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Semi-Connectedness and Pre-Connectedness in Biclosure Space

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Abstract. Our aim, in the present paper, is to introduce two new types of connectedness in biclosure space namely semi-connectedness in biclosure space and pre-connectedness in biclosure space. We also investigate the fundamental properties of these new types of connectedness by theorems.

Keywords: Closure space, connectedness in closure space, biclosure space, connectedness in biclosure space, semi-connectedness in biclosure space, pre- connectedness in biclosure space.

AMS Mathematics Subject Classification (2010): 54A40

1. Introduction

In 1963, bitopological space was introduced by Kelly [15] as triples $(X, \mathcal{J}_1, \mathcal{J}_2)$ where X is non empty set and $\mathcal{J}_1$ and $\mathcal{J}_2$ are topologies defined on X. After that, a larger number of papers have been written to generalize the topological concept to a bitopological spaces, by Aarts and MrŠevi´ [1], Deak [12] and Dvalishvili [14]. The concept of biclosure space was introduced and studied by Boonpok and Khampakdee [4] in 2010.

In 1966, Levine [18] introduced semi-open set and semi-continuous map in a topological space. The concepts of semi-open set and semi-continuous map in closure space were introduced by Khampakdee [17]. The concept of pre-open set was introduced by Mashhour et.al. [19] in 1982. The concept of pre open set in closure space was introduced by Rao, Gowri and Swaminathan [8], and the concept semi-open sets and pre-open sets was further generalized in biclosure space by Rao and Gowri [7] in 2006. Connectedness, semi-connectedness and pre-connectedness in closure space were introduced by our self [22, 24]. We have [23] generalized the concept of connectedness in biclosure space. Here we are using closure space in place of closure space for convenience. In this paper, we introduce semi-connectedness and pre-connectedness in biclosure space and study some of their fundamental properties.

2. Preliminaries

Definition 2.1. [3**]** Two maps u_1 and u_2 from power set of X to itself are called biclosure operators for X if they satisfies the following properties:

(1) $u_1\phi = \phi$, $u_2\phi = \phi$;

(2) $A \subseteq u_1A$, $A \subseteq u_2A$, for all $A \subseteq X$;

(3)$u_1(A \cup B) = u_1A \cup u_1B$, $u_2(A \cup B) = u_2A \cup u_2B$, for all $A \subseteq X$.

A structure (X, u_1, u_2) is called a biclosure space.

Definition 2.2. [13**]** A subset *A* in a biclosure space (X, k_1, k_2) is said to be

1. Semi open if $A \subseteq k_i(\text{int}_{k_i}(A))$, for all i = 1, 2.
2. Semi closed if $\text{int}_{k_i}(k_i(A)) \subseteq A$, for all i = 1, 2.
3. Pre open if $A \subseteq \text{int}_{k_i}(k_i(A))$, for all i = 1, 2.
4. Pre closed if $k_i(\text{int}_{k_i}(A)) \subseteq A$, for all i = 1, 2.

Definition 2.3. [16**]** Let (X, u_1, u_2) and (Y, v_1, v_2) are biclosure spaces and let $i \in \{1, 2\}$.Then a map $f : (X, u_1, u_2) \to (Y, v_1, v_2)$ is called:

(i) i-open (respectively, i-closed) if the map $f: (X, u_i) \to (Y, v_i)$ is open (respectively, closed).

(ii) Open (respectively, closed) if f is i-open (respectively, i-closed) for all $i \in \{1, 2\}$.

(iii) i-continuous if the map $f: (X, u_i) \to (Y, v_i)$ is continuous for all $i \in \{1, 2\}$.

(iv) continuous if f is i-continuous, for all $i \in \{1, 2\}$.

Definition 2.4. [16**]** Let (X, u_1, u_2) and (Y, v_1, v_2) are biclosure spaces. A map $f : (X, u_1, u_2) \to (Y, v_1, v_2)$ is called semi-continuous if $f^{-1}(G)$ is a semi-open subset of (X, u_1, u_2) for every open subset G of (Y, v_1, v_2).Clearly, if f is continuous, then f is semi-continuous. The converse need not be true.

Definition 2.5. [16**]** Let (X, u_1, u_2) and (Y, v_1, v_2) be biclosure spaces. A map $f : (X, u_1, u_2) \to (Y, v_1, v_2)$ is called pre-continuous if $f^{-1}(G)$ is a pre-open subset of (X, u_1, u_2) for every open subset G of (Y, v_1, v_2).

