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Almost Contra $\alpha\hat{g}$ Continuous Functions

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Abstract. In this paper, we introduce a new class of function called almost contra $\alpha\hat{g}$ continuous function. Some characterization are obtained and its relationship to connectedness, compactness and $\alpha\hat{g}$ regular graphs are obtained.

***Keywords*:** $\alpha\hat{g}$ closed sets, Contra $\alpha\hat{g}$ continuous, almost contra $\alpha\hat{g}$ continuous.

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1. Introduction

In 1996, Dontchev [5] introduced contra continuous function. Dontchev, Ganster and Reily[6] introduced a new class of function called regular set connected function. Jaffri and Noiri [10] introduced and studied a new form of function called contra pre continuous function. Many researchers have studied on Pre-continuous functions , almost contra pre-continuous functions on pre-topological spaces in [7],[8],[11],[15-18] and strong forms of continuous functions, called super continuous functions are studied in [20-22].In this paper, we introduce and study almost contra $\alpha\hat{g}$ continuous function. Moreover, we obtain basic properties and preservation theorems of almost contra $\alpha\hat{g}$ continuous function and relationship between almost contra $\alpha\hat{g}$ continuity and $\alpha\hat{g}$ regular graphs. Throughout this paper (X,τ) and (Y,σ) denote topological spaces where no separation axioms are assumed unless otherwise stated. They are simply denoted by X and Y.

In a topological space X, the interior of A and the closure of A are respectively denoted by int A and cl A.

2. Preliminaries

Definition 2.1. Let A be a subset of a topological space X. Then A is said to be
1) pre open if $A \subset$ int cl A and pre closed if cl int $A \subset A$[12]
2) regular open if A = int cl A and regular closed if A = cl int A [12]
3) semi open if $A \subset$ cl int A and semi closed if int cl $A \subset A$ [12]
4) α open if $A \subset$ int cl int A and α closed if cl int cl $A \subset A$ [19]

5) β open (semi pre open) if $A \subset \text{cl int cl } A$ and β closed (semi pre closed) if $\text{int cl int } A \subset A$ [2]
6) b open if $A \subset \text{int cl } A \cup \text{cl int } A$ and b closed if $\text{int cl } A \cap \text{cl int } A \subset A$ [1].

Definition 2.2. Let A be a subset of a topological space X. Then A is said to be
1) g closed if $\text{cl } A \subset U$ whenever $A \subset U$ and U is open[13]
2) sg closed if $\text{scl } A \subset U$ whenever $A \subset U$ and U is semi open [4]
3) gs closed if $\text{scl } A \subset U$ whenever $A \subset U$ and U is open [3]
4) w closed if $\text{cl } A \subset U$ whenever $A \subset U$ and U is semi open [24]

5) g^* closed if $\text{cl } A \subset U$ whenever $A \subset U$ and U is g open [12]
6) g^*p closed if $\text{pcl } A \subset U$ whenever $A \subset U$ and U is g open [25]
7) pg closed if $\text{pcl } A \subset U$ whenever $A \subset U$ and U is pre open[14]
8) gp closed if $\text{pcl } A \subset U$ whenever $A \subset U$ and U is open [14]
9) sgb closed if $\text{bcl } A \subset U$ whenever $A \subset U$ and U is semi open [9].

Definition 2.3. Let A be a subset of a topological X. Then A is said to be $\alpha\hat{g}$ closed if $\text{int cl int } A \subset U$ whenever $A \subset U$ and U is open [23].

The complements of the respective closed sets in X are respective open sets in X. The union of two $\alpha\hat{g}$ closed sets need not be $\alpha\hat{g}$ closed.
The intersection of two $\alpha\hat{g}$ closed sets need not be $\alpha\hat{g}$ closed.

Definition 2.4. A function $f : X \to Y$ is said to be
1) almost contra pre continuous if $f^{-1}(V)$ is pre closed in X for every regular open set V of Y.
2) almost contra semi continuous if $f^{-1}(V)$ is semi closed in X for every regular open set V of Y.
3) almost contra g continuous if $f^{-1}(V)$ is g closed in X for every regular open set V of Y.
4) almost contra sg continuous if $f^{-1}(V)$ is sg closed in X for every regular open set V of Y.
5) almost contra gs continuous if $f^{-1}(V)$ is gs closed in X for every regular open set V of Y.
6) almost contra w continuous if $f^{-1}(V)$ is w closed in X for every regular open set V of Y.
7) almost contra g^* continuous if $f^{-1}(V)$ is g^* closed in X for every regular open set V of Y.
8) almost contra g^*p continuous if $f^{-1}(V)$ is g^*p closed in X for every regular open set V of Y.
9) almost contra pg continuous if $f^{-1}(V)$ is pg closed in X for every regular open set V of Y.
10) almost contra gp continuous if $f^{-1}(V)$ is gp closed in X for every regular open set V of Y.
11) almost contra b continuous if $f^{-1}(V)$ is b closed in X for every regular open set V of Y.
12) almost continuous sgb continuous if $f^{-1}(V)$ is sgb closed in X for every regular open set V of Y.

