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Totally $\hat{\alpha}$g Continuous Functions

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Abstract. In this paper, $\hat{\alpha}$g closed sets and $\hat{\alpha}$g open sets are used to define and investigate a new class of function called totally $\hat{\alpha}$g continuous function. Relationships between the new class and other classes of existing known functions are established.

Keywords: $\hat{\alpha}$g clopen sets, totally $\hat{\alpha}$g continuous function

AMS Mathematics Subject Classification (2010): 54C10

1. Introduction

Levine [2] introduced the concept of semi continuous function. Senthilkumaran et al. [7] introduced $\hat{\alpha}$ generalised closed sets in topological spaces. The purpose of this paper is to introduce the concept of totally $\hat{\alpha}$g continuous function as a generalization of the concept of totally continuous function. Several properties of totally $\hat{\alpha}$g continuous function are obtained. The interior and closure of a subset A of a space X is denoted by int A and cl A respectively.

2. Preliminaries

Definition 2.1. A subset A of topological space X is said to be $\hat{\alpha}$ generalised closed ($\hat{\alpha}$g closed) if int clint A⊂A wherever A⊂U and U is open in X .The complement of $\hat{\alpha}$g closed set in X is $\hat{\alpha}$g open in X [7].

Definition 2.2. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be totally continuous if the inverse image of every open set of Y is clopen in X [1].

3. Totally $\hat{\alpha}$g continuous function

Definition 3.1. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called totally $\hat{\alpha}$g continuous if $f^{-1}(V)$ is $\hat{\alpha}$g clopen in X, for every open set V of Y.

The union of two $\hat{\alpha}$g clopen sets need not be $\hat{\alpha}$g clopen, but in the following theorem, we assume that arbitrary union of $\hat{\alpha}$g clopen sets in $\hat{\alpha}$g clopen .

Theorem 3.2. The following statements are equivalent for a function $f:(X,\tau)\to(Y,\sigma)$:

i. f is totally $\hat{\alpha}$g continuous.

ii. For each $x\in X$ and for each open set V of Y containing f(x) there exists a $\hat{\alpha}$gclopen set U of X such that $f(V)\subset V$.

Proof:

i) ⇒ ii)

Let $x\in X$ and V be open in Y containing f(x).Then $x\in f^{-1}(V)$ which is $\hat{\alpha}$g clopen in X. $f(f^{-1}(V))\subset V$.

ii)⇒i)

Let V be open in Y and $x\in f^{-1}(V)$.Then $f(x)\in V$. There exists a $\hat{\alpha}$g clopen set U_x of X such that $f(U_x)\subset V$. Hence $U_x\subset f^{-1}(V)$.

$f^{-1}(V)=\bigcup_{x\in f^{-1}(v)} U_x$, which is $\hat{\alpha}$g clopen in X.

Remark 3.3. It is clear that every totally $\hat{\alpha}$g continuous function is $\hat{\alpha}$g continuous. But the converse need not be true can be seen from the following example.

Example 3.4. Let X={a,b,c}, τ={Φ,{a},{a,b},X}, σ={ Φ,{a},{b},{a,b},X}
$\hat{\alpha}$g closed sets of (X,τ) ={ Φ,{b},{c},{a,c},{b,c},X}.
Let f: $(X,\tau)\to(X,\sigma)$ be the identity function.
f is $\hat{\alpha}$g continuous but not totally $\hat{\alpha}$g continuous as $f^{-1}(\{a,b\})=\{a,b\}$ is not $\hat{\alpha}$g closed.

Remark 3.5. It is clear that every totally continuous function is totally $\hat{\alpha}$g continuous. But the converse need not be true can be seen from the following example.

Example 3.6. Let X={a,b,c}, τ={Φ,{a},{a,b},X}

Define $f:(X,\tau)\to(X,\tau)$ by f(a)=a,f(b)=b,f(c)=a

f is totally $\hat{\alpha}$g continuous but not totally continuous as $f^{-1}(\{a\})=\{a,c\}$ is not closed.

Definition 3.7. A space (X,τ) is said to be $\hat{\alpha}$g space if every $\hat{\alpha}$g open set of X is open in X.

Theorem 3.8. A function $f:(X,\tau)\to(Y,\sigma)$ is totally $\hat{\alpha}$g continuous and X is a $\hat{\alpha}$g space, then f is totally continuous.
Proof: Let V be open in Y.Then $f^{-1}(V)$ is $\hat{\alpha}$g clopen in X.As X is a $\hat{\alpha}$g space, $f^{-1}(V)$ is clopen.

