

Modified New Operations for Symmetric Trapezoidal Intuitionistic Fuzzy Numbers: An Application of Diet Problem

R.Irene Hepzibah¹ and R.Vidhya²

¹Department of Mathematics, AVC College of Engineering, Mayiladuthurai
Tamil Nadu, India; E-mail: ireneraj74@gmail.com

²Department of Mathematics, AS-SALAM College of Engineering and Technology
Aduthurai, Tamil Nadu, India; E-mail: vidhya14m@gmail.com

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Abstract. Standard intuitionistic fuzzy arithmetic operations have certain problems in division and subtraction operations. Ezzati et al. (2014) defined a new operation on symmetric trapezoidal intuitionistic fuzzy numbers in which the properties $\frac{\tilde{A}}{\tilde{A}} \approx \tilde{1}$ and $\tilde{A} - \tilde{A} \approx \tilde{0}$ are satisfied. In this paper, the new operations according to Ezzati et al. (2014) are extended to symmetric trapezoidal intuitionistic fuzzy numbers and an intuitionistic fuzzy version of simplex algorithm for solving multi-objective linear programming problem (MOLPP) whose parameters are represented by symmetric trapezoidal intuitionistic fuzzy numbers is proposed. An illustrative numerical example is presented on diet problem to validate the effectiveness of the modified operations and the intuitionistic fuzzy outputs obtained by MATLAB software are also provided.

Keywords: Symmetric trapezoidal intuitionistic fuzzy number, modified arithmetic operations, ranking function, intuitionistic fuzzy multi-objective linear programming problem

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1. Introduction

The notion of fuzzy sets was introduced by Zadeh [9] and it was generalised to intuitionistic fuzzy sets by Atanassov [1,2]. The Intuitionistic fuzzy set (IFS) has received more and more attention since its appearance, because the information about attribute values is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking. In real life, due to the inevitable measurement inaccuracy, the exact values of the measured quantities are not known and so the parameters of the problem are usually defined by the decision maker in an uncertain way.

Therefore, it is desirable to consider the knowledge of experts about the parameters as intuitionistic fuzzy data. These intuitionistic fuzzy parameters are characterized by different intuitionistic fuzzy numbers as triangular, trapezoidal, etc. This paper is focused on the symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS). When we

consider the symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS), the arithmetic operations defined on them are of great influence. Bhardwaj et al [4] showed that according to fuzzy arithmetic operation used by Khan et al [7] for the fuzzy number \tilde{A} , the properties $\frac{\tilde{A}}{\tilde{A}} \approx \tilde{1}$ and $\tilde{A} - \tilde{A} \approx \tilde{0}$ could not be satisfied. Recently, Ezzati et al [5] defined a new operation on symmetric trapezoidal fuzzy numbers that the properties $\frac{\tilde{A}}{\tilde{A}} \approx \tilde{1}$ and $\tilde{A} - \tilde{A} \approx \tilde{0}$ are satisfied. In this paper, the same operations are extended to symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS) and the properties $\frac{\tilde{A}^I}{\tilde{A}^I} \approx \tilde{1}^I$ and $\tilde{A}^I - \tilde{A}^I \approx \tilde{0}^I$ are also satisfied. Linear programming (LP) is one of the most applicable optimization techniques. Most of the real world Problems is intrinsically characterized by multiple, conflicting and incommensurate aspects of evaluation. These axes of valuation are generally addressed by objective functions to be optimized in framework of multiple objective linear programming models. Fuzzy decision making was introduced by Bellman and Zadeh [3]. Furthermore, when captivating real world problems, frequently the parameters are imprecise numerical quantities and therefore the intuitionistic fuzzy attributes are very adequate for modeling these situations. Therefore, an intuitionistic fuzzy version of simplex algorithm is proposed for solving multi-objective problem whose parameters all are represented by symmetric trapezoidal intuitionistic fuzzy number without converting the given them into crisp linear programming problems by using the extended new arithmetic operations.

