In this paper, the researchers proposed a modified new weighted area method to rank fuzzy numbers using the area of trapezium. We also use some comparative examples to illustrate the advantage of the proposed method with the existing metric index ranking methods. If the comparison for negative numbers is not possible, we are using the comparison based on spread and weight. The process of this method is easier than that of other. A set of illustrative examples are also used.

Keywords: Area of trapezium, generalized trapezoidal fuzzy numbers, ranking of fuzzy numbers

AMS Mathematics Subject Classification (2010): 03E72, 90B99

1. Introduction

Ranking fuzzy numbers plays an important role in decision making. Most of the real world problems that exist in nature are fuzzy, than probabilistic or deterministic. Problems in which fuzzy theory is used, like fuzzy risk analysis, fuzzy optimization, etc., at one or the other stage fuzzy numbers must be ranked before an action is taken by a decision maker.

In this paper, we want to propose a new method for ranking in area of two generalized trapezoidal fuzzy numbers. A simpler and easier approach is proposed for the ranking of generalized trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches [17].

Since the inception of fuzzy sets by Zadeh [25] in 1965, many authors have proposed different methods for ranking fuzzy numbers. However, due to the complexity of the problem, a method which gives a satisfactory result to all situations is a challenging task. Most of the methods proposed so far are non discriminating, counter-intuitive and some produce different rankings for the same situation and some methods cannot rank crisp numbers. Ranking fuzzy numbers was firstproposed by Jain in the year 1976 for decision making in fuzzy situations by representing theill-defined quantity as a fuzzy set. Jain [13,14] proposed a method using the concept of maximizing set to order the fuzzy numbers and the decision maker considers only the right side membership function. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Yager [23,24] proposed four indices to order fuzzy quantities in [0, 1]. An Adamo [3] fuzzy decision tree was an important breakthrough in ranking fuzzy numbers. Dubois and Prade [12] proposed a complete set of comparison indices in the frame work of Zadeh's possibility theory.

Liou and Wang [16] presented ranking fuzzy numbers with integral value. Choobineh and Li [10] presented an index for ordering fuzzy numbers. Since then several methods have been proposed by various researchers which include distance method by Cheng [9]. Wang and Kerre [21,22] classified the existing ranking procedures into three classes. The first class consists of ranking procedures based on fuzzy mean and spread and second class consists ranking procedures based on fuzzy scoring whereas, the third class consists of methods based on preference relations and concluded that the ordering procedures associated with first class are relatively reasonable for the ordering of fuzzy numbers specially, the ranking procedure presented by Adamo [3] which satisfies all the reasonable properties for the ordering of fuzzy quantities. The methods presented in the second class are not doing well and the methods which belong to class three are reasonable. Stephen Dinagar and Kamalanathan [20] defined a distance based ranking procedure SD of PILOT to solve the maximize net present value.

Later on, ranking fuzzy numbers by area between the centroid point and original point by Chu andTsao [11], fuzzy risk analysis based on ranking of generalized trapezoidal fuzzy numbers by Chen and Chen [7], a new approach for ranking trapezoidal fuzzy numbers by Abbasbandy and Hajjari [1], fuzzy riskanalysis based on ranking generalized fuzzy numbers with different heights and different spreads by Chen and Chen [8] came into existence.Amit Kumar et al. [15] presented a procedure on ranking generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread.

In this paper, we have used area of trapezium as a new ranking procedure for generalized trapezoidal fuzzy numbers. The trapezium has two bases parallel to each other and two legs joining these bases at the points (a, 0), (b, 0), (c, w) and (d, w). The area covered by these four points is our ranking function. In addition to this [9] gives equal ranking for this case as the method calculate similar distance for bothfuzzy numbers as the centroid points are the samewhereas [9] also ranked equally ($C1 \approx C2$) for this casebecause both fuzzy numbers have similar area betweenthe centroids and original points. To overcome this, we can use mode, spread and weight method in addition [4,17].

