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A Multi-Criteria Decision Making Approach Based on Complement of Interval-Valued Fuzzy Soft Sets

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Abstract. Soft Set theory is proposed to deal with uncertainties embedded in decision making. In our daily life we often face some problems in which the correct decision making is essential. In this paper, to overcome this problem, the concept based on complement of interval-valued fuzzy soft sets have been discussed. Finally the algorithm based on complement of Interval-Valued fuzzy soft sets is proposed with an example to illustrate the new approach.

Keywords: Soft set, fuzzy soft set, interval-valued fuzzy soft set, complement of interval-valued fuzzy soft set, decision making problem

AMS Mathematics Subject Classification (2010): 15B15

1. Introduction

Multi-Criteria Decision Making (MCDM) is the study of method and procedure by which we concern about multiple conflicting criteria can be formally incorporated into the management planning process. Many scholars study the properties and applications on the soft set theory. Xiao et al. [8] studied synthetically evaluating method for business competitive capacity and also Xiao et al. [7] gave a recognition for soft information based on the theory of soft sets. Pie and Miao [5] showed that the soft sets are a class of special information systems. Mushrif et al. [4] presented a new algorithm based on the notions of soft set theory for classification of the natural textures. Kovkov et al. [2] considered the optimization problems in the framework of the theory of soft sets which is directed to formalize the concept of approximate object description. Zou and Xiao [9] presented data analysis approaches of soft sets under incomplete information. Majumdar and Samanta studied the similarity measure of soft sets. Ali et al. [1] introduced the analysis of several operations on soft sets. Mitra Basu et al. [3] presented the concept of Matrices in Interval-valued fuzzy soft set theory and its application. Dinagar and Rajesh [6] presented an application of interval-valued fuzzy soft sets on decision making problems. Also in this work the concept of a multi-criteria decision making approach based on complement of interval-valued fuzzy soft sets have been studied. In this paper the sections are organized as: In section 2,we considered some formal definitions and important notations that are very useful to develop the concept of this article. In section 3, we presented Algorithm based on Complement of interval-valued fuzzy soft sets. In section 4, Application of a decision making problem is discussed. In section 5, we conclude the paper with a summary and outlook for further research.

2. Preliminaries

In this section, we recall the basic definition of Soft Set, Fuzzy Soft Set, Interval-Valued Fuzzy Soft Set, Complement of Interval-Valued Fuzzy Soft Set,Complement of Interval-Valued Fuzzy Soft Matrix and Associated Real number of an interval number with example.

2.1. Soft set

Definition 2.1.[6] Let U be a universal set, E a set of parameters and $A \subset E$. Then a pair (F, A) is called soft set over U, Where F is a mapping from A to 2^U the power set of U **Example 2.1.**

Let U = $\{c_1, c_2, c_3\}$ be the set of three cars and E = $\{\cos tly(e_1), metallic$ $colour(e_2), cheap(e_3)\}$ be the set of parameters, where $A = \{e_1, e_2\} \subset E$. Then (F, A) ={ $F(e_1) = \{c_1, c_2, c_3\}$, $F(e_2) = \{c_1, c_3\}$}is the crisp soft set over U which describes the " attractiveness of the cars" which Mr.S (say) is going to buy.

2.2. Fuzzy soft set

Definition 2.2. [6] Let U be a universal set, E a set of parameters and $A \subset E$. Let $F(U)$ denotes the set of all fuzzy subsets of U. Then a pair (F, A) is called fuzzy soft set over U, Where F is a mapping from A to $F(U)$.

Example 2.2. Let $U = \{c_1, c_2, c_3\}$ be the set of three cars and E = $\{\cos tly(e_1), metallic$ $colour(e_2), getup(e_3)\}$ be the set of parameters, where A = $\{e_1, e_2\}$ $\subset$ E. Then $(G, A) = \{G(e_1) = \{c_1 / .6, c_2 / .4, c_3 / .3\},\ G(e_2) = \{c_1 / .5, c_2 / .7, c_3 / .8\}\}$ is the fuzzy soft set over U which describes the " attractiveness of the cars" in which Mr.S (say) is going to buy.

2.3. Interval-valued fuzzy soft set

Definition 2.3.1. [6] Let U be an initial universal set, E a set of parameters, a pair $(\tilde{F}, E)$ is called an interval valued fuzzy soft set over $\tilde{P}(U)$, Where $\tilde{F}$ is a mapping given by $\tilde{F} : E \to \tilde{P}(U)$.

Remark. [6] An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U , thus, its universe is the set of all interval-valued fuzzy sets of U, i.e. $\tilde{P}(U)$. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $\tilde{P}(U)$.