The paper is organized as follows: Section 2 introduces the preliminaries of symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS), the modified arithmetical operations for STIFNS, and the ranking function. Section 3 deals with the formulation of the intuitionistic fuzzy multi objective linear programming problem (IFMOLPP) and the associated definitions. In section 4, an intuitionistic fuzzy version of simplex algorithm is provided to solve the IFMOLPP. An application of these new operations with MATLAB outputs are provided by means of an example in section 5 and some concluding remarks are given in section 6.

2. Preliminaries

This section introduces the preliminary notations of the area of intuitionistic fuzzy set theory.

Definition 2.1. According to [6], a symmetric trapezoidal intuitionistic fuzzy number (STIFN) \tilde{A}^I is an intuitionistic fuzzy set in \mathbb{R} with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$.

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & , x < a_1 - h \\ \frac{x - (a_1 - h)}{h} & , a_1 - h \leq x \leq a_1 \\ 1 & , a_1 \leq x \leq a_2 \\ \frac{(a_2 + h) - x}{h} & , a_2 \leq x \leq a_2 + h \\ 0 & , a_2 + h \leq x \end{cases} \quad \text{and}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} 1 & , x < a_1 - h' \\ \frac{x - (a_1 - h')}{h'} & , a_1 - h' \leq x \leq a_1 \\ 0 & , a_1 \leq x \leq a_2 \\ \frac{(a_2 + h') - x}{h'} & , a_2 \leq x \leq a_2 + h' \\ 1 & , a_2 + h' \leq x \end{cases}$$

where $a_1 \leq a_2$ and $h, h' \geq 0$. This STIFN is denoted by $\tilde{A}^I = \{(a_1, a_2, h, h'); (a_1, a_2, h', h')\}$. For our convenience, STIFN is denoted by $\tilde{A}^I = (a_1, a_2, h, h')$ throughout this paper.

Definition 2.2. [1] A (α, β) - cut set of a intuitionistic fuzzy number is defined as $A_{\alpha, \beta}^I = \{x / \mu_{\tilde{A}^I}(x) \geq \alpha, \nu_{\tilde{A}^I}(x) \leq \beta\}$, where $0 \leq \alpha \leq 1; 0 \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$.

2.3. Modified arithmetic operations on symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS)

Ezzati et al. [5] has developed the modified arithmetic operations for symmetric trapezoidal fuzzy numbers. These operations are extended here for symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS).

Let $\tilde{A}^I = (a_1, a_2, h, h')$ and $\tilde{B}^I = (b_1, b_2, k, k')$ be two symmetric trapezoidal intuitionistic fuzzy numbers. Then

- (i) **Addition:** $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, h + k, h' + k')$
- (ii) **Subtraction:** $\tilde{A}^I - \tilde{B}^I = (a_1 - b_2, a_2 - b_1, h + k, h' + k')$
- (iii) **Multiplication:** $\tilde{A}^I \times \tilde{B}^I = (\frac{a_1 + a_2}{2} \times \frac{b_1 + b_2}{2} - w, \frac{a_1 + a_2}{2} \times \frac{b_1 + b_2}{2} - w, |w - w'|, |w - w'_1|)$

where $w = \min([\frac{a_1 + a_2}{2}][\frac{b_1 + b_2}{2}] - \min(m), \max(m) - [\frac{a_1 + a_2}{2}][\frac{b_1 + b_2}{2}])$,

$$w' = \min([\frac{a_1 + a_2}{2}][\frac{b_1 + b_2}{2}] - \min(n), \max(n) - [\frac{a_1 + a_2}{2}][\frac{b_1 + b_2}{2}])$$

$w'_1 = \min([\frac{a_1 + a_2}{2}][\frac{b_1 + b_2}{2}] - \min(n'), \max(n') - [\frac{a_1 + a_2}{2}][\frac{b_1 + b_2}{2}])$,

$$m = (a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2), n = ((a_1 - h)(b_1 - k), (a_1 - h)(b_2 + k),$$