Section 2 briefly introduces the basic concepts and definitions of fuzzy numbers. Section 3 presents the proposed new method. In Section 4, we have explained the comparative study made by the existing methods. In section 5, we have concluded the paper.

2. Preliminaries

Chen (1985, 1990) represented a Generalized Trapezoidal Fuzzy Number (GTrFN) $\tilde{A}$as $\tilde{A}$ = (a,b,c,d;w), 0 <w ≤ 1 and a, b, c and d are real numbers. The generalized fuzzy number $\tilde{A}$ is a fuzzy subset of real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0$,where $-\infty \le x \le a$.

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing with constant rate on $a \le x \le b$.

(iv) $\mu_{\tilde{A}}(x)$ = w, where $b \le x \le c$.

(v) $\mu_{\tilde{A}}(x)$ is strictly decreasing with constant rate on $c \le x \le d$ $\mu_{\tilde{A}}(x) = 0$, where $d \le x \le \infty$.

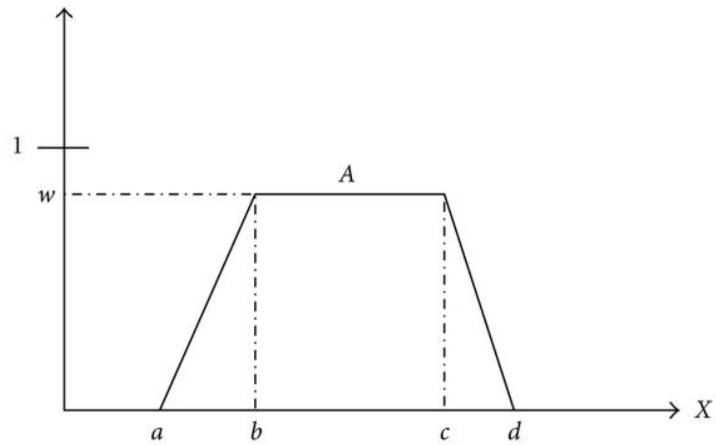

Figure 1: Generalized trapezoidal fuzzy number

A Generalized Trapezoidal Fuzzy Number $\tilde{A}$ = ($a,b,c,d;w$) is a fuzzy set of the real line R whose membership function $\mu_{\tilde{A}}(x):\ R \to [0,w]$is defined as

$$\mu^{w}_{\tilde{A}}(x) = \begin{cases} \mu^{w}_{L\tilde{A}}(x) = w\left(\dfrac{x-a}{b-a}\right), \ for a \le x \le b \\ w,\ for b \le x \le c \\ \mu_{R\tilde{A}}(x) = w\left(\dfrac{d-x}{d-c}\right), \ for c \le x \le d \\ 0, Otherwise \end{cases}$$

where $a < b < c < d$ and $w \in (0,1]$

3. Proposed method

In the given generalized trapezoidal number ADEF is the trapezium with the bases AD (= b_1) and EF (= b_2) and the legs AE and DF. The points covered by ADEF are A (a, 0), D (d, 0), E (a, w) and F (b, w).

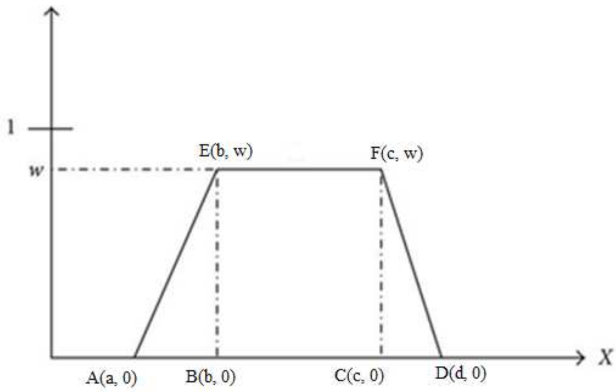

Figure 2: Generalized trapezoidal fuzzy number ABCDEF

The area of trapezium is give by

$$A = \frac{h}{2}(b_1 + b_2)$$

where h = height = w; b_1, b_2 = bases of trapezium

b_1 = distance between A(a, 0) and D(d,0) = $\sqrt{\left[(d-a)^2 + (0)^2\right]} = (d-a)$

b_2 = distance between E (b, w) and F (c, w) $= \sqrt{\left[(c-b)^2 + (w-w)^2\right]} = (c-b)$