2.4. Complement of interval - valued fuzzy soft set

Definition 2.4. [6] The complement of a interval interval-valued fuzzy soft set (F,A) is denoted by $(F,A)^C$ and is defined by $(F,A)^C=(F^c,\neg A)$, where $\forall\alpha\in A,\neg\alpha=$ not α, is the not set of the parameter α, which holds the opposite meanings of parameter α ; $F^c:\neg A\to\tilde{F}(U)$ is a mapping given by $F^C(\beta)=(F(\neg\beta))^C,\forall\beta\in\neg A$. **Example 2.4.**

Let $U=\{\mathrm{c}_1,c_2,c_3\}$ be the set of three cars and $E=\{\text{costly}(\mathrm{e}_1),\textit{metallic colour}(e_2),\textit{getup}(e_3)\}$ be the set of parameters, where $A=\{\mathrm{e}_1,e_2\}\subset E$. Then,
$(G,A)=\{G(e_1)=\{c_1/(0.3,0.8),c_2/(0.2,0.7),c_3/(0.5,0.8)\},$
$G(e_2)=\{c_1/(0.6,0.9),c_2/(0.5,0.7),c_3/(0.6,0.8)\}\}$ *is the interval-valued fuzzy soft set over* U *describes the " attractiveness of the cars "* which Mr.S (say) is going to buy. Then,
$(G,A)^C=\{G(\neg e_1)=\{c_1/(0.2,0.7),c_2/(0.3,0.8),c_3/(0.2,0.5)\},$
$G(\neg e_2)=\{c_1/(0.1,0.4),c_2/(0.3,0.5),c_3/(0.2,0.4)\}\}$
is the complement of (G,A).

2.5.Complement of interval - valued fuzzy soft matrix

Definition 2.5. [3] Let $(\bar{\mathrm{a}}_{ij})$ be an $m\times n$ IVFS-matrix, where $(\bar{\mathrm{a}}_{ij})$ is the matrix representation of the interval-valued fuzzy soft set $(\bar{F}_A,\mathrm{E})$. Then the complement of $(\bar{\mathrm{a}}_{ij})$ is denoted by $(\bar{\mathrm{a}}_{ij})^c$ and is defined by, $(\bar{\mathrm{a}}_{ij})^c=(\bar{\mathrm{c}}_{ij})$, where $(\bar{\mathrm{c}}_{ij})$ is also an IVFS-matrix of order $m\times n$ and it is the matrix representation of the interval-valued fuzzy soft set $(\bar{F}^c_{\neg A},\mathrm{E})$, ie., $c_{ij}=[\mu_{c_{ij}}{}^-,\mu_{c_{ij}}{}^+]=[1-\mu_{a_{ij}}{}^+,1-\mu_{a_{ij}}{}^-]$.

Example 2.5.

Let $$(\mathrm{a}_{ij})=\begin{bmatrix}(0.3,0.7) & (0.2,0.9) & (0.5,0.8)\\ (0.1,0.6) & (0.4,0.7) & (0.6,0.8)\\ (0.2,0.7) & (0.6,0.9) & (0.3,0.7)\end{bmatrix}_{3\times3}$$
be the interval-valued fuzzy soft matrix then complement of this matrix is

$$(\mathrm{a}_{ij})^c=\begin{bmatrix}(0.3,0.7) & (0.1,0.8) & (0.2,0.5)\\ (0.4,0.9) & (0.3,0.6) & (0.2,0.4)\\ (0.3,0.8) & (0.1,0.4) & (0.3,0.7)\end{bmatrix}_{3\times3}$$

2.6. Associated real number of an interval number

Definition 2.6. The associated real number of an interval number $A = (\mathrm{a}, \mathrm{b})$ is denoted by $R(A)$ and is defined as $R(\mathrm{A}) = \dfrac{a+b}{2}$.

Example 2.6. An Associated real number of the interval number $A = (2,3)$ is

$$R(A) = \frac{2+3}{2} = 2.5$$

3. Algorithm based on complement of interval-valued fuzzy soft sets

Step I:
Input the interval-valued fuzzy soft sets (F,E) and (G,E) ,Also write the interval-valued fuzzy soft matrices A and B corresponding to (F,E) and (G,E) respectively

Step II:
Write the complement of interval-valued fuzzy soft sets $(\mathrm{F},\mathrm{E})^C$ and $(\mathrm{G},\mathrm{E})^C$, Also write the complement of interval-valued fuzzy soft matrices $\overline{A}$ and $\overline{B}$ corresponding to $(\mathrm{F},\mathrm{E})^C$ and $(\mathrm{G},\mathrm{E})^C$ respectively.