$(a_2 + h)(b_1 - k), (a_2 + h)(b_2 + k)), n' = ((a_1 - h')(b_1 - k'),$

$(a_1 - h')(b_2 + k'), (a_2 + h')(b_1 - k'), (a_2 + h')(b_2 + k'))$

- (iv) **Division:** $\tilde{A}^I \div \tilde{B}^I = (\frac{a_1 + a_2}{b_1 + b_2} - w, \frac{a_1 + a_2}{b_1 + b_2} + w, |w - w'|, |w - w'_1|)$

where $w = \min\{[\frac{a_1 + a_2}{b_1 + b_2}] - \min(m), \max(m) - [\frac{a_1 + a_2}{b_1 + b_2}]\}$,

$w' = \min\{[\frac{a_1 + a_2}{b_1 + b_2}] - \min(n), \max(n) - [\frac{a_1 + a_2}{b_1 + b_2}]\}$,

$w'_1 = \min\{[\frac{a_1 + a_2}{b_1 + b_2}] - \min(n'), \max(n') - [\frac{a_1 + a_2}{b_1 + b_2}]\}$, $m = (\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2})$

$n = \{[\frac{a_1 - h}{b_1 - k}], [\frac{a_1 - h}{b_2 + k}], [\frac{a_2 + h}{b_1 - k}], [\frac{a_2 + h}{b_2 + k}]\}$, $n' = \{[\frac{a_1 - h'}{b_1 - k'}], [\frac{a_1 - h'}{b_2 + k'}], [\frac{a_2 + h'}{b_1 - k'}], [\frac{a_2 + h'}{b_2 + k'}]\}$

Definition 2.4. For any symmetric trapezoidal intuitionistic fuzzy number \tilde{A}^I , we define $\tilde{A}^I \succcurlyeq \tilde{0}^I$ if there exist $c, \delta, \delta' \geq 0$ such that $\tilde{A}^I \succcurlyeq (-c, c, \delta, \delta')$. We also denote $(-c, c, \delta, \delta')$ by $\tilde{0}^I$.

Definition 2.5. For any symmetric trapezoidal intuitionistic fuzzy number \tilde{A}^I , we define $\tilde{A}^I \succcurlyeq \tilde{1}^I$ if there exist $c, \delta, \delta' \geq 0$ such that $\tilde{A}^I \succcurlyeq (-c+1, c+1, \delta, \delta')$. We also denote $(-c+1, c+1, \delta, \delta')$ by $\tilde{1}^I$.

Definition 2.6. Let $\mathcal{F}(s)$ be the set of all symmetric trapezoidal intuitionistic fuzzy numbers. For $\tilde{A}^I = \{(a_1, a_2, h, h')\} \in \mathfrak{R}(s)$, we define a ranking function $F: \mathcal{F}(s) \rightarrow \mathbb{R}$ by $F(\tilde{A}^I) = \frac{(a_1+a_2)}{2} + (h'-h)$.

Definition 2.7. Let $\tilde{A}^I = (a_1, a_2, h, h')$ and $\tilde{B}^I = (b_1, b_2, h_1, h_1')$ be two symmetric trapezoidal intuitionistic fuzzy numbers.

Then (i) $\tilde{A}^I < \tilde{B}^I$ iff $F(\tilde{A}^I) < F(\tilde{B}^I)$.

(ii) $\tilde{A}^I > \tilde{B}^I$ iff $F(\tilde{A}^I) > F(\tilde{B}^I)$.

(iii) $\tilde{A}^I = \tilde{B}^I$ iff $F(\tilde{A}^I) = F(\tilde{B}^I)$.

3. Formulation of the problem

In this section, the description of the intuitionistic fuzzy multi-objective linear programming problem and the related definitions are discussed.