Definition 3.9. A topological space X is said to be $\hat{\alpha}$gconnected if it cannot be written as the union of two nonempty disjoint $\hat{\alpha}$g open sets.

Theorem 3.10. If f is a totally $\hat{\alpha}$g continuous function from a $\hat{\alpha}$g connected space X onto any space Y,then Y is an indiscrete space.
Proof: If possible, let Y be not indiscrete.
Let A be a proper nonempty open subset of Y.Thenf^{-1}(A) is a proper nonempty $\hat{\alpha}$g clopen subset of X, which is a contradiction to the fact that X is $\hat{\alpha}$g connected.

Theorem 3.11. A topological space (X,τ) is $\hat{\alpha}g$ connected if and only if every totally $\hat{\alpha}g$ continuous function from a space (X,τ) into any T_o space (Y,σ) is constant.

Proof: Let X be not $\hat{\alpha}g$ connected.

Let every totally $\hat{\alpha}g$ continuous function from (X,τ) to (Y,σ) be constant.

Since (X,τ) is not $\hat{\alpha}g$ connected, there exists a proper nonempty $\hat{\alpha}g$clopen subset A of X.

Let $Y=\{a,b\}$, $\sigma=\{\Phi,\{a\},\{b\},Y\}$ be a topology on Y.

Let $f:(X,\tau)\to(Y,\sigma)$ be a function such that $f(A)=\{a\}, f(Y-A)=\{b\}$. Then f is non constant and totally $\hat{\alpha}g$ continuous such that Y is T_o which is a contradiction. Hence X must be $\hat{\alpha}g$ connected.

Conversely, let X be $\hat{\alpha}g$ connected. Let $f:X\to Y$ be totally $\hat{\alpha}g$ continuous.

Let a,b be distinct points of X such that $f(a)=\alpha\neq\beta=f(b)$, $\alpha,\beta\in Y$ and they are distinct. As Y is T_o, there exists open set U containing α but not β. So U is a proper open subset of Y.

$f^{-1}(U)$ is a proper $\hat{\alpha}g$ clopen subset of X, which contradicts X is $\hat{\alpha}g$ connected.

Hence f must be constant.

Theorem 3.12. Let $f:(X,\tau)\to(Y,\sigma)$ be a totally $\hat{\alpha}g$ continuous function and Y is a T_1space.If A is a nonempty $\hat{\alpha}g$ connected subset of X, then f(A) is a single point.

Proof: Obvious.

Lemma 3.13. If $A\in\hat{\alpha}g\ O(X)$and $B\in\hat{\alpha}g\ O(Y)$, then $A\times B\in\hat{\alpha}g\ O(X\times Y)$.

Theorem 3.14. If the function $f_i : X_i \to Y_i$ is totally $\hat{\alpha}g$ continuous function for each i=1,2, then $f_1\times f_2 : X_1\times X_2\to Y_1\to Y_2$ defined by $(f_1\times f_2)(x_1,x_2)=(f_1(x_1),f_2(x_2))$, for each $x_1\in X_1$, $x_2\in X_2$ is totally $\hat{\alpha}g$ continuous.

Proof: Let $V_1\times V_2\in O(Y_1\times Y_2)$.Then $V_1\in O(Y_1)$, $V_2\in O(Y_2)$.

$f_1^{-1}(V_1)\in\hat{\alpha}gCO(X_1)$, $f_2^{-1}(V_2)\in\hat{\alpha}gCO(X_2)$

$(f_1\times f_2)^{-1}(V_1\times V_2)=(f_1^{-1}(V_1), f_2^{-1}(V_2))= f_1^{-1}(V_1)\times f_2^{-1}(V_2)\in\hat{\alpha}gCO(X_1\times X_2)$.Hence $f_1\times f_2$ is totally $\hat{\alpha}g$ continuous.

Definition 3.15. Let (X,τ) be a topological space. Then the set of all points y in X such that x and y cannot be separated by $\hat{\alpha}g$ separation of X is said to be the quasi $\hat{\alpha}g$ component of X.

Theorem 3.16. Let f: $(X,\tau)\to(Y,\sigma)$ be a totally $\hat{\alpha}g$ continuous function from a topological space X into a T_1 space Y. Then f is constant on each quasi $\hat{\alpha}g$ component of X.

