

Smarandache–Boolean–Near–Rings and Algorithms

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Abstract. In this paper we introduced Smarandache-2-algebraic structure of Boolean-near-ring namely Smarandache-Boolean-near-ring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exists a proper subset M of N , which is embedded with a stronger algebraic structure A_1 , stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-Boolean-near-ring and obtain some of its algorithms through Boolean-ring with left-ideals, direct summand, Boolean- l -algebra, Brouwerian algebra, Compatibility, maximal set and Polynomial Identities.

Keywords: Boolean-ring, Boolean-near-ring, Smarandache-Boolean-near-ring, left-ideal, direct summand, Boolean- l -algebra, Brouwerian algebra, Compatibility, maximal set and Polynomial Identities

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1. Introduction

In order that New notions are introduced in algebra to better study the congruence in number theory by Smarandache [4]. By \langle proper subset \rangle of a set A we consider a set P included in A , and different from A , different form the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship: They say that the algebraic structures $S_1 \ll S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1 .

For example: Semi group \ll Monoid \ll group \ll ring \ll field, or Semi group \ll commutative semi group, ring \ll unitary, ring etc. They define a General special structure to be a structure SM on a set A , different form a structure SN , such that a proper subset of A is an structure, where $SM \ll SN \ll$

2. Preliminaries

Definition 2.1. A left near-ring A is a system with two binary operations, addition and multiplication, such that

- (i) the elements of A form a group $(A, +)$ under addition,

- (ii) the elements of A form a multiplicative semi-group,
- (iii) $x(y + z) = xy + xz$, for all $x, y, z \in A$

In particular, if A contains a multiplicative semi-group S whose elements generate $(A, +)$ and satisfy

- (iv) $(x+y)s = xs + ys$, for all $x, y \in A$ and $s \in S$, then we say that A is a distributively generated near-ring.

Definition 2.2. A near-ring $(B, +, \cdot)$ is Boolean-Near-Ring if there exists a Boolean-ring $(A, +, \wedge, 1)$ with identity such that \cdot is defined in terms of $+$, \wedge and 1 , and for any $b \in B$, $b \cdot b = b$.

Definition 2.3. A near-ring $(B, +, \cdot)$ is said to be idempotent if $x^2 = x$, for all $x \in B$. If $(B, +, \cdot)$ is an idempotent ring, then for all $a, b \in B$, $a + a = 0$ and $a \cdot b = b \cdot a$

Definition 2.4. A Boolean-near-ring $(B, +, \cdot)$ is said to be Smarandache-Boolean-near-ring whose proper subset A is a Boolean-ring with respect to same induced operation of B .

Definition 2.5. (Alternative definition for S-Boolean-near-ring) If there exists a non-empty set A which is a Boolean-ring such that its superset B of A is a Boolean-near-ring with respect to the same induced operation, then B is called Smarandache-Boolean-near-ring. It can also be written as S-Boolean-near-ring.

3. Algorithms

Left – Ideal: Clay and Lawver [2] have introduced the left-ideals of $(B, +, \cdot)$ in $P(x)$ are the subgroups of the groups $(P(x), +)$, where $P(x) = \{b \in B / b \wedge x = b\} = B_x$ is a maximal sub-z-ring. It also contained in an ideal. Let $A = I_0$. Now to construct a set B as follows.

B contains a unique minimal ideal I_0 contained in all other non – zero ideals. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache -Boolean-near-ring.

Algorithm 3.1.

- Step 1: Consider a Boolean-ring A
- Step 2: Let $A = I_0$, be an ideal
- Step 3: Let $I_i, i = 0, 1, 2, 3, \dots$ be supersets of I_0 .
- Step 4: Let $B = \bigcup I_0$
- Step 5: Choose the sets I_j from I_i 's subject to a, b and $c \in B$ such that

$$(a + b) \cdot c + a \cdot c + b \cdot c = x \wedge c \in I_j \text{ and } x \in B \text{ we have } P(x) \subseteq I$$
- Step 6: Verify that $\bigcap I_j = I_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Direct Summand

Clay and Lawver [2] has introduced the concept of direct summand. Let A be an ideal of B , then A is a direct summand if and only if $A = P(x)$. Now to construct a set B as

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follows. B contains a unique minimal direct summand M_0 contained in all other non – zero direct summands. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.2.