3.1. Formulation of the intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP)

The general form of multi-objective optimization problem with 'k' intuitionistic fuzzy objective functions $\tilde{Z}_1^I, \tilde{Z}_2^I, \dots, \tilde{Z}_k^I$ and 'm' intuitionistic fuzzy constraints is given by

Maximize or Minimize: $\tilde{Z}_p^I = \sum_{j=1}^n \tilde{c}_j^I \tilde{x}_j^I$

Subject to

$\sum_{j=1}^n \tilde{a}_{ij}^I \tilde{x}_j^I \{ \leq, \geq, \approx \} \tilde{b}_i^I$ (or) $\tilde{A}^I \tilde{x}^I \{ \leq, \geq, \approx \} \tilde{b}^I$ (a)

$\tilde{x}_j^I \succcurlyeq \tilde{0}^I$ (or) $\tilde{x}^I \succcurlyeq \tilde{0}^I$.

where $p = 1, 2, \dots, k$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$ where $\tilde{A}^I = (\tilde{a}_{ij}^I)$, \tilde{C}_j^I , $\tilde{b}^I = \tilde{b}_i^I$, are symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS) and \tilde{x}_j^I whose states are also given by symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS).

Definition 3.2. Let the i-th intuitionistic fuzzy constraint of the intuitionistic fuzzy linear programming problem be $\sum_{j=1}^n \tilde{a}_{ij}^I \tilde{x}_j^I \{ \leq, \geq, \approx \} \tilde{b}_i^I$, $\tilde{x}_j^I \succcurlyeq \tilde{0}^I$ where $\tilde{b}_i^I \succcurlyeq \tilde{0}^I$. Then a intuitionistic fuzzy variable $\tilde{s}_i^I \succcurlyeq \tilde{0}^I$ and $\sum_{j=1}^n \tilde{a}_{ij}^I \tilde{x}_j^I + \tilde{s}_i^I$ (or $-\tilde{s}_i^I$) $\approx \tilde{b}_i^I$ is called a intuitionistic fuzzy slack (or surplus) variable.

Definition 3.3. According to Ganesan et al. [6], given a system of m intuitionistic fuzzy linear equations involving symmetric trapezoidal intuitionistic fuzzy numbers in n unknowns ($n \geq m$), $\tilde{A}^I \tilde{x}^I \approx \tilde{b}^I$ is a $m \times n$ matrix and rank of \tilde{A}^I is m. Let the columns of \tilde{A}^I corresponding of fuzzy variables $\tilde{x}_{k_1}^I, \tilde{x}_{k_2}^I, \dots, \tilde{x}_{k_m}^I$ are linearly independent then $\tilde{x}_{k_1}^I, \tilde{x}_{k_2}^I, \dots, \tilde{x}_{k_m}^I$ are said to be intuitionistic fuzzy basic variables and remaining (n-m) variables are called intuitionistic fuzzy non basic variables. Let \tilde{B}^I the basis matrix

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formed by linearly independent columns of \tilde{A}^I . The value of $\tilde{X}_B^I = (\tilde{x}_{k_1}^I, \tilde{x}_{k_2}^I, \dots, \tilde{x}_{k_m}^I)$ is obtained by using $\tilde{X}_B^I = \tilde{B}^{-1} \tilde{b}^I$, where $k_j \in \{1, 2, \dots, n\}$, $k_i \neq k_j$, $i, j = 1, 2, \dots, m$ and the values of non basic variables are assumed to be zero. The combined solution formed by using the values of intuitionistic fuzzy basic variables and intuitionistic fuzzy non basic variables are called intuitionistic fuzzy basic solution.

Definition 3.4. Any \tilde{X}^I which satisfies all the constraints $\tilde{A}^I \tilde{x}^I \approx \tilde{b}^I$ and nonnegative restricted of $\tilde{x}^I \geq \tilde{0}^I$ of (a) is said to be an intuitionistic fuzzy feasible solution of (a).

Definition 3.5. Let Q be the set of all intuitionistic fuzzy feasible solution of (a). An intuitionistic fuzzy feasible solution $\tilde{X}_0^I \in Q$ said to be an intuitionistic fuzzy optimal solution of (a) if $\tilde{c}^I \tilde{X}_0^I \geq \tilde{c}^I \tilde{X}^I$.

Definition 3.6. Preemptive optimization [8] which performs multi-objective optimization by considering objectives one at a time. The most important one is optimized, then the second most important one is optimized subject to a requirement that the first achieve its optimal value in the preemptive optimization and so on. If each stage of preemptive optimization yields a single objective optimum, the final solution is an efficient point of the full multi-objective model.