Definition 2.6. [24] A closure space (X, u) is said to be semi-connected if and only if any semi-continuous map f from X to the discrete space {0, 1} is constant. A subset A in a closure space (X, u) is said to be semi-connected if A with the subspace topology is semi-connected closure space.

Definition 2.7. [24] A closure space (X, u) is called pre-connected if and only if there exists a pre-continuous map f from X to the discrete space {0, 1} is constant. A subset A in a closure space (X, u) is said to be pre-connected if A with the subspace topology is pre-connected closure space.

3. Semi-connectedness in biclosure space

Definition 3.1. A biclosure space (X, u_1, u_2) is called semi-connected if there exists a semi-continuous mapping f from X to discrete space {0, 1} is constant.

Example 3.2. Consider a biclosure space (X, u_1, u_2), where X = {a, b, c}, and u_1 and u_2 are two closure operators which are defined by

u_1: P(X) $\rightarrow$ P(X) such that

$u_1\{b\}=u_1\{c\}=u_1\{b, c\}=\{b, c\}$,

$u_1\{a\}=u_1\{a, b\}=u_1\{a, c\}=u_1\{X\}=X$, $u_1\{\phi\}=\phi$.

Then underlying topology for (X, u_1) is t (u_1) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X,ϕ}

Hence (X, u_1) is a closure space.

Open sets = {{a}, {b}, {c}, {a, b}, {a, c}, X, ϕ}.

SO sets of (X, u_1) = {{a}, {b}, {c}, {a, b}, {a, c}, X, ϕ}.

And u_2: P(X) $\rightarrow$ P(X) such that

$u_2\{a\}=\{a, b\}$, $u_2\{b\}=\{b, c\}$, $u_2\{c\}= \{c, a\}$,

$u_2\{a, b\}= u_2\{b, c\}= u_2\{a, c\}=X= u_2\{X\}$, $u_2\{\phi\}=\phi$.

Then underlying topology for (X, u_2) is t (u_2) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X, ϕ}

Hence (X, u_2) is a closure space.

Open sets = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X, ϕ},

SO sets of (X, u_2) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X, ϕ},

Semi-open sets of biclosure space (X, u_1, u_2) are {{a}, {b}, {c}, {a, b}, {a, c}, X, ϕ}.

Let f: X $\rightarrow$ {0, 1} is a semi-continuous mapping such that

$f^{-1}\{1\}$={a}={b}={c}={a, b}={b, c}={a, c}=X.

$f^{-1}\{0\} = \phi$, i. e. f{a}=f{b}=f{c}=f{a, b}= f{b, c}=f{a, c}=f{X}=1,f{ϕ}=0.

Here semi-continuous mapping f is constant.

Hence (X, u_1, u_2) is a semi-connected biclosure space.

Example 3.3. Consider a biclosure space (X, u_1, u_2), where X = {a, b, c, d}, and u_1 and u_2 are two closure operators which are defined by u_1: P(X) $\rightarrow$ P(X) such that

$u_1\{a\}=\{a, b\}$,$u_1\{b\}=\{a, b\}$, $u_1\{c\}=\{b, c\}$,

$u_1\{d\}=\{c, d\}$, $u_1\{X\}=X$, $u_1\{\phi\}=\phi$.

For all subsets A contained in X, let

$$u_1(A)=\begin{cases}\phi,\text{if } A=\phi;\\ \cup\{u_1(a): a\in A\}, otherwise.\end{cases}$$

Then underlying topology for (X, u_1) is t (u_1) = {{a}, {b}, {c}, {d}, X, ϕ}

Hence (X, u_1) is a closure space.

Open sets = {{a}, {b}, {c}, {d}, X, ϕ}.

SO sets of (X, u_1) = {{a}, {b}, {c}, {d}, X, ϕ}.

And u_2: P(X) $\rightarrow$ P(X) such that

u_2{a} = {a, b, c}, u_2{b} ={b, c, d}, u_2{c}= {c, a, d},

u_2{d} = {d, a, b}, u_2{X}= X, u_2{ϕ}=ϕ.

For all subset A contained in X, let

$$u_2(A)=\begin{cases}\phi,\text{if } A=\phi;\\ \cup\{u_2(a): a\in A\}, otherwise.\end{cases}$$

Then underlying topology for (X, u_2) is t (u_2) = {{a}, {b}, {c}, {d}, X,ϕ}

Hence (X, u_2) is a closure space.

