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2–Domination in Fuzzy Graphs

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Abstract. In this paper, 2–dominating set and 2–domination number of a fuzzy graph are introduced. The 2–domination number γ_2(G), of the fuzzy graph G is the minimum cardinality taken over all 2–dominating sets of G. We also prove some results on 2-dominating set. The exact values of γ_2(G) for some standard fuzzy graphs are found.

Keywords: Strong neighbours, 2 – dominating set, 2 – domination number

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

The study of dominating sets in graphs was started by Ore and Berge [1,9]. The domination number and the independent domination number were introduced by Cockayne and Hedetniemi [3]. The n-domination in graphs was introduced by Fink and Jacobson [4] in the year 1985. The concept of fuzzy relation was introduced by Zadeh [12] in his classical paper in 1965. Rosenfeld [10] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness.

Somasundram and Somasundram [11] discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graphs. NagoorGani and Chandrasekaran [5] discussed domination in fuzzy graph using strong arcs. NagoorGani and Vadivel [7,8] discussed domination, independent domination and irredundance in fuzzy graphs using strong arcs. NagoorGani and Prasanna Devi [6] discussed edge domination and edge independence in fuzzy graphs.

2. Preliminaries

A *fuzzy graph* G $=<\sigma,\mu>$is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all x,y ∈ V, we have $\mu(x,y) \le \sigma(x) \wedge \sigma(y)$. A fuzzy graph H $=<\tau,\rho>$ is called a *fuzzy subgraph* of G if $\tau(v_i) \le \sigma(v_i)$ for all v_i ∈ V and $\rho(v_i,v_j) \le \mu(v_i,v_j)$ for all v_i,v_j ∈V. The *underlying crisp graph* of a fuzzy graph G $=<\sigma,\mu>$is denoted by G^* $=<\sigma^*, \mu^*>$, where $\sigma^* = \{v_i \in V / \sigma(v_i)>0\}$ and $\mu^* = \{(v_i,v_j) \in V\times V / \mu(v_i,v_j) > 0\}$. An edge in G is called an *isolated edge* if it is not adjacent to any edge in G. A node in G is called an *isolated node* if it is not adjacent to any node in G. A *path* with n vertices in a fuzzy graph is denoted asP_n . A fuzzy graph G $=<\sigma,\mu>$ is a *complete fuzzy graph* if

$\mu(v_i, v_j) = \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \sigma^*$. An arc (x, y) in a fuzzy graph G =<σ,μ> is said to be *strong* if $\mu^\infty(x, y) = \mu(x, y)$ then x, y are called *strong neighbours*. The *strong neighbourhood* of the node u is defined as $N_S(u) = \{v \in V: (u, v)$ is a strong arc$\}$. A subset D of V is called a *dominating set* of a fuzzy graph G if for every $v \in$ V− D, there exist $u \in$ D such that u dominates v. The *domination number*, γ(G), of a fuzzy graph G, is the smallest number of nodes in any dominating set of G.

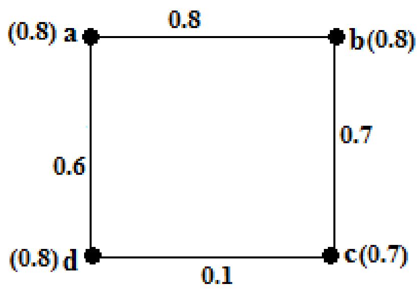
The *2-dominating set* D of a graph is defined as if for every node $v \in$ V−D there exist atleast two neighbours in D. In this paper we discuss about 2-dominating set and 2-domination number of a fuzzy graph.

3. 2-dominating set

In this section, we define 2-dominating set and 2-domination number of a fuzzy graph with suitable examples. We also derive some results on the 2-domination number of the fuzzy graphs.

Definition 3.1. A subset D of V is called a ***2-dominating set*** of G if for every node $v \in$ V−D there exist atleast two strong neighbours in D.

Example 3.2.