4. Proposed method

In this section, the intuitionistic fuzzy version of the simplex algorithm is proposed.

Step 1: Consider the IFMOLPP Maximize or Minimize $[\tilde{c}_1^I \tilde{x}^I, \tilde{c}_2^I \tilde{x}^I, \dots, \tilde{c}_n^I \tilde{x}^I]$

Subject to the constraints $\sum_{j=1}^n \tilde{a}_{ij}^I \tilde{x}_j^I \{ \leq, \geq, \approx \} \tilde{b}_i^I, \tilde{x}_j^I \geq \tilde{0}^I$.

First select the single most important objective function which is to be maximized or minimized with respect to the given constraints by preemptive optimization method.

Step 2: Use simplex table, compute the net evaluation $\tilde{z}_j^I - \tilde{c}_j^I$ ($j = 1, 2, 3, \dots, n$) by using the relation $\tilde{z}_j^I - \tilde{c}_j^I \approx \tilde{c}_B^I \tilde{a}_j^I - \tilde{c}_j^I$. Examine the sign of $\tilde{z}_j^I - \tilde{c}_j^I$.

(a) If all $\tilde{z}_j^I - \tilde{c}_j^I \geq \tilde{0}^I$ then the current basic feasible solution \tilde{X}_B^I is optimal.

(b) If at least one $\tilde{z}_j^I - \tilde{c}_j^I < \tilde{0}^I$ then the current basic feasible solution is not optimal, go to the next step.

Step 3: The entering variable is the non-basic variable corresponding to the most negative value of $\tilde{z}_j^I - \tilde{c}_j^I$. Let it be \tilde{x}_r^I for some $j = r$. The entering variable column is known as the key column (or) pivot column.

Step 4: Compute the ratio $R \approx \text{Min} \left\{ \frac{\tilde{x}_{B_i}^I}{\tilde{a}_{ir}^I}, \tilde{a}_{ir}^I > \tilde{0}^I \right\}$. If $R \approx \frac{\tilde{x}_{B_k}^I}{\tilde{a}_{kr}^I}$, then the basic variable leaves the basis. The leaving variable row is called the key row or pivot row and the element at the intersection of the pivot column and pivot row is called the pivot element.

Step 5: Drop the leaving variable and introduce the entering variable along with associated value under \tilde{c}_B^I column. Convert the pivot element to unity ($\tilde{1}^I$) by dividing the pivot equation by the pivot element and all other elements in its column to zero ($\tilde{0}^I$).

Step 6: Go to step:2 and repeat the procedure until either an optimum solution is obtained.

Step 7: Consider the second most important objective function as per preemptive optimization. Then optimize it with respect to the given set of constraints together with the new constraint until all the objective functions of the given IFMOLPP have been considered. The final solution obtained is the overall solution of the given IFMOLPP.

5. Numerical example

For an illustration of the proposed method, consider the following problem:
 (Diet problem) A person wants to decide the constituents of a diet which will fulfill his daily requirements of vitamins A and B at the minimum cost. The choice is to be made from two different types of food. The yields per unit of these foods are given in the following table:

Food type	Yield/Unit		Cost/Unit	Procurement Cost/Unit
	Vitamin-A	Vitamin-B		
1	5	2	3	2
2	1	2	1	3
Minimum Requirements	5	6		

Find the minimum cost of the food consumption and the minimum cost of procuring the food items?

Let x_1 and x_2 be the units of foods type 1 and 2 used respectively.

The above problem can be written as

Minimize $z_1 = 3x_1 + x_2$

Minimize $z_2 = 2x_1 + 3x_2$

Subject to the constraints

$5x_1 + 2x_2 \geq 5, x_1 + 2x_2 \geq 6, x_1, x_2 \geq 0$

Since the cost of the food consumption and the cost of procurement are uncertain, the number of units to be consumed on each product will also be uncertain. So, the above problem is formulated as a bi-level multi-objective intuitionistic fuzzy linear programming problem with symmetric trapezoidal intuitionistic fuzzy numbers for each indecisive value. Similarly the other parameters are also modelled as symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS) by taking into account the nature of the problem and other requirements. So, the given bi-level multi-objective intuitionistic fuzzy linear programming problem is formulated as

