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A Modified Approach for Solving Intuitionistic Fuzzy Assignment Problems

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Abstract. The Assignment Problem is one of the most studied, well known and important problem in mathematics and is used very often in solving problems of engineering and management sciences. In this problem c_{ij} denotes the cost for assigning the j^{th} job to the i^{th} person. This cost is usually deterministic in nature. In this paper c_{ij} has been considered to be generalized trapezoidal intuitionistic fuzzy numbers denoted by $\tilde{c}_{ij}$which are more realistic and general in nature. Here, we present a new method for solving assignment problems whose costs as generalized trapezoidal intuitionistic fuzzy numbers.

Keywords: Assignment problem, intuitionistic fuzzy numbers, generalized trapezoidal intuitionistic fuzzy numbers, ranking of fuzzy numbers.

AMS Mathematics Subject Classification (2010): 03E72, 90C08

1. Introduction

An Assignment Problem (AP) plays an important role in industry and other applications. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In this problem c_{ij} denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one person can be assigned exactly one job; also each person can do atmost one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum.

To find solutions to assignment problems, various algorithms such as linear programming, Hungarian algorithm, neural network and genetic algorithm have been developed. Over the past 50 years, many variations of the classical assignment problems are proposed, e.g., bottleneck assignment problem, quadratic assignment problem etc. Lin and Wen [5] proposed an efficient algorithm based on the labeling method for solving the linear fractional programming case. Sakawa et al. [9] solved the problems on production and work force assignment in a firm using interactive fuzzy programming for two level linear and linear fractional programming models. Chen [3] projected a fuzzy assignment model that considers all persons to have same skills. Chen Liang-Hsuan and Lu Hai-Wen

[4] developed a procedure for solving assignment problems with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model to determine the assignments with maximum efficiency. Linzhong Liu and XinGao [6] considered the genetic algorithm for solving the fuzzy weighted equilibrium and multi-job assignment problem. Majumdar and Bhunia [7] developed an exclusive genetic algorithm to solve a generalized assignment problem with imprecise cost(s)/ time(s), in which the impreciseness of cost(s)/time(s) are represented by interval valued numbers. Ye and Xu [15] developed a priority based genetic algorithm to a fuzzy vehicle routing assignment model with connection network. Mukherjee and Basu [12] developed a method for solving intuitionistic fuzzy assignment problems by using similarity measures and score function. Pandian and Kavitha [11], Jose and Kuriakose [13], Thorani and Shankar [14] and Nirmala and Anju [10] developed various algorithms for solving assignment problems in the fuzzy context. Here we are considering assignment problems having generalized trapezoidal intuitionistic fuzzy numbersas costs. We apply a ranking method [8] defined on generalized intuitionistic trapezoidal fuzzy numbers to rank the fuzzy costs present in the assignment problem.

This paper is organized as follows. In section 2, we present the basic concepts of generalized trapezoidal intuitionistic fuzzy numbers and its arithmetic operations. In section 3, a method of ranking is given to rank the generalized trapezoidal intuitionistic fuzzy numbers and in section 4, intuitionistic fuzzy assignment problem and its mathematical formulation are reviewed. A new method for solving an assignment problem with costs as generalized trapezoidalintuitionistic fuzzy numbers is given in section 5. In section 6, a numerical example is presented to show the application of the proposed method and finally, the conclusion is given in section 7.

2. Preliminaries

In this section we will review the basic concepts of intuitionistic fuzzy sets and intuitionistic fuzzy numbers.

2.1. Intuitionistic fuzzy sets

Definition 2.1.1. [1, 2] Let X be the universal set. An intuitionistic fuzzy set (IFS) A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where the functions $\mu_A(x), \nu_A(x)$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Obviously, every fuzzy set has the form

$$A = \{(x, \mu_A(x), \mu_{A^c}(x)) : x \in X\}.$$

For each intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ in X,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is called the hesitancy degree of x to lie in A. If A is a fuzzy set, then $\pi_A(x) = 0$ for all $x \in X$.

2.2. Intuitionistic fuzzy numbers

Here we will introduce the concepts of intuitionistic fuzzy number (IFN), generalized trapezoidal intuitionistic fuzzy number (GTIFN) and its arithmetic operations.

Definition 2.2.1. An IFS $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ of the real line $\mathbb{R}$ is called an intuitionistic fuzzy number (IFN) if

a) A is convex for the membership function $\mu_A(x)$, i.e., if $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$.

b) A is concave for the non-membership function $\nu_A(x)$, i.e., if $\nu_A(\lambda x_1 + (1-\lambda)x_2) \leq \nu_A(x_1) \vee \nu_A(x_2)$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$.

c) A is normal, that is, there is some $x_0 \in \mathbb{R}$ such that $\mu_A(x_0) = 1$ and $\nu_A(x_0) = 0$.