Proof: Let $x,y\in X$ that lie in the same quasi $\hat{\alpha}g$ component of X. Let $f(x)=\alpha\neq\beta=f(y)$. Since Y is T_1 ,$\{\alpha\}$ is closed in Y and $Y-\{\alpha\}$ is open in Y.

Since f is totally $\hat{\alpha}g$ continuous, $f^{-1}(\{\alpha\})$and $f^{-1}(Y-\{\alpha\})$ are disjoint $\hat{\alpha}g$ clopen subsets of X.

Further $x\in f^{-1}(\{\alpha\})$and $y\in f^{-1}(Y-\{\alpha\})$ which is a contradiction to the fact that x and y belongs to the same quasi $\hat{\alpha}g$ component of X. Hence the theorem.

Definition 3.17. A $\hat{\alpha}g$ frontier of a subset A of X is $\hat{\alpha}gfrA=\hat{\alpha}gclA\cap\hat{\alpha}gcl(X-A)$.

Theorem 3.18. The set of all points $x \in X$ in which a function $f:(X,\tau)\to(Y,\sigma)$ is not totally $\hat{\alpha}g$ continuous is the union of $\hat{\alpha}g$ frontier of the inverse images of open sets containing f(x) if arbitrary intersection of $\hat{\alpha}g$ closed sets in X is $\hat{\alpha}g$ closed in X.

Proof: Let $A = \{x \in X$: f is not totally $\hat{\alpha}g$ continuous at $x\}$. Let B be the union of $\hat{\alpha}g$ frontier of the inverse images of open sets containing f(x).

Let $x \in A$. Then there exists an open set V of Y containing f(x) such that f(U) is not contained in V for each $U \in \hat{\alpha}gO(X)$ containing x. Hence $x \in \hat{\alpha}gcl(X\text{-}f^{-1}(V))$.

On the other hand $x \in f^{-1}(V)\subset\hat{\alpha}gclf^{-1}(V)$.So $x \in \hat{\alpha}g$ fr $f^{-1}(V)$. Hence $A\subset B$.

Conversely, let f be totally $\hat{\alpha}g$ continuous at $x \in X$. Let V be open in Y containing f(x). Then there exists $U\in\hat{\alpha}gCO(X)$containing x such that $f(U)\subset V$.

That is $U\subset f^{-1}(V)$. Hence $x \in \hat{\alpha}gintf^{-1}(V)$

$x\notin\hat{\alpha}gcl(X\text{-}f^{-1}(V)$.Hence $x\notin \hat{\alpha}gfr\ f^{-1}(V)$. So $x\notin A$ implies $x\notin B$. Hence $B\subset A$.

Theorem 3.19. Let $\{X_\lambda:\lambda \in \Lambda\}$ be any family of topological spaces. If $f:X\to\Pi X_\lambda$ is a totally $\hat{\alpha}g$ continuous function. Then $P_\lambda of: X\to X_\lambda$ is totally $\hat{\alpha}g$ continuous function for each $\lambda \in \Lambda$, where P_λ is the projection of ΠX_λ onto X_λ.

Proof: We shall consider a fixed $\lambda \in \Lambda$. Suppose U_λ is an arbitrary open set in X_λ. Then

$P_\lambda^{-1}(U_\lambda)$ is open in Π X . Since f is totally $\hat{\alpha}g$ continuous, we have by

$f^{-1}(P_\lambda^{-1}(U_\lambda))=(P_\lambda of)^{-1}(U_\lambda)$ is $\hat{\alpha}g$ clopen in X. Hence the assertion.

Definition 3.20.

i) A filter base Λ is said to be $\hat{\alpha}g$ co-convergent to a point $x\in X$ of for any $U\in\hat{\alpha}gCO(X)$ containing x, there exists $B\in\Lambda$ such that $B\subset U$.

ii) A filter base Λ is said to be convergent to a point $x\in X$ of for any $U\in O(X)$ containing x, there exists $B\in\Lambda$ such that $B\subset U$.

Theorem 3.21. If a function $f:(X,\tau)\to(Y,\sigma)$ is totally $\hat{\alpha}g$ continuous, then for each points $x \in X$ and each filter base Λ in X $\hat{\alpha}g$ co-convergent to x, the filter base $f(\Lambda)$ is convergent to f(x).