Minimize $\tilde{Z}_1^I = (2, 4, 2, 2)\tilde{x}_1^I + (1, 1, 0, 0)\tilde{x}_2^I$

Minimize $\tilde{Z}_2^I = (1, 3, 2, 2)\tilde{x}_1^I + (2, 4, 2, 2)\tilde{x}_2^I$

Subject to the constraints $(4, 6, 2, 2)\tilde{x}_1^I + (1, 3, 2, 2)\tilde{x}_2^I \geq (4, 6, 2, 2)$
 $(1, 1, 0, 0)\tilde{x}_1^I + (1, 3, 2, 2)\tilde{x}_2^I \geq (5, 7, 2, 2), \tilde{x}_1^I, \tilde{x}_2^I \geq 0.$

Suppose we decide that the first objective is the single most important one by 2.7. Preemptive optimization is used to begin by minimizing the single objective LPP.

Minimize $\tilde{Z}_1^I = (2, 4, 2, 2)\tilde{x}_1^I + (1, 1, 0, 0)\tilde{x}_2^I$

subject to the constraints $(4, 6, 2, 2)\tilde{x}_1^I + (1, 3, 2, 2)\tilde{x}_2^I \geq (4, 6, 2, 2)$

$(1, 1, 0, 0)\tilde{x}_1^I + (1, 3, 2, 2)\tilde{x}_2^I \geq (5, 7, 2, 2), \tilde{x}_1^I, \tilde{x}_2^I \geq 0.$

Using duality principle, we have the dual problem as

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Maximize $\widetilde{Z}_1^{I*} = (4,6,2,2)\widetilde{y}_1^I + (5,7,2,2)\widetilde{y}_2^I$
 subject to the constraints $(4,6,2,2)\widetilde{y}_1^I + (1,1,0,0)\widetilde{y}_2^I \leq (2,4,2,2)$
 $(1,3,2,2)\widetilde{y}_1^I + (1,3,2,2)\widetilde{y}_2^I \leq (5,7,2,2)$ and $\widetilde{y}_1^I, \widetilde{y}_2^I \geq 0$

Enter into simplex table and from Table: 1, We observe that all $\widetilde{z}_j^I - \widetilde{c}_j^I \geq 0$, then the current solution is optimal.i.e., $\widetilde{y}_1^I = (0,0,0,0)$, $\widetilde{y}_2^I = (0.33,0.67,0.33,0.33)$ with maximize $\widetilde{Z}_1^{I*} = (1.65,4.35,1.65,1.65)$.

By Duality theory, we get the solution to the primal problem as $\widetilde{x}_1^I = (0,0,0,0)$,
 $\widetilde{x}_2^I = (1.65, 4.35, 1.65, 1.65)$ with minimize $\widetilde{Z}^I = (1.65,4.35,1.65,1.65)$.

Next, we impose an extra constraint

$(2,4,2,2)\widetilde{x}_1^I + (1,1,0,0)\widetilde{x}_2^I \leq (1.65,4.35,1.65,1.65)$ and minimize the second objective,

Minimize $\widetilde{Z}_2^I = (1,3,2,2)\widetilde{x}_1^I + (2,4,2,2)\widetilde{x}_2^I$

subject to the constraints $(2,4,2,2)\widetilde{x}_1^I + (1,1,0,0)\widetilde{x}_2^I \leq (1.65,4.35,1.65,1.65)$

$(4,6,2,2)\widetilde{x}_1^I + (1,3,2,2)\widetilde{x}_2^I \geq (4,6,2,2)$

$(1,1,0,0)\widetilde{x}_1^I + (1,3,2,2)\widetilde{x}_2^I \geq (5,7,2,2), \widetilde{x}_1^I, \widetilde{x}_2^I \geq 0$.

Using duality principle, we have the dual problem as

$(-1,-1,0,0)\widetilde{y}_1^I + (1,3,2,2)\widetilde{y}_2^I + (1,3,2,2)\widetilde{y}_3^I \leq (2,4,2,2)$ & $\widetilde{y}_1^I, \widetilde{y}_2^I, \widetilde{y}_3^I \geq 0$.