Definition 2.2.2. (**Generalized trapezoidal intuitionistic fuzzy number**) A generalized trapezoidal intuitionistic fuzzy number (GTIFN) A is an IFS in $\mathbb{R}$ with membership and non-membership functions are as follows:

$$
\begin{aligned}
\mu_A(x) &= 0 && \text{if} \quad x < a_1 \\
&= \omega_A\left(\frac{x-a_1}{a_2-a_1}\right) && \text{if} \quad a_1 \leq x \leq a_2 \\
&= \omega_A && \text{if} \quad a_2 \leq x \leq a_3 \\
&= \omega_A\left(\frac{a_4-x}{a_4-a_3}\right) && \text{if} \quad a_3 \leq x \leq a_4 \\
&= 0 && \text{if} \quad x > a_4
\end{aligned}
$$

and

$$
\begin{aligned}
\nu_A(x) &= 1 && \text{if} \quad x < a_1 \\
&= \frac{(a_2-x) + u_A(x-a_1)}{a_2-a_1} && \text{if} \quad a_1 \leq x \leq a_2 \\
&= u_A && \text{if} \quad a_2 \leq x \leq a_3 \\
&= \frac{(x-a_3) + u_A(a_4-x)}{a_4-a_3} && \text{if} \quad a_3 \leq x \leq a_4 \\
&= 1 && \text{if} \quad x > a_4.
\end{aligned}
$$

where

$$0 < \omega_A \leq 1, 0 \leq u_A \leq 1$$

and

$$0 < \omega_A + u_A \leq 1.$$

It is denoted by

$$A = ((a_1, a_2, a_3, a_4); \omega_A, u_A).$$

2.3. Arithmetic operations on generalized trapezoidal intuitionistic fuzzy numbers

Let $A = ((a_1, a_2, a_3, a_4); \omega_A, u_A)$ and $B = ((b_1, b_2, b_3, b_4); \omega_B, u_B)$ be two GTIFNs and λ be a real number. Then

(***i***) $A + B = ((a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \omega, u)$, where $\omega = \min\{\omega_A, \omega_B\}$ and $u = \max\{u_A, u_B\}$.

(***ii***) $A - B = ((a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \omega, u\}$,where
$\omega = \min\{\omega_A, \omega_B\}andu = \max\{u_A, u_B\}$.

(***iii***) $\lambda A = ((\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \omega_A, u_A)$ if $\lambda > 0$
$= ((\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); \omega_A, u_A)$ if $\lambda < 0$.

3. Ranking of generalized trapezoidal intuitionistic fuzzy numbers

The ranking order relation between two GTIFNs is a difficult problem. However, GTIFNs must be ranked before the action is taken by the decision maker. In this paper we use the following method for ranking generalized trapezoidal intuitionistic fuzzy numbers.

If $A = ((a_1, a_2, a_3, a_4); \omega_A, u_A)$, then

$$\Re(A) = \frac{\omega_A S(\mu_A) + u_A S(\nu_A)}{\omega_A + u_A},$$

where

$$S(\mu_A) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right)\left(\frac{7\omega_A}{18}\right)$$

and

$$S(\nu_A) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right)\left(\frac{11 + 7u_A}{18}\right).$$

4. Intuitionistic fuzzy assignment problems

Suppose there are n jobs to be performed and n persons are available for doing the jobs. Assume that each person can do each job at a time, depending on their efficiency to do the job. Let $\tilde{c}_{ij}$ be the intuitionistic fuzzy cost if the i^{th}person is assigned the j^{th}job. The objective is to minimize the total intuitionistic fuzzy cost of assigning all the jobs to the available persons (one job to one person).

The intuitionistic fuzzy assignment problem can be stated in the form ofan $n \times n$ cost matrix $[\tilde{c}_{ij}]$ of intuitionistic fuzzy numbers as given in the following table:

Persons	Jobs				
	1	2	3	….	n
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\tilde{c}_{13}$	….	$\tilde{c}_{1n}$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\tilde{c}_{23}$	….	$\tilde{c}_{2n}$
⋮					
n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	$\tilde{c}_{n3}$	….	$\tilde{c}_{nn}$

Mathematically an intuitionistic assignment problem can be stated as

Minimize $\tilde{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2, \dots n.$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2, \dots n.$$

where

$$x_{ij} = 1, \text{if the } i^{th} \text{person is assigned the} j^{th} \text{job}$$
$$= 0, \text{ otherwise}$$

is the decision variable denoting the assignment of the person i to job *j*. $\tilde{c}_{ij}$ is the cost of assigning the j^{th} job to the i^{th} person.

5. The proposed method

Let $P_1, P_2, \dots, P_n$ be the n persons and $J_1, J_2, \dots, J_n$ be the n jobs to be performed. Also let $\tilde{c}_{ij}$ be the cost of assigning i^{th} person to the j^{th} job, where $\tilde{c}_{ij}$'s are generalized trapezoidal intuitionistic fuzzy numbers. We perform the following steps for solving the assignment problem.

Step 1: Construct the matrix of ranks$\left[\Re\left(\tilde{c}_{ij}\right)\right]$, by using the given ranking method.

Step 2: Find the minimum unit costof each row of the matrix $\left[\Re\left(\tilde{c}_{ij}\right)\right]$.