3. Almost Contra $\alpha\hat{g}$ Continuous Functions.

In this section, we define almost contra $\alpha\hat{g}$ continuous function and discuss some of its properties.

Definition 3.1. A function $f : (X,\tau) \to (Y,\sigma)$ is called almost contra $\alpha\hat{g}$ continuous if $f^{-1}(V)$ is $\alpha\hat{g}$ closed in (X,τ) for every regular open set V in (Y,σ)

Example 3.2. Let X = Y = {a,b,c} with τ = {ϕ,{a},{a,b},{a,c},X}
σ = {ϕ,{a},{b},{a,b},Y}.

Define f : (X,τ) → (Y,σ) by f(a) = c, f(b) = b, f(c) = a. Clearly f is almost contra $\alpha\hat{g}$ continuous

Theorem 3.3. If f: X → Y is contra $\alpha\hat{g}$ continuous, then it is almost contra $\alpha\hat{g}$ continuous.

Proof : The proof is obvious, as every regular open set is open set.

The converse of the above theorem need not be true can be seen from the following example.

Example 3.4. Let X= Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X}
σ = {ϕ,{a},{a,b},{a,c},X}. Define f : (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c
f is almost contra $\alpha\hat{g}$ continuous but not contra $\alpha\hat{g}$ continuous as f^{-1}({a,b}) = {a,b} is not $\alpha\hat{g}$ closed in X.

Theorem 3.5.

i) Every almost contra pre continuous function is almost contra $\alpha\hat{g}$ continuous.
ii) Every almost contra semi continuous function is almost contra $\alpha\hat{g}$ continuous.
iii) Every almost contra g continuous function is almost contra $\alpha\hat{g}$ continuous.
iv) Every almost contra sg continuous function is almost contra $\alpha\hat{g}$ continuous.
v) Every almost contra gs continuous function is almost contra $\alpha\hat{g}$ continuous.
vi) Every almost contra *w c*ontinuous function is almost contra $\alpha\hat{g}$ continuous.
vii) Every almost contra g^* continuous function is almost contra $\alpha\hat{g}$ continuous.
viii) Every almost contra g^*p continuous function is almost contra $\alpha\hat{g}$ continuous.
ix) Every almost contra pg continuous function is almost contra $\alpha\hat{g}$ continuous.
x) Every almost contra gp continuous function is almost contra $\alpha\hat{g}$ continuous.
xi) Every almost contra b continuous function is almost contra $\alpha\hat{g}$ continuous .
xii) Every almost contra sgb continuous function is almost contra $\alpha\hat{g}$ continuous.

Proof : The proof directly follows from the definition of almost contra $\alpha\hat{g}$ continuous function.

The converse of the above results need not be true can be seen from the following examples.

Example 3.6. Let X= Y = {a,b,c}, τ = {ϕ,{a},{a,b},X}
σ = {ϕ,{a},{b},a,b},X}. Define f : (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = b
f is almost contra $\alpha\hat{g}$ continuous but not almost contra pre continuous or semi continuous as f^{-1}({b}) = {a,c} is not pre closed or semi closed.

Example 3.7. Let X = Y = {a,b,c}. Let τ and σ be as above.
Define f : (X,τ) → (Y,σ) by f(a) = c, f(b) = b, f(c) = a
f is almost contra $\alpha\hat{g}$ continuous but not almost contra g continuous as f^{-1}({b}) = {b} is not g closed.

Example 3.8. Let X = Y = {a,b,c}. Let τ and σ be as above.
Define f : $(X,\tau) \to (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = b
f is almost contra $\alpha\hat{g}$ continuous but not almost contra sg continuous as $f^{-1}(\{b\}) = \{a,c\}$ is not sg closed.

Example 3.9. Let X = Y = {a,b,c}. Let τ and σ be as above.
Define f : $(X,\tau) \to (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = b
f is almost contra $\alpha\hat{g}$ continuous but not almost contra gs continuous as $f^{-1}(\{b\}) = \{a,c\}$ is not gs closed.