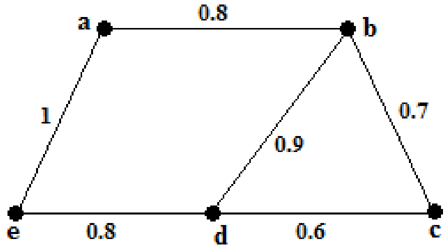
(G)

Figure 3.1:

$\{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ and $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$are 2-dominating sets of the fuzzy graph G.

Definition 3.3. The ***2-domination number*** of a fuzzy graph G denoted by γ_2(G), is the minimum cardinality of a 2-dominating set of G.

Example 3.4.



(G)

Figure 3.2:

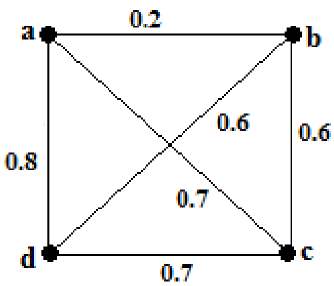
$\{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{e}\}, \{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{e}\}, \{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}\}$ and $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}\}$ are 2-dominating set of the fuzzy graph G.

$\Longrightarrow \ \gamma_2(\mathrm{G}) = 3.$

Definition 3.5. A 2-dominating set D of a fuzzy graph G such that $|\mathrm{D}| = \gamma_2(\mathrm{G})$ is called a ***minimum2-dominating set*** of G.

A 2-dominating set D is called a ***minimal 2-dominatingset*** if no proper subset of D is a 2-dominating set of G.

Example 3.6.



(G)

Figure 3.3:

Here $\{\boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{d}\}$and $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ are all 2-dominating set of the fuzzy graph G.

The sets $\{\boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\ and\ \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{d}\}$ are minimal 2-dominating sets of G.

The set $\{\boldsymbol{c}, \boldsymbol{d}\}$ is the minimum 2-dominating set of G.

Theorem 3.7. If $|N_S(v)| \leq 1$, then v belongs to every 2-dominating set of the fuzzy graph G.

Proof: Let G be a fuzzy graph and v ∈V has atmost one strong neighbour in G. Let D be a 2-dominating set of G.

Case (i):

Suppose v has no strong neighbours in G. i.e.,$N_S(v) = \emptyset$.

Then v is not dominated by any node in D, since v is an isolated node in G. Thus v should be dominated by itself.

Hence v belongs to every 2-dominating set of G.

Case (ii):

Suppose v has only one strong neighbour in G.

Suppose $v \notin$ D.

Then v has atmost one strong neighbour in D. But D is a 2-dominating set of G i.e., every node $v \in$ V−D has atleast two strong neighbours in D. Since $v \notin$ D and has atmost one strong neighbour in D then D is not a 2-dominating set of G, which is a contradiction to our assumption.

Therefore, $v \in$ D, for every 2-dominating set D of G.

Theorem 3.8. Every 2-dominating set of a fuzzy graph G is a dominating set of G.

Proof: Let D be a 2-dominating set of the fuzzy graph G. Then every node in V− D has atleast two strong neighbours in D i.e., for every node $v \in$ V−D, there exist minimum two nodes in D and both dominates v.

Every node in V − D is dominated by atleast two nodes in D. Thus D is a dominating set of G.

Corollary 3.9. If G is a fuzzy graph then γ_2(G) $\geq$ γ(G).

Proof: By the above theorem, every 2-dominating set of a fuzzy graph G is a dominating set of G. Thus every minimum 2-dominating set of G is also a dominating set of G.

Therefore, γ_2(G) $\geq$ γ(G).

Theorem 3.10. Every connected fuzzy graph G has a minimum 2-dominating set D then V – D need not be a 2-dominating set of G.

Proof: Let D be a 2-dominating set of G and let $v \in$ V.

Suppose $|N_S(v)| = 1$, then v belongs to every 2-dominating set of G.

Thus v belongs to every minimum 2-dominating set of G. Then V − D has either no strong neighbour of v or only one strong neighbour of v. Thus V – D does not has two strong neighbours for v.

⇒ V – D is not a 2-dominating set of G.

Suppose every node in D has atleast two strong neighbours in V − D.