\widetilde{c}_B^I	Basis	\widetilde{y}_1^I	\widetilde{y}_2^I	\widetilde{s}_1^I	\widetilde{s}_2^I	\widetilde{b}_i^I	Ratio
(0,0,0,0)	\widetilde{s}_1^I	(4,6,2,2)	(1,1,0,0)	(1,1,0,0)	(0,0,0,0)	(2,4,2,2)	(2,4,2,2)
(0,0,0,0)	\widetilde{s}_2^I	(1,3,2,2)	(1,3,2,2)	(0,0,0,0)	(1,1,0,0)	(1,1,0,0)	
	\widetilde{z}_j^I	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)		
	$\widetilde{z}_j^I - \widetilde{c}_j^I$	(-6,-4,2,2)	(-7,-5,2,2)	(0,0,0,0)	(0,0,0,0)		
\widetilde{c}_B^I	Basis	\widetilde{y}_1^I	\widetilde{y}_2^I	\widetilde{s}_1^I	\widetilde{s}_2^I	\widetilde{b}_i^I	Ratio
(0,0,0,0)	\widetilde{s}_1^I	(2.33,5.67, 4,4)	(-0.67,0.67, 0.67,0.67)	(1,1,0,0)	(-0.67,-0.33, 0.33,0.33)	(1.33,4.67, 4,4)	*
(5,7,2,2)	\widetilde{y}_2^I	(0.33,1.67, 0.67,0.67)	(0.33,1.67, 0.67,0.67)	(0,0,0,0)	(0.33,0.67, 0.33,0.33)	(0.33,0.67, 0.33,0.33)	*
	\widetilde{z}_j^I	(1.65,10.35, 4.71,4.71)	(1.65,10.35, 4.71,4.71)	(0,0,0,0)	(1.65,4.35, 1.65,1.65)		
	$\widetilde{z}_j^I - \widetilde{c}_j^I$	(-4.35,6.35, 6.71,6.71)	(-5.35,5.35, 6.71,6.71)	(0,0,0,0)	(1.65,4.35, 1.65,1.65)		

Table 1: The MATLAB outputs of the optimal solutions of \widetilde{Z}_1^I (Figure.1) and \widetilde{Z}_2^I (figure.2) .

	\tilde{c}_j^I	(-4.35,-1.65,1.65,1.65)	(4,6,2,2)	(5,7,2,2)	(0,0,0,0)	(0,0,0,0)		
\tilde{c}_B^I	Basis	\tilde{y}_1^I	\tilde{y}_2^I	\tilde{y}_3^I	\tilde{s}_1^I	\tilde{s}_2^I	\tilde{b}_i^I	Ratio
(0,0,0,0)	\tilde{s}_1^I	(-4,-2,2,2)	(4,6,2,2)	(1,1,0,0)	(1,1,0,0)	(0,0,0,0)	(1,3,2,2)	(2,4,2,2)
(0,0,0,0)	\tilde{s}_2^I	(-1,-1,0,0)	(1,3,2,2)	(1,3,2,2)	(0,0,0,0)	(1,1,0,0)	(2,4,2,2)	(0.67,2.33,1.13,1.13)
	\tilde{z}_j^I	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)		
	$\tilde{z}_j^I - \tilde{c}_j^I$	(1.65,4.35,1.65,1.65)	(-6,-4,2,2)	(-7,-5,2,2)	(0,0,0,0)	(0,0,0,0)		
\tilde{c}_B^I	Basis	\tilde{y}_1^I	\tilde{y}_2^I	\tilde{y}_3^I	\tilde{s}_1^I	\tilde{s}_2^I	\tilde{b}_i^I	Ratio
(0,0,0,0)	\tilde{s}_1^I	(-3.67,-1.33,2.47,2.47)	(2.33,5.67,4,4)	(-0.67,0.67,0.67,0.67)	(1,1,0,0)	(-0.67,-0.33,0.33,0.33)	(-1.33,2.33,3.13,3.13)	*
(5,7,2,2)	\tilde{y}_2^I	(-0.67,-0.33,0.47,0.47)	(0.33,1.67,0.67,0.67)	(0.33,1.67,0.67,0.67)	(0,0,0,0)	(0.33,0.67,0.33,0.33)	(0.67,2.33,1.13,1.13)	*
	\tilde{z}_j^I	(-4.35,-1.65,2.91,2.91)	(1.65,10.35,4.71,4.71)	(1.65,10.35,4.71,4.71)	(0,0,0,0)	(1.65,4.35,1.65,1.65)		
	$\tilde{z}_j^I - \tilde{c}_j^I$	(-2.7,2.7,4.56,4.56)	(-4.35,6.35,6.71,6.71)	(-5.35,5.35,6.71,6.71)	(0,0,0,0)	(1.65,4.35,1.65,1.65)		
	\tilde{c}_j^I	(4,6,2,2)	(5,7,2,2)	(0,0,0,0)	(0,0,0,0)			