Example 3.10. Let X= Y = {a,b,c}, $\tau = \{\phi,\{a,b\},X\}$
Let σ be as above. Define f : $(X,\tau) \to (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = c
f is almost contra $\alpha\hat{g}$ continuous but not almost contra w continuous as $f^{-1}(\{b\}) = \{a\}$ is not w closed.

Example 3.11. Let X = Y = {a,b,c}. Let τ and σ be as in 3.6. Define f as in 3.9.
f is almost contra $\alpha\hat{g}$ continuous but not almost contra g^* continuous as $f^{-1}(\{b\}) = \{a,c\}$ is not g^* closed.

Example 3.12. Let X= Y = {a,b,c}, Let $\tau = \sigma = \{\phi,\{a\},\{b\},\{a,b\},X\}$
Define f : $(X,\tau) \to (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = c
f is almost contra $\alpha\hat{g}$ continuous but not almost contra g^*p continuous as $f^{-1}(\{b\}) = \{a\}$ is not g^*p closed.

Example 3.13. Let X = Y = {a,b,c}. Let τ and σ be as in previous example. Define f as in the previous example. f is almost contra $\alpha\hat{g}$ continuous but not almost contra pg continuous as $f^{-1}(\{b\}) = \{a\}$ is not pg closed.

Example 3.14. Let X = Y = {a,b,c}. Let τ and σ be as in previous example.
Define f : $(X,\tau) \to (Y,\sigma)$ by f(b) = b, f(b) =a, f(c) =c
f is almost contra $\alpha\hat{g}$ continuous but not almost contra gp continuous as $f^{-1}(\{b\}) = \{a\}$ is not gp closed.

Example 3.15. Let X = Y = {a,b,c}. Let τ and σ be as in 3.6.
Define f : $(X,\tau) \to (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = b
f is almost contra $\alpha\hat{g}$ continuous but not almost contra b continuous as $f^{-1}(\{b\}) = \{a,c\}$ is not b closed.

Example 3.16. Let X = Y = {a,b,c}. Let τ and σ be as in 3.6.
Define f : $(X,\tau) \to (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = b
f is almost contra $\alpha\hat{g}$ continuous but not almost contra sgb continuous as $f^{-1}(\{b\}) = \{a,c\}$ is not sgb closed.

Theorem 3.17. Let arbitrary union of $\alpha\hat{g}$ open sets be $\alpha\hat{g}$ open in X.
The following are equivalent for a function f : X $\to$ Y.
1) f is almost contra $\alpha\hat{g}$ continuous

2) For every closed set F of Y, $f^{-1}(F)$ is $\alpha\hat{g}$ open in X.
3) For each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists $\alpha\hat{g}$ open set U containing x in X such that $f(U) \subset F$.
4) For each $x \in X$ and each regular open set V of Y not containing $f(x)$, there exists a $\alpha\hat{g}$ closed set K in X not containing x such that $f^{-1}(V) \subset K$.

Proof :
1) $\Leftrightarrow$ 2) is obvious.
2) $\Rightarrow$3) Let F be a regular closed set in Y containing $f(x)$. This implies $x \in f^{-1}(F)$. By (2) $f^{-1}(F)$ is $\alpha\hat{g}$ open in X containing x. Let $U = f^{-1}(F)$. This implies U is $\alpha\hat{g}$ open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$.
3)$\Rightarrow$2) Let F be regular closed in Y containing $f(x)$. This implies $x \in f^{-1}(F)$. From (3), there exists $\alpha\hat{g}$ open set U_x in X containing x such that $f(U_x) \subset F$. That is $U_x \subset f^{-1}(F)$.
Thus $f^{-1}(F) = \cup \{U_x : x \in f^{-1}(F)\}$.
This is union of $\alpha\hat{g}$ open sets. So $f^{-1}(F)$ is $\alpha\hat{g}$ open in X.
3) $\Rightarrow$ 4) Let V be regular open set in Y not containing $f(x)$. Then Y-V is a regular closed set in Y containing $f(x)$. From (3) there exists a $\alpha\hat{g}$ open set U in X containing x such that $f(U) \subset Y - V$.
This implies $U \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Hence $f^{-1}(V) \subset X-U$. Let $K = X - U$. Then K is $\alpha\hat{g}$ closed not containing x such that $f^{-1}(V) \subset K$.
(4) $\Rightarrow$ (3). Let F be regular closed set in Y containing $f(x)$. Then $Y - F$ is a regular open set in Y not containing $f(x)$. From (4), there exists a $\alpha\hat{g}$ closed set K not containing x such that $f^{-1}(Y-F) \subset K$.
That is $X - f^{-1}(F) \subset K$. Hence $X - K \subset f^{-1}(F)$. That is $f(X - K) \subset F$. Let $U = X - K$. U is $\alpha\hat{g}$ open containing x such that $f(U) \subset F$.