Step 3: Form two columns, where column 1 represents the persons and in column 2, each row contains those jobs having minimum unit cost in that row of $\left[\Re\left(\tilde{c}_{ij}\right)\right]$.

Step 4: For each person, if there is a unique job in column 2, we are done.

Step 5: If there is no unique job for each persons, then the assignment is done in the following way:

(***i***) Find any one person having a unique job. Assign that job for the corresponding person. Next delete that row and its corresponding column from $\left[\Re\left(\tilde{c}_{ij}\right)\right]$.

(***ii***) Again find the minimum unit cost for the remaining rows and perform step 3 for the reduced matrix. Check if it satisfy step 4 then perform it. Otherwise, repeat 5(***i***) and 5(***ii***).

Step 6: If there is no person having a unique job, find the difference between minimum and next to the minimum unit cost for all those rows having the same job. Find the row with maximum difference. Assign that person the corresponding job. If there is tie in difference for two or more than two jobs, then further take the difference between minimum and next to next minimum unit cost for those rows. Check the row having maximum difference. Assign that person the corresponding job. Delete the row and the corresponding column from the reduced matrix.

Step 7: Repeat steps 3, 4, 5, 6 until all the persons are assigned uniquely to a job.

Remarks:

(i) If the problem is of maximization type then find the rank of each element of thechosen fuzzy cost matrix $\left[\tilde{c}_{ij}\right]$ by using the given ranking procedure and determine the element with the highest rank. Subtract each element of the cost matrix from this element. Then the problem with the modified matrix is a minimization problem.

(ii) Sometimes technical, legal or other restrictions do not permit the assignment of a particular facility to a particular job. In such cases also we use the algorithm by assigning a very high intuitionistic fuzzy cost to the cells which do not permit the assignment so that the activity will be automatically excluded from the optimal solution.

6. Numerical example

To illustrate the proposed algorithm, let us consider an intuitionistic fuzzy assignment problem with rows representing 4 persons A, B, C, D and columns representing the 4 jobs Job1, Job2, Job3 and Job4. The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are generalized trapezoidal intuitionistic fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

Persons	Jobs			
	1	2	3	4
A	(5, 8,10,13; 0.5,0.1)	(8,10,12,14; 0.7,0.2)	(8, 11,13,16: 0.5,0.3)	(8,10,12,14; 0.7,0.2)
B	(3,5,7,9; 0.7,0.1)	(1,2,4,5; 0.4,0.3)	(4,7,9,13; 0.7,0.1)	(2,4,6,8; 0.8,0.1)
C	(5,6,8,9; 0.6,0.1)	(2,5,7,10; 0.7,0.1)	(10,12,14,16; 0.6,0.2)	(8,10,12,14; 0.7,0.2)
D	(5,6,8,9; 0.6,0.1)	(7,9,11,13; 0.7,0.1)	(7,9,11,13; 0.7,0.1)	(5, 8, 10, 13;0.5,0.1)

We first form the matrix of ranks $[\Re(\tilde{c}_{ij})]$ by using the given ranking method. It is given by

Persons	Jobs			
	1	2	3	4
A	**2.433**	4.013	4.733	4.013
B	1.917	**1.202**	2.591	1.744
C	2.05	**1.917**	4.686	4.013
D	2.05	3.194	3.194	**2.433**

Form two columns, where column 1 represents the persons and in column 2, each row contains those jobs having minimum unit cost in that row of $[\Re(\tilde{c}_{ij})]$.

Persons	Jobs
A	1
B	2
C	2
D	4

Here we get a unique job for D, so assign D for job 4. Delete the corresponding row and column. The corresponding reduced matrix is

2.433	4.013	4.733
1.917	**1.202**	2.591
2.05	**1.917**	4.686

Form two columns, where column 1 represents the persons and in column 2, each row contains those jobs having minimum unit cost in that row of the reduced matrix. It is given by

Persons	Jobs
A	1
B	2
C	2

Here we get a unique job for A, so assign A for job 1. Delete the corresponding row and column. The corresponding reduced matrix is

1.202	2.591
1.917	4.686

Form two columns, where column 1 represents the persons and in column 2, each row contains those jobs having minimum unit cost in that row of the reduced matrix. It is given by

Persons	Jobs
B	2
C	2

Since no person has a unique job, we find the difference between minimum and next to minimum for job 2. The maximum difference is for B. Hence assign B for job 2. So the optimal assignment is

$$A \to 1, B \to 2, C \to 3, D \to 4.$$

The corresponding optimal minimum fuzzy cost is given by

$$\tilde{c}_{11} + \tilde{c}_{22} + \tilde{c}_{33} + \tilde{c}_{44} = (21, 28, 36, 43; 0.4, 0.3).$$

7. Conclusion

In this paper, a new algorithm has been developed for solving assignment problems with costs as generalized trapezoidal intuitionistic fuzzy numbers by using the given ranking method. There are several papers in the literature for solving assignment problems with intuitionistic fuzzy costs, but no one has used generalized intuitionistic fuzzy costs. The algorithm is easy to understand and can be used for all types of assignment problems with costs as fuzzy as well as intuitionistic fuzzy numbers.

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