- Step 1: Consider a Boolean-ring A
- Step 2: Let $A = M_0$, be a direct summand.
- Step 3: Let $M_i, i = 0,1,2,3,\dots$ be supersets of M_0 .
- Step 4: Let $B = \bigcup M_i$
- Step 5: Choose the sets M_j from M_i 's subject to for all $x \in B$ such that M_0 is a direct summand we have $M_0 = P(x) + P(x^1)$, where $P(x)$ and $P(x^1)$ are ideals of B and $x, x^1 \in B$.
- Step 6: Verify that $\bigcap M_j = M_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Boolean- l -Algebra

Rao has introduced the notions of Boolean- l -algebra and lattice ordered groups. In [8] he proved A is a Boolean-ring if and only if A is a Boolean- l -algebra such that $x \leq a$ implies $x \cap (a-x) = 0$. He has established that the class of Boolean- l -algebra is a subclass of DRI semigroups also. Let $A = I_0$. Now to construct a set B as follows. B contains a unique minimal Boolean- l -algebra I_0 contained in all other non – zero Boolean- l -algebras. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.3.

- Step 1: Consider a Boolean-ring A
- Step 2: Let $A = I_0$, be a Boolean- l -algebra
- Step 3: Let $I_i, i = 0,1,2,3,\dots$ be supersets of I_0 .
- Step 4: Let $B = \bigcup I_i$
- Step 5: Choose the sets I_j from I_i 's subject to for all $i_1, i_2 \in I_j$ such that $i_1 \leq i_2$ implies $i_1 \cap (i_2 - i_1) = 0$
- Step 6: Verify that $\bigcap I_j = I_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Brouwerian Algebra

Rao has established that the class of Brouwerian algebras. Brouwerian algebras being a subclass of Boolean- l -algebras. If $(B; -)$ is a Boolean-ring then $(B; -)$ is a Boolean- l -algebra if and only if B is a Brouwerian such that that $x \leq a$ then $a = x \cup (a-x)$.

Let A be a Boolean – ring. Let $A = M_0$. Now to construct a set B as follows. B contains a unique minimal Brouwerian algebra contained in all other non – zero Brouwerian algebras. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.4.

- Step 1: Consider a Boolean-ring A
- Step 2: Let $A = M_0$
- Step 3: Let $M_i, i = 0, 1, 2, 3, \dots$ be the supersets of M_0 .
- Step 4: Let $B = \bigcup M_i$
- Step 5: Choose the sets M_j from M_i 's subject to for all x and $a \in B$ such that $x \leq a$ then $a = x \cup (a-x)$.
- Step 6: Verify that $\bigcap M_j = M_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Compatibility: A subset A of Boolean-near-ring B is said to be compatibility $a \sim b$ if $ab^2 = a^2b$. Let $A = I_0$. Now to construct a set B as follows. B contains a unique minimal compatibility I_0 contained in all other non – zero compatibilities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.5.

- Step 1: Consider a Boolean-ring A
- Step 2: Let $A = I_0$, be a compatibility
- Step 3: Let $I_i, i = 0, 1, 2, 3, \dots$ be the supersets of I_0 .
- Step 4: Let $B = \bigcup I_i$
- Step 5: Choose the sets I_j from I_i 's subject to for all $a, b \in A$ such that $ab^2 = a^2b \in I_j$
- Step 6: Verify that $\bigcap I_j = I_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean near-ring.

Maximal Set: Let B be a Boolean-near-ring and let $A = (\dots, a, b, c, \dots)$ be a set of pairwise compatible elements of an associate ring R. Let A be maximal in the sense that each element of A is compatible with every other element of A and no other such elements may be found in R. Then A is called maximal compatible set or a maximal set. Let $A = I_0$. Now to construct a set B as follows. B contains a unique minimal maximal set I_0 contained in all other non – zero maximal sets. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.6.

- Step 1 : Consider a Boolean-ring A
- Step 2 : Let $A = I_0$, be a maximal set
- Step 3 : Let $I_i, i = 0, 1, 2, 3, \dots$ be the supersets of I_0
- Step 4 : Let $B = \bigcup I_i$
- Step 5 : Choose the sets I_j from I_i 's subject to for all $a, b \in I_j$ such that $a \vee b = a + b - 2a^0b = (a \cup b) - (a \cap b)$ and $a \wedge b = a^0b = ab^0 = a \cap b \in I_j$, for all $a, b \in I_j$
- Step 6 : Verify that $\bigcap I_j = I_0 \neq \{0\}$

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Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Given two numbers $m > n \geq 1$, a ring B is said to be (m,n) – Boolean if $x^m = x^n$, for all x in B. Let $A = I_0$. Now to construct a set B as follows. B contains a unique minimal Polynomial identity I_0 contained in all other non – zero Polynomial identities. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.7.