Theorem 3.18. The following are equivalent for a function $f : X \rightarrow Y$
1) f is almost contra $\alpha\hat{g}$ continuous
2) $f^{-1}(\text{int cl } G)$ is $\alpha\hat{g}$ closed in X for every open set G of Y.
3) $f^{-1}(\text{cl int } F)$ is $\alpha\hat{g}$ open in X for every closed set F of Y.

Proof :
(1) $\Rightarrow$ (2). Let G be open in Y. Then int cl G is regular open in Y. By (1) $f^{-1}(\text{int cl } G)$ is $\alpha\hat{g}$ closed in X.
(2) $\Rightarrow$ (1). Let V be regular open in Y. Then $f^{-1}(V) = f^{-1}(\text{int cl } V)$ is $\alpha\hat{g}$ closed in X, as V is open in Y. So, f is almost contra $\alpha\hat{g}$ continuous.
(1) $\Rightarrow$ (3). Let F be closed in Y. Then cl int F is regular closed in Y. By (1) $f^{-1}(\text{cl int } F)$ is $\alpha\hat{g}$ open in X.
(3) $\Rightarrow$ (1) is obvious.

Definition 3.19. A function $f : X \rightarrow Y$ is said to be R-map if $f^{-1}(V)$ is regular open for each regular open set V of Y.

Theorem 3.20. If $f : X \rightarrow Y$ is almost contra $\alpha\hat{g}$ continuous and almost continuous, then f is an R-map.

Proof: Let $V \in RO(Y)$. Then $f^{-1}(V)$ is $\alpha\hat{g}$ closed and open. Then $f^{-1}(V)$ is regular open in X. So, f is an R-map.

Definition 3.21. A function f : X → Y is said to be perfectly continuous if $f^{-1}(V)$ is clopen for each open set V of Y.

Theorem 3.22. For two functions f : X → Y and g : Y → Z, let gof ; X → Z be a composition function. Then the following hold.
1) If f is almost contra $\hat{\alpha g}$ continuous and g is an R-map, then gof is almost contra $\hat{\alpha g}$ continuous.
2) If f is almost contra $\hat{\alpha g}$ continuous and g is perfectly continuous, then gof is almost $\hat{\alpha g}$ continuous and almost contra $\hat{\alpha g}$ continuous.
3) If f is contra $\hat{\alpha g}$ continuous and g is almost continuous, then gof is almost contra $\hat{\alpha g}$ continuous.
Proof :
1) Let V be regular open in Z. Then $g^{-1}(V)$ is regular open in Y. As f is almost contra $\hat{\alpha g}$ continuous,
$(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha g}$ closed in X.
2) Let V be regular open in Z. Then $g^{-1}(V)$ is clopen in Y. That is $g^{-1}(V)$ is regular open and regular closed in Y. So, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha g}$ open and $\hat{\alpha g}$ closed in X.
3) Let V be regular open in Z. $g^{-1}(V)$ is open in Y. As f is contra $\hat{\alpha g}$ continuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha g}$ closed in X.

Definition 3.23. A topological space X is said to be $T_{\hat{\alpha g}}$ space if every $\hat{\alpha g}$ open in X is open in X.

Theorem 3.24. Let f : X → Y be contra $\hat{\alpha g}$ continuous and g : Y → Z be $\hat{\alpha g}$ continuous. If Y is a $T_{\hat{\alpha g}}$ space, then gof : X → Z is almost contra $\hat{\alpha g}$ continuous.
Proof : Let V be regular open in Z. Then $g^{-1}(V)$ is $\hat{\alpha g}$ open in Y. As Y is $T_{\hat{\alpha g}}$ space, $g^{-1}(V)$ is open in Y.
So, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha g}$ closed in X.

Definition 3.25. A function f : X → Y is said to be strongly $\hat{\alpha g}$ open (strongly $\hat{\alpha g}$ closed) if f(U) is $\hat{\alpha g}$ open ($\hat{\alpha g}$ closed) for every $\hat{\alpha g}$ open ($\hat{\alpha g}$ closed) set U of X.

Theorem 3.26. If f : X → Y is surjective and strongly $\hat{\alpha g}$ open (strongly $\hat{\alpha g}$ closed) and g : Y → Z is a function such that gof : X → Z is almost contra $\hat{\alpha g}$ continuous, then g is almost contra $\hat{\alpha g}$ continuous.
Proof: Let V be regular closed (regular open) set in Z. As gof is almost contra $\hat{\alpha g}$ continuous $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha g}$ open ($\hat{\alpha g}$ closed).
Since f is surjective and strongly $\hat{\alpha g}$ open (strongly $\hat{\alpha g}$ closed) $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $\hat{\alpha g}$ open ($\hat{\alpha g}$ closed).
Hence g is almost contra $\hat{\alpha g}$ continuous.