Table 2:

Maximize $\tilde{Z}_2^{I*} = (-4.35,-1.65,1.65,1.65) \tilde{y}_1^I + (4,6,2,2) \tilde{y}_2^I + (5,7,2,2) \tilde{y}_3^I$
 subject to the constraints $(-4,-2,2,2) \tilde{y}_1^I + (4,6,2,2) \tilde{y}_2^I + (1,1,0,0) \tilde{y}_3^I \leq (1,3,2,2)$

Enter into simplex table and from Table:2, We observe that all $\tilde{z}_j^I - \tilde{c}_j^I \geq 0$, then the current solution is optimal. i.e., $\tilde{y}_1^I = (0,0,0,0)$ $\tilde{y}_2^I = (0, 0,0,0)$, $\tilde{y}_3^I = (0.67,2.34,1.13,1.13)$ with maximize $\tilde{Z}_2^{I*} = (3.35,14.65,7.49,7.49)$. Therefore the solution of primal problem is $\tilde{x}_1^I = (0, 0,0,0)$, $\tilde{x}_2^I = (1.65,4.35,1.65,1.65)$ with minimize $\tilde{Z}_2^I = (3.35,14.65,7.49,7.49)$.

If each stage of preemptive optimization yields a single objective optimum, the final solution is an efficient point of the full multi-objective model.

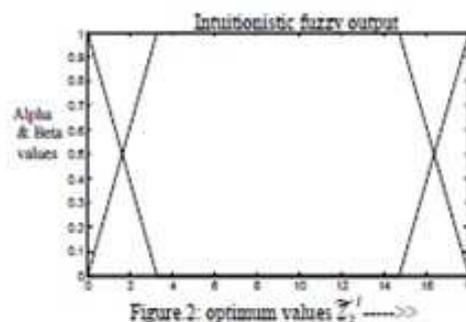
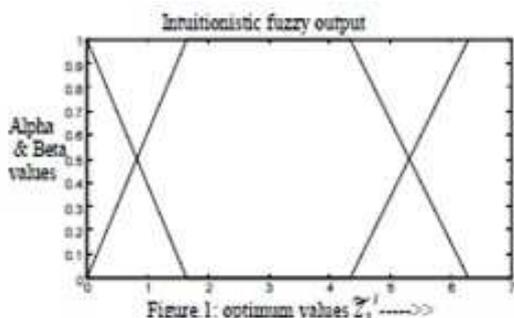
Finally, the food type 1 is taken by the person to obtain the minimum requirements of vitamins A and B and by spending minimum procurement cost.

6. Conclusion

This paper reveals how the modified operations can be efficiently used for solving bi-level multi-objective linear programming problem with symmetric trapezoidal intuitionistic fuzzy numbers on a diet problem. An intuitionistic fuzzy version of simplex

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algorithm for solving IFMOLPP is also discussed in this paper. Finally, the MATLAB outputs of using the modified arithmetic operations on STIFNS are given. Based on the MATLAB outputs, the modified arithmetic operations on STIFNS are effectively verified.



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