The area of the trapezium ADFE is, $A = \frac{w}{2}\left[(d-a) + (c-b)\right]$

The ranking function of the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as:

$$\Re(\tilde{A}) = \frac{w}{2}\left[(c+d) - (a+b)\right]$$

The mode of the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ is defined as:

$$Mode = \frac{1}{2}\int_0^w (b+c)dx = \frac{w}{2}(b+c)$$

The spread of the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ is defined as:

$$Spread = \int_0^w (d-a)dx = w(d-a)$$

The left spread of the generalized trapezoid dal fuzzy number $A = (a, b, c, d; w)$ is defined as: $Left\,Spread = \int_0^w (b-a)dx = w(b-a)$

The right spread of the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ is defined as: $Right\ Spread = \int_0^w (d - c)dx = w(d - c)$

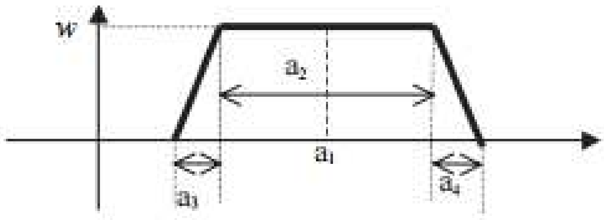

Figure 3:

In exercise 3, the fuzzy number A is mirror image of B (non-overlapping fuzzy number). The existing ranking methods including the defined our proposed method does not consider this case. [9] gives equal ranking for this case as the method calculate similar distance for both fuzzy numbers as the centroid points are the same whereas [9] also ranked equally ($C1 \approx C2$) for this case because both fuzzy numbers have similar area between the centroids and original points. To overcome this, we can use the method based on mode, spread and weight as an additional tool.

Using the above definitions we now define the ranking procedure of two generalized trapezoidal fuzzy numbers $A = (a_1, b_1, c_1, d_1; w_1)$ and $B = (a_2, b_2, c2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers.

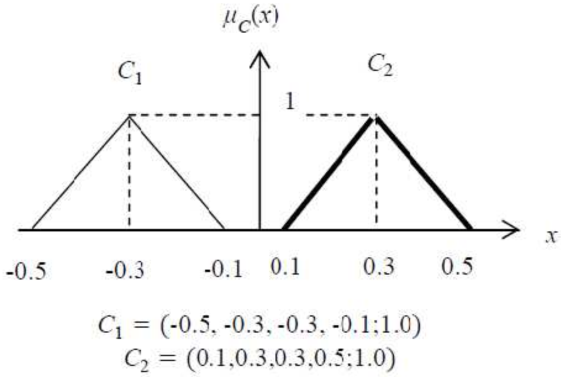

Figure 4: Non-overlapping Fuzzy Numbers (Mirror image case).

The working procedure to compare A ad B are as follows [17]:

Step 1. Find $\Re(A)$ and $\Re(B)$

Case (i) If $\Re(A) < \Re(B)$ then $A \prec B$

Case (ii) If $\Re(A) > \Re(B)$ then $A \succ B$

Case (iii) If $\Re(A) = \Re(B)$, comparison is not possible, then go to step 2.

Step 2. Find Mode (A) and Mode (B)

Case (i) If Mode (A) < Mode (B) then $A \prec B$

Case (ii) If Mode (A) > Mode (B) then $A \succ B$

Case (iii) If Mode (A) = Mode (B) then go to Step 3.