Open sets = {{a}, {b}, {c}, {d}, X,ϕ},

SO sets of(X, u_2) = {{a}, {b}, {c}, {d}, X,ϕ},

Semi-open sets of biclosure space (X, u_1, u_2) are {{a}, {b}, {c}, {d}, X,ϕ}.

Let f: X $\rightarrow$ {0, 1} is a semi-continuous mapping such that

f^{-1}{1}={a}={b}={c}={d}={a, b}={b, c}={c, d}={d, a}=

{a, b, c}={b, c, d}={c, d, a}= {d, a, b} = X,

f^{-1}{0} = ϕ.

Here semi-continuous mapping f is constant.

Hence (X, u_1, u_2) is a semi-connected biclosure space.

Definition 3.4. A biclosure space (X, u_1, u_2) is called semi-disconnected if there exists a semi-continuous mapping f from X to discrete space {0, 1} is surjective.

Theorem 3.5. A biclosure space (X, u_1, u_2) is semi connected if and only if every semi continuous mapping f from X into a discrete space Y= {0, 1} with at least two points is constant.

Proof: Necessary: Let (X, u_1, u_2) is a semi-connected biclosure space. Then there exists a semi continuous mapping f from the X into the discrete space Y= {0, 1}, for each $y\in Y$,

$f^{-1}\{y\} = \phi$ or X. If $f^{-1}\{y\} = \phi$ for all $y \in Y$, then f ceases to be a mapping. Therefore $f^{-1}\{y_0\} = X$ for a unique $y_0 \in Y$. This implies that $f(X) = \{y_0\}$ and hence f is a constant mapping.

Sufficiency: Let every semi continuous mapping f from X into a discrete space Y= {0, 1} is constant. Suppose U is a semi open set in a biclosure space (X, u_1, u_2).If $U \neq \phi$, we will show that $U = X$. Otherwise, choose two fixed points y_1 and y_2 in Y. Define $f: X \to Y$ by

$$f(x) = \begin{cases} y_1, & if x \in U; \\ y_2, & otherwise \end{cases}$$

Then for any open set V in Y,

$$f^{-1}(V) = \begin{cases} U, & \text{if V contains } y_1 \text{ only}, \\ X/U, & \text{if V contains } y_2 \text{ only}, \\ X, & \text{if V contains both } y_1 \text{ and } y_2, \\ \phi, & otherwise. \end{cases}$$

In all the cases $f^{-1}(V)$ is semi open in X. Hence f is not constant semi-continuous mapping. This is a contradiction to our assumption. This proves that the only semi–open subset of X is ϕ and X. Hence (X, u_1, u_2) is semi-connected biclosure space.

Theorem 3.6. The following assertions are equivalent:

1. (X, u_1, u_2) is semi-connected biclosure space.
2. The only subsets of X both semi-open and semi-closed are ϕ and X.
3. No semi-continuous mapping $f: X \to \{0, 1\}$ is surjective.

Proof: [1] ⇒ [2]

Let (X, u_1, u_2) is semi-connected biclosure space. Suppose $G \subset X$ is both semi-open and semi-closed such that $G \neq \phi$ and $G \neq X$, then $X = G \cup G^C$, Where G^C is complement of G in X. Hence Semi-continuous mapping $f: X \to \{0, 1\}$ is not constant i. e. (X, u_1, u_2) is not semi-connected biclosure space, which is a contradiction to our initial assumption. Hence the only subsets of X both semi-open and semi-closed are ϕ and X.

[2] ⇒ [3]

Suppose the only subsets of X both semi-open and semi-closed are ϕ and X. Let $f: X \to \{0, 1\}$ is a semi-continuous surjection. Then $f^{-1}\{0\} \neq \phi$ and $f^{-1}\{0\} \neq X$. But {0} is both open and closed in {0, 1}.Hence $f^{-1}\{0\}$ is semi-open and semi-closed in X. This is a contradiction to our assumption. Hence no semi-continuous mapping $f: X \to \{0, 1\}$ is surjective.

[3]⇒ [1]
Let no semi-continuous mapping $f: X \to \{0, 1\}$ is surjective. If possible let biclosure space
(X, u_1, u_2) is not semi-connected biclosure space. So $X = A \cup B$, A and B are also semi closed sets. Then

$$\chi_A(x) = \begin{cases} 1, \text{if } x \in A, \\ 0, \text{if } x \notin A. \end{cases}$$

is semi-continuous surjection which is a contradiction to our initial assumption. Hence biclosure space (X, u_1, u_2) is semi-connected biclosure space.