Proof: Let $x \in X$ and Λ be any filter base in X $\hat{\alpha}g$ co-convergent to x.

Since f is totally $\hat{\alpha}g$ continuous , then for any $V \in O(Y)$ containing f(x), there exists a $U \in\hat{\alpha}gCO(X)$ containing x such that $f(U)\subset V$.

Since Λ is $\hat{\alpha}g$ co-convergent to x, there exists a $B \in\Lambda$ such that $B\subset U$. This implies $f(B)\subset V$.

Hence the filter base $f(\Lambda)$ converges to f(x).

4. Covering properties

Definition 41. A space (X,τ) is said to be $\hat{\alpha}gT_2$ if for any two distinct points x and y of X, there exists disjoint $\hat{\alpha}g$ open sets U and V such that $x \in U$ and $y \in V$.

Theorem 4.2. If arbitrary intersection of $\hat{\alpha}g$ closed sets is $\hat{\alpha}g$ closed in a topological space X, then X is $\hat{\alpha}gT_2$ if and only if for any two distinct points x and y of X, there exists a $\hat{\alpha}g$ neighbourhood N_y of y such that $x\notin \hat{\alpha}gclN_y$.

Proof: Let X be $\hat{\alpha}gT_2$. Let x and y be distinct points of X. Then there exists $\hat{\alpha}g$ open sets U and V such that $x\in U,y\in V$ and $U\cap V=\phi$.

But $U\cap V=\phi$ implies $V\subset X\text{-}U$

So $y \in V \subset X\text{-}U$.Put $X\text{-}U=N_y$, We have $\hat{\alpha}gclN_y=\hat{\alpha}gcl(X\text{-}U)= X\text{-}U=N_y$, as X-U is $\hat{\alpha}$g closed. N_y is a $\hat{\alpha}$g neighbourhood of y such that $x \notin \hat{\alpha}gclN_y$.

Conversely, let X be a topological space such that for any two distinct points x and y of X,there exists $\hat{\alpha}$g neighbourhood N_y of y such that $x \notin \hat{\alpha}gclN_y$.

$\hat{\alpha}gclN_y$ is also a $\hat{\alpha}$g neighbourhood of y. Since $\hat{\alpha}gclN_y$ $\hat{\alpha}$g closed, X- $\hat{\alpha}gclN_y$ is $\hat{\alpha}$g open. $x \notin \hat{\alpha}gclN_y$ implies $x \in X\text{-}\hat{\alpha}gclN_y$.

As N_y is a $\hat{\alpha}$g neighbourhood of y, there exists a $\hat{\alpha}$g open set U such that $y \in U$ and$(X\text{-}\hat{\alpha}gclN_y) \cap U=\boldsymbol{\phi}$.

Hence X is $\hat{\alpha}gT_2$.

Theorem 4.3. If arbitrary intersection of $\hat{\alpha}$g closed sets is $\hat{\alpha}$g closed in a topological space X, then X is $\hat{\alpha}gT_2$ if for any two distinct points x and y of X, there exists a $\hat{\alpha}$g open sets U and V such that $x \in U$, $y \in V$ and $\hat{\alpha}gclU \cap \hat{\alpha}gclV=\boldsymbol{\phi}$.

Proof: Let X be a topological space. Let x and y be distinct points of X.

Then there exists $\hat{\alpha}$g open sets U and V such that $x \in U$, $y \in V$ and $\hat{\alpha}gclU \cap \hat{\alpha}gclV=\boldsymbol{\phi}$.

V is a $\hat{\alpha}$g neighbourhood of y such that $x \notin \hat{\alpha}gclV$, as $x \in \hat{\alpha}gclU$. Hence by the above theorem X is $\hat{\alpha}gT_2$.

Lemma 4.4. Let arbitrary intersection of $\hat{\alpha}$g closed sets be $\hat{\alpha}$g closed in a topological space X and Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a totally $\hat{\alpha}$g continuous injective function. If Y is T_o, then X is $\hat{\alpha}gT_2$.

Proof: Let x and y be any pair of distinct points of X. Then $f(a)\neq f(b)$. Since Y is T_o,there exists an open sets U containing f(x) but not f(y).Then $x \in f^{-1}(U)$and $y \notin f^{-1}(U)$. As f is totally $\hat{\alpha}$g continuous, $f^{-1}(U)$ is $\hat{\alpha}$g clopen in X. Also $x \in f^{-1}(U)$ and $y \in x - f^{-1}(U)$. By the above theorem, X is $\hat{\alpha}gT_2$.