Step 3. Find left spread (A) and left spread (B)

Case (i) If Spread (A) < Spread (B) then $A \succ B$

Case (ii) If Spread (A) > Spread (B) then $A \prec B$

Case (iii) If Spread (A) = Spread (B) then go to Step 4.

Step 4. Find Left Spread (A) and Left Spread (B)

Case (i) If left spread (A) < left spread (B) then $A \prec B$

Case (ii) If left spread (A) > left spread (B) then $A \succ B$

Case (iii) If left spread (A) = left spread (B) then go to Step 5.

Step 5. Examine w_1 and w_2

Case (i) If left w_1<w_2, then $A \prec B$

Case (ii) If w_1< w_2, then $A \succ B$

Case (iii) If $w_1 = w_2$, then $A \approx B$.

4. Results and discussion

Example 1. Let A = (0.2,0.4,0.6,0.8;0.35) and B =(0.1,0.2,0.3,0.4;.7) be two generalized trapezoidal fuzzy number, then

$\Re(A) = \frac{0.35}{2}[(0.6+0.8)-(0.2+0.4)] = 0.14$ $\Re(B) = \frac{0.7}{2}[(0.3+0.4)-(0.1+0.2)] = 0.14$

since $\Re(A) = \Re(B) = 0.14$, then go to step 2.

Step 2: Mode (A) = $\frac{0.35}{2}(0.4+0.6) = 0.175$ Mode (B) = $\frac{0.35}{2}(0.2+0.3) = 0.09$

$$Mode(A) > Mode(B) \Rightarrow A \succ B$$

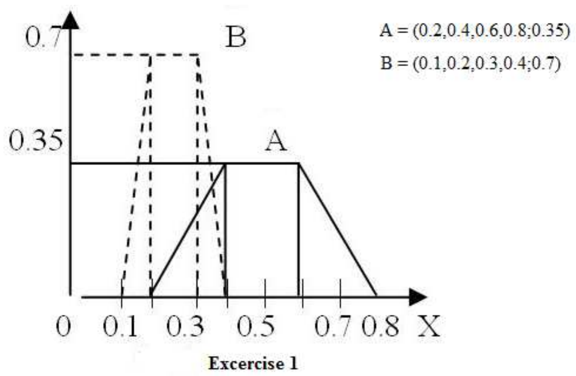

Figure 5:

Example 2. Let A =(0.1,0.2,0.4,0.5;1) and B =(0.1,0.3,0.3,0.5;1) be two generalized trapezoidal fuzzy number, then

$$\Re(A)=\frac{1.0}{2}\left[(0.4+0.5)-(0.1+0.2)\right]=0.3 \quad \Re(B)=\frac{1.0}{2}\left[(0.3+0.5)-(0.1+0.3)\right]=0.2$$

so $\Re(A)>\Re(B)\Rightarrow A\succ B$

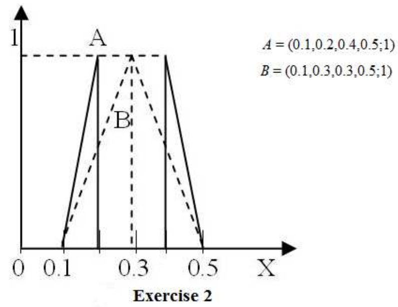

Figure 6:

Example 3. Let A =(0.1,0.2,0.4,0.5;1) and B =(1,1,1,1;1) be two generalized trapezoidal fuzzy number, then

$$\Re(A)=\frac{1.0}{2}\left[(0.4+0.5)-(0.1+0.2)\right]=0.3$$

$$\Re(B)=\frac{1.0}{2}\left[(1+1)-(1+1)\right]=0$$

so $\Re(A)>\Re(B)\Rightarrow A\succ B$

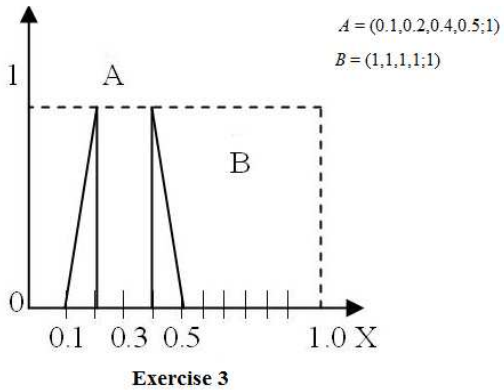

Figure 7:

Example 4. Let A =(-0.5,- 0.3, -0.3,- 0.1;1) and B =(0.1,0.3,0.3,0.5;1) be two generalized trapezoidal fuzzy number, then

$$\Re(A)=\frac{1.0}{2}\left[(-0.3-0.1)-(-0.5-0.3)\right]=0.2$$

$$\Re(B)=\frac{1.0}{2}\left[(0.3+0.5)-(0.1+.03)\right]=0.2$$

since $\Re(A)$ = $\Re(B)$ = 0.2, then go to step 2.

Step 2: Mode (A) = $\frac{1.0}{2}[(-0.3)+(-0.3)]=-0.3$

Mode (B) = $\frac{1.0}{2}[0.3+0.3]=0.3$ $\quad Mode(A)<Mode(B)\Rightarrow A\prec B$

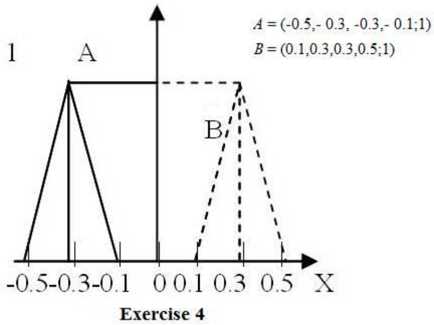

Figure 8:

Example 5. Let A =(0.3,0.5,0.5,1;1) and B =(0.1,0.6,0.6,0.8;1) be two generalized trapezoidal fuzzy number, then

$\Re(A)=\frac{1.0}{2}[(1.0+0.5)-(0.3+0.5)]=0.35$

$\Re(B)=\frac{1.0}{2}[(0.6+0.8)-(0.1+0.6)]=00.35$

since $\Re(A)$ = $\Re(B)$= 0.35, then go to step 2.

Step 2: Mode (A) = $\frac{1.0}{2}[0.5+0.5]=0.5$ Mode (B) = $\frac{1.0}{2}[0.6+0.6]=0.6$

$Mode(A)<Mode(B)\Rightarrow A\prec B$

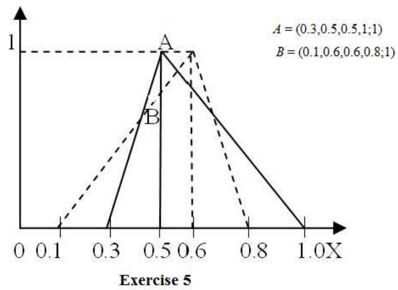

Figure 9:

Example 6. Let A =(0,0.4,0.6,0.8;1) and B =(0.2,0.5,0.5,0.9;1) and C = (0.1,0.6,0.7,0.8;1) be two generalized trapezoidal fuzzy number, then

$\Re(A)=\frac{1.0}{2}[(0.8+0.6)-(0.0+0.4)]=0.5$ $\Re(B)=\frac{1.0}{2}[(0.9+0.5)-(0.2+0.5)]=0.35$

$\Re(C)=\frac{1.0}{2}[(0.8+0.7)-(0.1+0.6)]=0.4$ $\quad \Re(A)>\Re(C)>\Re(B)\Rightarrow A>C>B$