Then in this case, every node in D has atleast two strong neighbours in V− D. Thus V− D is a 2-dominating set of G.

Hence V – D need not be a 2-dominating set of G.

4. 2-domination number for standard fuzzy graphs

In this section we discuss about the 2-dominating set and 2-domination number of some standard fuzzy graphs.

Theorem 4.1. If $e = (u, v)$ is an isolated edge in a fuzzy graph G then both u and v are in every 2-dominating set of G.

Proof: Let $e = (u, v)$ is an isolated edge in a fuzzy graph G. then e is a strong arc in G i.e., u and v are strong neighbours in G.

⇒ $N_S(u) = \{v\}$and$N_S(v) = \{u\}$

⇒ $|N_S(u)| = 1$and$|N_S(v)| = 1$

Since$|N_S(v)| = 1$, then u belongs to every 2-dominating set of G. Simillarly v belongs to every 2-dominating set of G.

Thus both u and v are in every 2-dominating set of G.

Corollary 4.2. If G is a fuzzy graph and G*= nK_2 then γ_2(G) $= 2n$.

Proof: Let G be fuzzy graph and G*= nK_2.

K_2 is a complete fuzzy graph with two nodes i.e., a strong arc with its nodes.

By above theorem, both the vertices of K_2 is in the 2-dominating set of G.

Thus the fuzzy graph G has all the $2n$ nodes in the 2-dominating set of G and also it will be in the minimum 2-dominating set of G.

Therefore, γ_2(G) $= 2n$.

Theorem 4.3. If K_n is a complete fuzzy graph, $n \geq 2$, then $\gamma_2(K_n) = 2$.

Proof: K_n is a complete fuzzy graph with n nodes. Here every node in K_n is a strong neighbour to all other nodes in it.

Thus any two nodes in K_n will form a 2-dominating set of K_n and it will be a minimum 2-dominating set of K_n.

Therefore $\gamma_2(K_n) = 2$.

Theorem 4.4. If G is a fuzzy graph and G* is a cycle with n nodes then the 2-domination number of G,

$$\gamma_2(\mathrm{G}) = \begin{cases} \left\lfloor \frac{n+1}{2} \right\rfloor, & \text{if } G \text{ has more than one weakest arc.} \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & \text{if } G \text{ has only one weakest arc.} \end{cases}$$

Proof: Let G be a fuzzy graph and G* be a cycle with n nodes. Let $v_1, v_2, \ldots\ldots, v_n$ be the n nodes of G.

Case (i): If G has more than one weakest arc then all the arcs of G are of strong arcs. Then $\left\lfloor \frac{n+1}{2} \right\rfloor$ nodes in G will form a 2-dominating set of G.

Therefore $\gamma_2(G) = \left\lfloor \frac{n+1}{2} \right\rfloor$.

Case (ii): If G has only one weakest arc then G has $n-1$ strong arcs and only one non strong arcs.

If $e = (v_1, v_2)$ is the only one non strong arc in G then $|N_S(v_1)| = 1$ and $|N_S(v_2)| = 1$. Thus both v_1 and v_2 will be in every 2-dominating set of G. Therefore $\left\lfloor \frac{n}{2} \right\rfloor + 1$ nodes in G will form a 2-dominating set of G.

Therefore $\gamma_2(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Corollary 4.5. If P_n is a fuzzy path with n nodes then $\gamma_2(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Theorem 4.6. Let G be a fuzzy graph and G* is a star with $n+1$ nodes, $n \geq 2$, then $\gamma_2(G) = n$.

Proof: Let G be a fuzzy graph and G* is a star with $n+1$ nodes.

The nodes except the centre node will have only one strong neighbour and it will be in every 2-dominating set of G. And the centre will have all other n nodes as its strong neighbours. The nodes except the centre node will form a 2-dominating set and it will be the minimum. Therefore $\gamma_2(G) = n$.

5. Conclusion

We defined 2-dominating set and 2-domination number of a fuzzy graph. For some standard fuzzy graphs, we have given the exact value of the 2-domination number. Further works are to find the relation between 2- domination number with edge domination number of fuzzy graphs.

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