Definition 4.5. A space X is said to be $\hat{\alpha}$g compact if every $\hat{\alpha}$g open cover of X has a finite subcover.

Definition 4.6. A subset A of a space X is said to be $\hat{\alpha}$gcocompact relative to X if every cover of A by $\hat{\alpha}$g clopen sets of X has a finite subcover.

Definition 4.7. A subset A of a space X is said to be $\hat{\alpha}$g cocompact if the subspace A is $\hat{\alpha}$g cocompact.

Theorem 4.8. If arbitrary union of $\hat{\alpha}$g clopen sets is $\hat{\alpha}$g clopen for a space X and a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is totally $\hat{\alpha}$g continuous and A is $\hat{\alpha}$g cocompact relative to X, then f(A) is compact in Y.

Proof: Let $\{H_\alpha:\alpha \in I\}$be any cover of f(A) by open sets of the subspace f(A).For each $\alpha \in I$, there exists an open set A_α of Y such that $H_\alpha=A_\alpha \cap f(A)$.

For each $x \in A$, there exists $\alpha_x \in I$ such that $f(x) \in A_{\alpha x}$ and there exists $U_x \in \hat{\alpha}gCO(X)$ containing x such that $f(U_x) \subset A_{\alpha x}$.

Since the family $\{U_x :x \in A\}$ is a cover of A by $\hat{\alpha}$g clopen sets of X, there exists a finite subset $A_o$of A such that $A \subset \cup\{ U_x :x \in A_o\}$.

Therefore we obtain $f(A) \subset \cup\{f(U_x):x \in A_o\}$ which is a subsets of $\{ A_{\alpha x} :x \in A_o\}$.

Thus $f(A)=\cup\{A_{\alpha x} \cap f(A) : x \in A_o\}=\cup\{H_{\alpha x} : x \in A_o\}$.
Hence f(A) is compact.

Corollary 4.9. If arbitrary union of $\hat{\alpha}$g clopen sets is $\hat{\alpha}$g clopen in topological space X and if $f:(X,\tau)\rightarrow(Y,\sigma)$ is totally $\hat{\alpha}$g continuous surjective function and X is $\hat{\alpha}$g cocompact, then Y is compact.
Proof: Follows from the above theorem.

Definition 4.10. A space X is said to be
Countably $\hat{\alpha}$g cocompact if every $\hat{\alpha}$g clopen countable cover of X has a finite subcover.
i) $\hat{\alpha}$g co-Lindelof if every $\hat{\alpha}$g clopen cover of X has a countable subcover.

Theorem 4.11. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a totally $\hat{\alpha}$g continuous surjective function. Then the following statements hold:
i) If X is $\hat{\alpha}$g co-Lindelof, then Y is Lindelof
ii) If X is countably $\hat{\alpha}$g cocompact,, then Y is countably compact.

Proof:
i) Let$\{V_\alpha:\alpha \in I\}$ be an open cover of Y. Since f is totally $\hat{\alpha}$g continuous , then $\{f^{-1}(V_{\alpha)}:\alpha \in I\}$ is a $\hat{\alpha}$g clopen cover of X. Since X is $\hat{\alpha}$g co-Lindelof, there exists a countable subset I_o of I such that $X=\cup\{f^{-1}(V_{\alpha)}:\alpha \in I_o\}$.
Then $Y=\cup\{V_\alpha:\alpha \in I_o\}$ and hence Y is Lindelof.
ii) Similar to (i).

Definition 4.12. A space X is said to be
i) $\hat{\alpha}$gcoT_1, if for each pair of distinct points x and y of X,there exist $\hat{\alpha}$g clopen sets U and V containing x and y respectively such that $y\notin U$ and $x\notin V$;
ii) $\hat{\alpha}$gcoT_2, if for each pair of distinct points x and y of X,there exist disjoint $\hat{\alpha}$g clopen sets U and V in X such that $x\in U$ and $y\in V$.