Step III:
Compute $\overline{A} + \overline{B}$

Step IV:
Then compute the weight of each object (O_i) by adding the membership values of the entries of its concerned row $(i^{th} row)$ and denote it as $W(\mathrm{O}_i)$.

Step V:
$\forall O_i \in U$, Compute the score r_i of O_i such that,

$$r_i = \sum_{o_i \in U} ((\mu_i^- - \mu_j^-) + (\mu_i^+ - \mu_j^+))$$

Step VI:
The object having the highest score becomes the jointly selected object according to all decision makers. If more than one objecthave the highest score then any one of these highest scores may be chosen as the jointly selected object.

Step VII:
Select the candidate according to the maximum value of $R(\mathrm{c})$ and verify that he will get the high score (r_i).

4. Application of a decision making problem
Mr. X and Mrs. X are facing the problem for choosing the suitable candidate for their company, suppose $U = \{c_1, c_2, c_3, c_4\}$ is the collection of four candidates appearing in a

interview for appointment of managerial level. Mr. X and Mrs. X are willing to find out the candidates performance at the time of interview considering the parameters which are denoted by e_1, e_2, e_3 and e_4 respectively.

ie) $E = \{e_1, e_2, e_3, e_4\}$ =**{high percentage, good knowledge, fluency in language, friendly approach}** is the set of parameters which would vary with interval-values.

1) Let (F, E) and (G, E) be two interval-valued fuzzy soft sets representing the candidates performance at the time of interview.

$(F,E) = \{F(e_1) = \{c_1/(0.6,0.9), c_2/(0.7,1), c_3/(0.5,0.8), c_4/(0.3,0.6)\},$

$F(e_2) = \{c_1/(0.2,0.6), c_2/(0.5,0.7), c_3/(0.6,1), c_4/(0.4,0.7)\},$

$F(e_3) = \{c_1/(0.3,0.6), c_2/(0.2,0.5), c_3/(0.4,0.8), c_4/(0.5,0.9)\},$

$F(e_4) = \{c_1/(0.4,0.7), c_2/(0.5,0.9), c_3/(0.6,1), c_4/(0.1,0.4)\}\}.$

$(G,E) = \{G(e_1) = \{c_1/(0.4,0.6), c_2/(0.3,0.7), c_3/(0.6,0.8), c_4/(0.5,0.9)\},$

$G(e_2) = \{c_1/(0.1,0.4), c_2/(0.6,0.9), c_3/(0.8,1), c_4/(0.2,0.4)\},$

$G(e_3) = \{c_1/(0.2,0.5), c_2/(0.4,0.6), c_3/(0.6,0.9), c_4/(0.1,0.4)\},$

$G(e_4) = \{c_1/(0.3,0.5), c_2/(0.6,0.9), c_3/(0.2,0.4), c_4/(0.5,0.8)\}\}.$

These two interval-valued fuzzy soft sets are represented by the following fuzzy matrices respectively.

$$A = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} (0.6,0.9) & (0.2,0.6) & (0.3,0.6) & (0.4,0.7) \\ (0.7,1) & (0.5,0.7) & (0.2,0.5) & (0.5,0.9) \\ (0.5,0.8) & (0.6,1) & (0.4,0.8) & (0.6,1) \\ (0.3,0.6) & (0.4,0.7) & (0.5,0.9) & (0.1,0.4) \end{bmatrix} \end{array} \text{ and}$$

$$B = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} (0.4,0.6) & (0.1,0.4) & (0.2,0.5) & (0.3,0.5) \\ (0.3,0.7) & (0.6,0.9) & (0.4,0.6) & (0.6,0.9) \\ (0.6,0.8) & (0.8,1) & (0.6,0.9) & (0.2,0.4) \\ (0.5,0.9) & (0.2,0.4) & (0.1,0.4) & (0.5,0.8) \end{bmatrix} \end{array}$$

2) Let A and B be the interval-valued fuzzy soft matrices then complement of interval-valued fuzzy soft sets are given by

$(F,E)^C = \{F^C(e_1) = \{c_1/(0.1,0.4), c_2/(0,0.3), c_3/(0.2,0.5), c_4/(0.4,0.7)\},$

$F^C(e_2) = \{c_1/(0.4,0.8), c_2/(0.3,0.5), c_3/(0,0.4), c_4/(0.3,0.6)\},$

$F^C(e_3) = \{c_1/(0.4,0.7), c_2/(0.5,0.8), c_3/(0.2,0.6), c_4/(0.1,0.5)\},$

$F^C(e_4) = \{c_1/(0.3,0.6), c_2/(0.1,0.5), c_3/(0,0.4), c_4/(0.6,0.9)\}\}$.