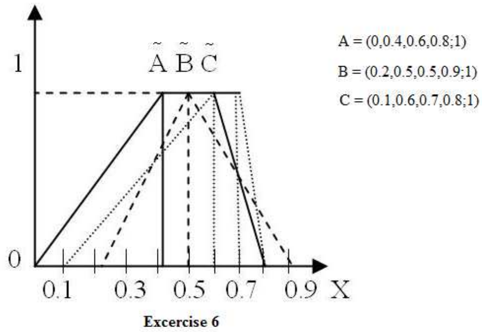

Figure 10:

Example 7. Let *A* =(0.1,0.2,0.4,0.5;1) and *B* =(-2,0,0, 2;1) be two generalized trapezoidal fuzzy number, then

$\Re(A)=\frac{1.0}{2}[(0.4+0.5)-(0.1+0.2)]=0.3$ $\Re(A)=\frac{1.0}{2}[(0+2.0)-(-2.0+0)]=2.0$

so $\Re(A)\prec\Re(B)\Rightarrow A<B$

Now consider the example that A = (0.1,0.2,0.4,0.5;1.0) and
B = (0.2,0.3,0.3,0.8;1.0) be two generalized trapezoidal fuzzy number, then

$$\Re(A)=\frac{1.0}{2}[(0.4+0.5)-(0.1+0.2)]=0.3$$

$$\Re(A)=\frac{1.0}{2}[(0.3+0.8)-(0.2+0.3)]=0.3$$

since $\Re(A)$ = $\Re(B)$ = 0.35, then go to step 2.

Step 2: Mode (*A*) = $\frac{1.0}{2}[0.2+0.4]=0.3$ Mode (*B*) = $\frac{1.0}{2}[0.3+0.3]=0.3$

Since Mode (*A*) = Mode (*B*) = 0.3, then go to step 3.

Step 3: Spread (*A*) = 1.0 (0.5 – 0.1) = 0.4 Spread (*B*) = 1.0 (0.8 – 0.2) = 0.6

Since Spread (*A*) < Spread (*B*)

$\Rightarrow A\succ B$

The following Table 1 indicates the comparison of this proposed method with various ranking methods.

Approach	**Ex 1**	**Ex 2**	**Ex 3**	**Ex 4**	**Ex 5**	**Ex 6**	**Ex 7**
Cheng [9]	A $\prec$ B	A $\approx$ B	Error	A $\approx$ B	A $\succ$ B	A $\prec$ B $\prec$ C	Error
Chu [11]	A $\prec$ B	A $\prec$ B	A $\prec$ B	A $\prec$ B	A $\succ$ B	A $\prec$ B $\prec$ C	Error
Chen [7]	A $\prec$ B	A $\prec$ B	A $\prec$ B	A $\prec$ B	A $\succ$ B	A $\prec$ C $\prec$ B	A $\succ$ B
Abbasbandy [1]	Error	A $\approx$ B	A $\prec$ B	A $\approx$ B	A $\prec$ B	A $\prec$ B $\prec$ C	A $\succ$ B
Chen [8]	A $\prec$ B	A $\prec$ B	A $\prec$ B	A $\prec$ B	A $\succ$ B	A $\prec$ B $\prec$	A $\succ$ B

						C	
Kumar [15]	$A \succ B$	$A \approx B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \prec B \prec C$	$A \succ B$
Singh [19]	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \prec B \prec C$	$A \succ B$
Rezvani [18]	$A \prec B$	$A \succ B$	$A \succ B$	$A \approx B$	$A \approx B$	$A \succ C \succ B$	$A \prec B$
Proposed method	$A \prec B$	$A \succ B$	$A \succ B$	$A \prec B$	$A \prec B$	$A \succ C \succ B$	$A \prec B$

Table 1. A comparison of the ranking results for different approaches

5. Conclusion

It is clear from Table 1 that the results of the proposed approach are same as obtained by using the existing approach (Chen and Chen, 2009). The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

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