Theorem 4.13. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a totally $\hat{\alpha}$g continuous injective function and Y is T_1, then X is $\hat{\alpha}$gcoT_1.
Proof: Suppose Y is T_1, For any distinct points x and y in X, there exists $V,W \in O(Y)$ such that $f(x) \in V, f(y)\notin V$ and $f(y) \in W$, $f(x)\notin W$. Since f is totally $\hat{\alpha}$g continuous, $f^{-1}(V)$and $f^{-1}(W)$ are $\hat{\alpha}$g clopen subsets of (X,τ) such that $x \in f^{-1}(V), y\notin f^{-1}(V)$ and $y\in f^{-1}(W)$, $x \notin f^{-1}(W)$. This shows X is $\hat{\alpha}$gcoT_1.

Theorem 4.14. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a totally $\hat{\alpha}$g injective function and Y is T_2, then X is $\hat{\alpha}$gcoT_2.
Proof: For any pair of distinct points x and y in X, there exist disjoint open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$.
Since f is totally $\hat{\alpha}$g continuous,$f^{-1}(U)$ and $f^{-1}(V)$ are $\hat{\alpha}$g clopen in X containing x and y respectively.
Therefore $f^{-1}(U) \cap f^{-1}(V)=\boldsymbol{\phi}$ because $U\cap V=\boldsymbol{\phi}$. This shows X is $\hat{\alpha}$gcoT_2.

Definition 4.15. A space X is called $\hat{\alpha}$g coregular if for each $\hat{\alpha}$g clopen set F and each point $x\notin F$, there exists disjoint open sets U and V such that $F\subset U$ and $x \in V$.

Definition 4.16. A space X is said to be $\hat{\alpha}$g conormal if for any pair of distinct $\hat{\alpha}$g clopen sets F_1 and F_2, there exists disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 4.17. If f is totally $\hat{\alpha}$g continuous injective open function from a $\hat{\alpha}$g coregular space X onto a space Y, then Y is regular.
Proof: Let F be a closed set of Y and $y \notin F$.
Take y=f(x). Since f is totally $\hat{\alpha}$g continuous, $f^{-1}(F)$ is a $\hat{\alpha}$gclopen set.
Take G= $f^{-1}(F)$. We have $x \notin G$. Since X is $\hat{\alpha}$g coregular, there exists disjoint open sets U and V such that $G \subset U$ and $x \in V$.
We obtain that $F=f(G) \subset f(U)$ and $y= f(x) \in f(V)$ such that f(U) and f(V) are disjoint open sets. Hence Y is regular.

Theorem 4.18. If f is totally $\hat{\alpha}$g continuous injective open function from a $\hat{\alpha}$g conormal space X onto a space Y, then Y is normal.
Proof: Similar to the above proof.

Definition 4.19. For a function $f:(X,\tau) \rightarrow (Y,\sigma)$, the subset $\{(x,f(x)):x \in X\} \subset X \times Y$ is called the graph of f and is denoted by G(f).

Definition 4.20. A graph G(f) of a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be strongly $\hat{\alpha}$g co-closed if for each $(x,y) \in (X \times Y)$- G(f), there exist $U \in \hat{\alpha}gCO(X)$ containing x and $V \in O(Y)$ containing y such that $(U \times V) \cap G(f)=\boldsymbol{\phi}$.

Lemma 4.21. A graph G(f) of a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is strongly $\hat{\alpha}$g co-closed in $X \times Y$ if and only if for each $(x,y) \in (X \times Y)$- G(f), there exist $U \in \hat{\alpha}gCO(X)$ containing x and $V \in O(Y)$ containing y such that $f(U) \cap V=\boldsymbol{\phi}$.
Proof: Let G(f) be strongly $\hat{\alpha}$g- co-closed.
Let $(x,y) \in (X \times Y)$- G(f).Then there exist $\hat{\alpha}$g clopen set U containing x and $V \in O(Y)$ containing y such that $(U \cap V) \times G(f)=\boldsymbol{\phi}$. That is $V \cap f(X)=\boldsymbol{\phi}$.
That is $V \cap f(U)=\boldsymbol{\phi}$.
Conversely, let for each $(x,y) \in (X \times Y)$- G(f), there exist $U \in \hat{\alpha}gCO(X)$ containing x and $V \in O(Y)$ containing y such that $f(U) \cap V=\boldsymbol{\phi}$.
Let $y \in V$. $y \in Y\text{-}f(X)$. That is $y \neq f(x)$ for any x.That is $V \cap f(X)=\boldsymbol{\phi}$. This implies $(U \times V) \cap (X \times f(X))=\boldsymbol{\phi}$. That is$(U \times V) \cap G(f)=\boldsymbol{\phi}$.