Theorem 3.7. The semi-continuous image of a semi-connected biclosure space is semi-connected biclosure space.
Proof: Let biclosure space (X, u_1, u_2) is a semi-connected biclosure space. Consider a semi-continuous mapping $f: X \to f(X)$ is surjective. If f(X) is not semi-connected biclosure space, there would be a semi-continuous surjection $g: f(X) \to \{0, 1\}$ so that the composite function g o $f: X \to \{0, 1\}$ would also be a semi-continuous surjection, which is a contradiction to semi-connectedness of biclosure space (X, u_1, u_2). Hence f(X) is a semi-connected biclosure space.

4. Pre-Connectedness in Biclosure Space:
Definition 4.1. A biclosure space (X, u_1, u_2) is called pre-connected if there exists a pre-continuous mapping f from X to discrete space {0, 1} is constant.

Example 4.2. Consider a biclosure space (X, u_1, u_2), where X = {a, b, c}, and u_1 and u_2 are two closure operators which are defined by
u_1: P(X) $\to$ P(X) such that
$u_1\{b\} = u_1\{c\} = u_1\{b, c\} = \{b, c\}$,
$u_1\{a\} = u_1\{a, b\} = u_1\{a, c\} = u_1\{X\} = X$, $u_1\{\phi\} = \phi$.
Then underlying topology for (X, u_1) is $t(u_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X, \phi\}$

Hence (X, u_1) is a closure space.

Open sets = {{a}, {b}, {c}, {a, b}, {a, c}, X, ϕ}.

PO sets of (X, u_1) = {{a}, {a, b}, {a, c}, X, ϕ}.

And $u_2 : P(X) \to P(X)$ such that

$u_2\{a\}=\{a, b\}$, $u_2\{b\}=\{b, c\}$, $u_2\{c\}= \{c, a\}$,

$u_2\{a, b\}= u_2\{b, c\}= u_2\{a, c\}=X= u_2\{X\}$, $u_2\{\phi\}=\phi$.

Then underlying topology for (X, u_2) is t (u_2) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X, ϕ}

Hence (X,u_2) is a closure space.

Open sets = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X, ϕ},

PO sets of (X, u_2) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X, ϕ},

Pre-open sets of biclosure space (X, u_1, u_2) are {{a}, {a, b}, {a, c}, X, ϕ}.

Let f: X $\to$ {0, 1} is a pre-continuous mapping such that

$f^{-1}\{1\}=\{a\}=\{b\}=\{c\}=\{a, b\}=\{b, c\}=\{a, c\}=X$,

$f^{-1}\{0\} =\phi$.

Hence pre-continuous mapping f is constant.

Hence (X, u_1, u_2) is a pre-connected biclosure space.

Example 4.3. Consider a biclosure space (X, u_1, u_2), where X = {a, b, c, d}, and u_1 and u_2 are two closure operators which are defined by

u_1: P(X) $\to$ P(X) such that

$u_1\{a\}=\{a, b\}$,$u_1\{b\}=\{a, b\}$, $u_1\{c\}=\{b, c\}$,

$u_1\{d\}=\{c, d\}$, $u_1\{X\}=X$, $u_1\{\phi\}=\phi$.

For all subsets A contained in X, let $u_1(A)= \begin{cases} \phi, \text{if } A=\phi; \\ \cup\{u_1(a) : a\in A\}, \textit{otherwise.} \end{cases}$

Then underlying topology for (X, u_1) is t (u_1) = {{a}, {b}, {c}, {d}, X, ϕ}

Hence (X, u_1) is a closure space.

Open sets = {{a}, {b}, {c}, {d}, X, ϕ}.

PO sets of (X, u_1) = {{a}, {b}, {c}, {d}, X, ϕ}.

And u_2: P(X) $\to$ P(X) such that

$u_2\{a\}=\{a, b, c\}$, $u_2\{b\}=\{b, c, d\}$, $u_2\{c\}= \{c, a, d\}$,

$u_2\{d\}= \{d, a, b\}$, $u_2\{X\}=X$, $u_2\{\phi\}=\phi$.

For all subsets A contained in X, let

$$u_2(A)=\begin{cases}\phi, \text{if } A=\phi;\\ \cup\{u_2(a): a\in A\}, otherwise.\end{cases}$$

Then underlying topology for (X, u_2) is t (u_2) = {{a}, {b}, {c}, {d}, X, ϕ}

Hence (X, u_2) is a closure space.