$(G,E)^C = \{G^C(e_1) = \{c_1/(0.4,0.6), c_2/(0.3,0.7), c_3/(0.2,0.4), c_4/(0.1,0.5)\}$,

$G^C(e_2) = \{c_1/(0.6,0.9), c_2/(0.1,0.4), c_3/(0,0.2), c_4/(0.6,0.8)\}$,

$G^C(e_3) = \{c_1/(0.5,0.8), c_2/(0.4,0.6), c_3/(0.1,0.4), c_4/(0.6,0.9)\}$,

$G^C(e_4) = \{c_1/(0.5,0.7), c_2/(0.1,0.4), c_3/(0.6,0.8), c_4/(0.2,0.5)\}\}$.

These two complement of interval-valued fuzzy soft sets are represented by the following fuzzy matrices in order.

$$\overline{A} = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} (0.1,0.4) & (0.4,0.8) & (0.4,0.7) & (0.3,0.6) \\ (0,0.3) & (0.3,0.5) & (0.5,0.8) & (0.1,0.5) \\ (0.2,0.5) & (0,0.4) & (0.2,0.6) & (0,0.4) \\ (0.4,0.7) & (0.3,0.6) & (0.1,0.5) & (0.6,0.9) \end{bmatrix} \end{array} \text{ and}$$

$$\overline{B} = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} (0.4,0.6) & (0.6,0.9) & (0.5,0.8) & (0.5,0.7) \\ (0.3,0.7) & (0.1,0.4) & (0.4,0.6) & (0.1,0.4) \\ (0.2,0.4) & (0,0.2) & (0.1,0.4) & (0.6,0.8) \\ (0.1,0.5) & (0.6,0.8) & (0.6,0.9) & (0.2,0.5) \end{bmatrix} \end{array}$$

3) The sum of two complement of interval-valued fuzzy soft matrices is $\overline{A}+\overline{B}$, Which represents the maximum membership function of choosing the suitable candidate.

$$\overline{A}+\overline{B} = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} (0.4,0.6) & (0.6,0.9) & (0.5,0.8) & (0.5,0.7) \\ (0.3,0.7) & (0.3,0.5) & (0.5,0.8) & (0.1,0.5) \\ (0.2,0.5) & (0,0.4) & (0.2,0.6) & (0.6,0.8) \\ (0.4,0.7) & (0.6,0.8) & (0.6,0.9) & (0.6,0.9) \end{bmatrix} \end{array}$$

4) Now the weights of the each candidate are,

$(i)W(\mathrm{c}_1) = [(0.4+0.6+0.5+0.5),(0.6+0.9+0.8+0.7)] = [2,3]$

$(ii)W(c_2) = [(0.3+0.3+0.5+0.1),(0.7+0.5+0.8+0.5)] = [1.2,2.5]$

$(iii)W(c_3) = [(0.2+0+0.2+0.6),(0.5+0.4+0.6+0.8)] = [1,2.3]$

$(iv)W(c_4) = [(0.4+0.6+0.6+0.6),(0.7+0.8+0.9+0.9)] = [2.2,3.3]$

5) Now the scores for the candidate are,

$(i)r_1 = (0.8+1-0.2)+(0.5+0.7-0.3) = 2.5$

$(ii) r_2 = (-0.8 + 0.2 - 1) + (-0.5 + 0.2 - 0.8) = -2.7$

$(iii) \mathrm{r}_3 = (-1 - 0.2 - 1.2) + (-0.7 - 0.2 - 1) = -4.3$

$(iv) r_4 = (0.2 + 1 + 1.2) + (0.3 + 0.8 + 1) = 4.5$

6) Since the score r_4 is maximum (4.5) ,the highest scorer candidate c_4 will be their selected candidate which fulfill the choice parameters of Mr.X and Mrs.X as much as possible.

7) Finally by using the definition 2.6, we can verify that c_4 is the suitable candidate for the company, as $R(c_4) > R(c_1) > R(c_2) > R(c_3)$ .Hence c_4 is selected who has got the highest score and hence it is verified.

Remark: If two interval-valued fuzzy soft sets (F, E) and (G, E) are taken directly without using the notion of complement which is applied in the algorithm section 3. As $R(c_3) > R(c_2) > R(c_4) > R(c_1)$, Mr.X and Mrs.X will select candidate c_3 for their company.

5. Conclusion

In this paper, we have discussed the concept based on complement of interval-valued fuzzy soft sets in a decision making problem. Also the algorithm based on complement of interval-valued fuzzy soft sets is proposed to solve the discussed notion with a new approach and relevant illustration is added to justify the above said concept.

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