Theorem 4.22. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ has a strongly $\hat{\alpha}$g co-closed graph G(f). If f is injective, then X is $\hat{\alpha}gcoT_2$.
Proof: Let x and y be any two distinct points of X.
Then, we have$(x,f(y)) \in (X \times Y)$- G(f).
By the above lemma, there exist $\hat{\alpha}$g clopen set U of X and $V \in O(Y)$ such that $(x,f(y)) \in (U \times V)$ and $f(U) \cap V=\boldsymbol{\phi}$.
Hence $U \cap f^{-1}(V)=\boldsymbol{\phi}$, $x \in U$ and $y \in f^{-1}(V)$.
Hence X is $\hat{\alpha}gcoT_2$.

Theorem 4.23. If arbitrary union of $\hat{\alpha}$g clopen sets is $\hat{\alpha}$g clopen in a space X and $f:(X,\tau) \rightarrow (Y,\sigma)$ is totally $\hat{\alpha}$g continuous and Y is T_2, then G(f) is strongly $\hat{\alpha}$g co-closed in the product space $X \times Y$.

Proof: Let $(x,y) \in (X\times Y)$- G(f). Then $y\neq f(x)$ and there exist open sets V_1 and V_2 such that $f(x)\in V_1, y\in V_2$ and $V_1\cap V_2=\boldsymbol{\phi}$.

From hypothesis, there exists $U\in\hat{\alpha}gCO(X,x)$ such that $f(U)\subset V_1$.

Therefore, we obtain $f(U)\cap V_2=\boldsymbol{\phi}$. So G(f) is strongly $\hat{\alpha}$g co-closed graph.

Definition 4.24. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be :

i) Totally $\hat{\alpha}$g irresolute if the preimage of $\hat{\alpha}$g clopen subset of Y is $\hat{\alpha}$g clopen in X.

ii) Totally pre $\hat{\alpha}$g clopen if the image of every $\hat{\alpha}$g clopen subset of X is $\hat{\alpha}$g clopen in Y

Theorem 4.25. Let $f:(X,\tau)\to(Y,\sigma)$ be surjective and totally $\hat{\alpha}$g irresolute and totally pre $\hat{\alpha}$g clopen and $g:(Y,\sigma)\to(Z,\eta)$ be any function. Then gof: $(X,\tau)\to(Z,\eta)$ is totally $\hat{\alpha}$g continuous if and only if g is totally $\hat{\alpha}$g continuous.

Proof: Let g be totally $\hat{\alpha}$g continuous. Let V be open in Z. $g^{-1}(V)$ $\hat{\alpha}$g clopen in Y.$f^{-1}((g^{-1}(V))$ is $\hat{\alpha}$g clopen in X.

Hence gof is totally $\hat{\alpha}$g continuous.

Conversely, let gof: $(X,\tau)\to(Z,\eta)$ be totally $\hat{\alpha}$g continuous. Let V be open in Z. Then $(gof)^{-1}(V)$ is $\hat{\alpha}$g clopen in X. That is $f^{-1}((g^{-1}(V))$ is $\hat{\alpha}$g clopen.

Since f is totally pre $\hat{\alpha}$g clopen, $f(f^{-1}((g^{-1}(V)))$ is $\hat{\alpha}$g clopen in Y.

That is $g^{-1}(V)$ is clopen in Y. Hence g is totally $\hat{\alpha}$g continuous.

Theorem 4.26. Let $f:(X,\tau)\to(Y,\sigma)$ has a strongly $\hat{\alpha}$gco-closed graph G(f). If f is surjective totally pre $\hat{\alpha}$g clopen function,then Y is $\hat{\alpha}$g T_2 space

Proof: Let y_1 and y_2 be distinct points of Y. Since f is surjective $f(x)=y_1$, for some $x\in X$. $(x,y_2) \in (X\times Y)$- G(f). There exist $U\in\hat{\alpha}gCO(X)$ and $V\in O(Y)$ such that $(x,y_2)\in U\times V$ and $(U\times V)\cap G(f)=\boldsymbol{\phi}$. Then we have $f(U) \cap V= \boldsymbol{\phi}$

Since f is totally pre $\hat{\alpha}$g clopen such that $f(x)=y_1\in f(U)$. Hence Y is $\hat{\alpha}$g T_2.

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