Open sets are {{a}, {b}, {c}, {d}, X, ϕ},

PO sets of (X, u_2) = {{a}, {b}, {c}, {d}, X, ϕ},

Pre-open sets of biclosure space (X, u_1, u_2) are {{a}, {b}, {c}, {d}, X, ϕ}.

Let $f: X \to \{0, 1\}$ is a pre-continuous mapping such that

$f^{-1}\{1\}$={a}={b}={c}={d}={a, b}={b, c}={c, d}={d, a}={a, b, c}=

{b, c, d}={c, d, a}= {d, a, b}=X.

$f^{-1}\{0\} = \phi$,

Hence pre-continuous mapping f is constant.

Then (X, u_1, u_2) is a pre-connected biclosure space.

Definition 4.4. A biclosure space (X, u_1, u_2) is called pre-disconnected biclosure space if and only if any pre-continuous map f from X to the discrete space {0, 1} is surjective.

Theorem 4.5. If $\{A_i : i\in \wedge\}$ is a family of pre-connected biclosure subsets of Pre-connected biclosure space (X, u_1, u_2), then $\cup A_i$ is also a pre-connected biclosure subset of (X, u_1, u_2), where Λ is any index set.

Proof: Each $A_i, i\in \wedge$ is a pre-connected biclosure subset of pre-connected biclosure space (X, u_1, u_2) so there exists pre-continuous mapping $f_i: A_i \to \{0, 1\}$ is constant. Let a pre-continuous mapping f: $\cup A_i \to \{0, 1\}$ is not constant, $f^{-1}\{1\} \neq A_i$ which is a contradiction to each A_i is pre-connected subsets of (X, u_1, u_2), i.e. pre-continuous mapping f is constant. Hence $\cup A_i$ is pre-connected biclosure space.

Theorem 4.6. Let (X, u_1, u_2) and (Y, v_1, v_2) are two biclosure spaces and $f: X \to Y$ is a bijection. Then

1) f is pre-continuous mapping and X is a pre-connected biclosure space then Y is connected biclosure space.
2) f is continuous mapping and X is pre-connected biclosure space then Y is a connected biclosure space.
3) f is pre-open mapping and Y is pre-connected biclosure space then X is connected biclosure space.
4) f is open mapping and X is connected biclosure space then Y is pre-connected biclosure space.

Proof: 1. Let (Y, v_1, v_2) is a biclosure space and X is a pre-connected biclosure space then there exists a pre-continuous mapping $fog: X \to \{0, 1\}$ is constant. Consider a Pre-continuous mapping $g: Y \to \{0, 1\}$, given that $f: X \to Y$ is pre- continuous mapping and f is bijection so that g is also a constant mapping. Hence Y is connected biclosure space.

2. Given that X is a pre-connected biclosure space, i.e. $g: X \to \{0, 1\}$ pre-continuous mapping is constant. $f^{-1}: Y \to X$ is continuous bijection, so that $f^{-1}og: Y \to \{0, 1\}$ continuous mapping is constant. Hence Y is connected biclosure space.

3. Given that Y is pre-connected biclosure space i.e. $g: Y \to \{0, 1\}$ pre-continuous mapping is constant. Since $f: X \to Y$ is pre-open and bijection mapping so that continuous mapping $fog: X \to \{0, 1\}$ is constant. Hence X is connected biclosure space.

4. Given that X is connected biclosure space i.e. a continuous mapping $g: X \to \{0, 1\}$ is constant and $f^{-1}: Y \to X$ is open mapping so that it is a pre-open mapping then $f^{-1}og: Y \to \{0, 1\}$ is a pre-continuous constant mapping. Hence Y is a pre-connected biclosure space.

Theorem 4.7. A biclosure space (X, u_1, u_2) is pre-disconnected if and only if there exists a pre-continuous map f from X onto a discrete two point space Y = {0, 1}.

Proof: Given that biclosure space (X, u_1, u_2) is pre-disconnected i.e. there exists a pre-continuous map $f: X \to \{0, 1\}$ is not constant and $f^{-1}\{0\} \neq \phi$.If a pre-continuous map $f: X \to \{0, 1\}$ is onto, so that mapping is not constant. Hence (X, u_1, u_2) is pre-disconnected biclosure space.

5. Conclusion

In this paper, the idea of semi-connectedness and pre-connectedness in biclosure space were introduced and studied some of their fundamental properties by theorems.

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