- Step 1 : Consider a Boolean-ring A
- Step 2 : Let $A = I_0$
- Step 3 : Let $I_i, i = 0,1,2,3,\dots$ be the supersets of I_0 .
- Step 4 : Let $B = \bigcup I_i$
- Step 5 : Choose the sets I_j from I_i 's subject to for all $m, n \in B$ and for all $x \in B$ such that $x^m = x^n \in I_j$
- Step 6 : Verify that $\bigcap I_j = I_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and n be two positive integers such that $x^{2^{n+1}+2^n} = x$, for all x in B. Let $A = M_0$. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.8.

- Step 1 : Consider a Boolean-ring A
- Step 2 : Let $A = M_0$
- Step 3 : Let $M_i, i = 0,1,2,3,\dots$ be the supersets of M_0 .
- Step 4 : Let $B = \bigcup M_i$
- Step 5 : Choose the sets M_j from M_i 's subject to for all two positive integers m and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^{n+1}+2^n} = x, \in M_j$
- Step 6 : Verify that $\bigcap M_j = M_0 \neq \{0\}$
- Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and q be two fixed positive integers and $x^{2^{q(m+1)}+2^m} = x$, for all x in B. Then B is known as a Smarandache-boolean-near-ring.

Let $A = P_0$.

Now to construct a set B as follows. B contains a unique minimal Polynomial identity P_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.9.

- Step 1 : Consider a Boolean-ring A
 Step 2 : Let $A = P_0$
 Step 3 : Let $P_i, i = 0, 1, 2, 3, \dots$ be the supersets of P_0 .
 Step 4 : Let $B = \bigcup P_i$
 Step 5 : Choose the sets P_j from P_i 's subject to for all two positive integers m and q such that $x^{2^{q(m+1)}+2^m} = x, \in P_j$ and for all $x \in P_j$
 Step 6 : Verify that $\bigcap P_j = P_0 \neq \{0\}$
 Step 7 : If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and n be two positive integers such that $x^{2^{m+1}+2^n} = x$, for all x in B. Let $A = M_0$.

Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.10.

- Step 1 : Consider a Boolean-ring A
 Step 2 : Let $A = M_0$
 Step 3 : Let $M_i, i = 0, 1, 2, 3, \dots$ be the supersets of M_0 .
 Step 4 : Let $B = \bigcup M_i$
 Step 5 : Choose the sets M_j from M_i 's subject to for all two positive integers m and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^{m+1}+2^n} = x, \in M_j$
 Step 6 : Verify that $\bigcap M_j = M_0 \neq \{0\}$
 Step 7 : If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and n be two positive integers such that $x^{2^{m+1}+2^n} = x$, for all x in B. Let $A = M_0$. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.11.

- Step 1 : Consider a Boolean-ring A
 Step 2 : Let $A = M_0$
 Step 3 : Let $M_i, i = 0, 1, 2, 3, \dots$ be the supersets of M_0 .
 Step 4 : Let $B = \bigcup M_i$

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Step 5 : Choose the sets M_j from M_i 's subject to for all two positive integers m and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^{n+1}+2^n} = x, \in M_j$

Step 6 : Verify that $\bigcap M_j = M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity : Let m and n be two positive integers such that $x^{2^{n+1}+2^n} = x$, for all x in B . Let $A = M_0$. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.12.

Step 1 : Consider a Boolean-ring A

Step 2 : Let $A = M_0$

Step 3 : Let $M_i, i = 0,1,2,3,\dots$ be the supersets of M_0 .

Step 4 : Let $B = \bigcup M_i$

Step 5 : Choose the sets M_j from M_i 's subject to for all two positive integers m and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^{n+1}+2^n} = x, \in M_j$

Step 6 : Verify that $\bigcap M_j = M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let B be a Boolean-near-ring and let m, q and r be fixed positive integers with $r < m+1$ such that $x^{2^{q(m+1)+r}+2^m} = x$, for all x in B and $x^{2^{r+1}} = x$, then B is Smarandache-Boolean-near-ring. Let $A = M_0$. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.13.

Step 1 : Consider a Boolean-ring A

Step 2 : Let $A = M_0$

Step 3 : Let $M_i, i = 0,1,2,3,\dots$ be the supersets of M_0 .

Step 4 : Let $B = \bigcup M_i$

Step 5 : Choose the sets M_j from M_i 's subject to for all two positive integers m, q and r be three fixed positive integers with $r < m+1$ and for all $x \in M_j$ such that $x^{2^{q(m+1)+r}+2^m} = x$, and $x^{2^{r+1}} = x, \in M_j$

Step 6 : Verify that $\bigcap M_j = M_0 \neq \{0\}$

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Step 7: If step (6) is true, then we write B is a Smarandache-Boolean-near-ring.

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