Definition 3.27. A function f : X → Y is said to be weakly $\hat{\alpha g}$ continuous, if for each $x \in X$ and each open set V of Y, containing $f(x)$ there exists a $\hat{\alpha g}$ open set U of X containing x such that $f(U) \subset cl\, V$

Theorem 3.28. If a function f : X → Y is almost contra $\alpha\hat{g}$ continuous, then f is weakly $\alpha\hat{g}$ continuous function.
Proof : Let $x \in$ X and V be an open set containing f(x). Then cl V is regular closed in Y containing f(x).
As f is almost contra $\alpha\hat{g}$ continuous, f^{-1}(cl V) is $\alpha\hat{g}$ open in X containing x. Let U = f^{--1}(cl V).
Then f(U) ⊂ f(f^{-1}(cl V)) ⊂ cl V. Hence f is almost weakly $\alpha\hat{g}$ continuous.

Definition 3.29. A space X is called locally $\alpha\hat{g}$ indiscrete, if every $\alpha\hat{g}$ open set is closed in X.

Theorem 3.30. If a function f : X → Y is almost contra $\alpha\hat{g}$ continuous and X is locally $\alpha\hat{g}$ indiscrete, then f is almost continuous.
Proof : Let V be regular closed in Y. So f^{-1}(V) is $\alpha\hat{g}$ open in X. As X is locally $\alpha\hat{g}$ indiscrete, f^{-1}(V) is closed in X. Hence f is almost continuous.

4. $\alpha\hat{g}$ regular graphs

Definition 4.1. For a function f : X → Y, the subset {(x,f(x)) : $x \in$ X} ⊂ XxY is called the graph of f and is denoted by G(f).

Definition 4.2. A graph G(f) of a function f :X → Y is said to be $\alpha\hat{g}$ regular if for each (x,y)∈ (XxY) – G(f), there exists a $\alpha\hat{g}$ closed set U in X containing x and V ∈ RO(Y) containing y such that (UxV) ∩ G(f) = ϕ.

Lemma 4.3. The following properties are equivalent for a graph G(f)of a function:
1) G(f) is $\alpha\hat{g}$ regular
2) for each point (x,y))∈ (XxY) – G(f), there exist a $\alpha\hat{g}$ closed set U in X containing x and V ∈ RO(Y) containing y such that f(U)∩V = ϕ.
Proof:
(1) ⇒ (2). Let (x,y)∈ (XxY) – G(f). Then there exists a $\alpha\hat{g}$ closed set U in X containing x and V ∈ RO(Y) containing y such that (UxV) ∩ G (f) = ϕ. That is V ∩ f(X) = ϕ. That is V ∩ f(U) = ϕ .
(2) ⇒ (1) : Assume (2). y ∈ V. y ∈ Y – f(X). That is y≠ f(x) for any x∈ X. That is V∩ f(X) = ϕ.
This implies (UxV) ∩ (Xxf(X)) = ϕ. That is (UxV) ∩ G(f) = ϕ.

Theorem 4.4. If f : X → Y is almost contra $\alpha\hat{g}$ continuous and Y is T_2, then G(f) is $\alpha\hat{g}$ regular in XxY.
Proof : Let Y be T_2. Let (x,y)∈ (XxY) – G(f). It follows f(x) ≠ y. As Y is T_2, there exist open sets V and W containing f(x) and y respectively such that V∩ W = ϕ. Then int cl V ∩ int cl W = ϕ. Since f is almost contra $\alpha\hat{g}$ continuous, f^{-1}(int cl V) is $\alpha\hat{g}$ closed in X, as int cl V is regular open in Y.
Let U = f^{-1}(int cl V).Then f(V) ⊂ int cl V .So, f(U) ∩ int cl W = ϕ. Hence G(f) is $\alpha\hat{g}$ regular in XxY. The intersection of two $\alpha\hat{g}$ open sets need not be $\alpha\hat{g}$ open. But in the following theorem, we assume that intersection of two $\alpha\hat{g}$ open sets is $\alpha\hat{g}$ open.

Theorem 4.5. Let f : (X,τ) → (Y,σ) be a function and g : (X,τ) → (XxY,τxσ), the graph function defined by g(x) = (x,f(x)), for every x∈X. Then f is almost $\hat{\alpha g}$ continuous if and only if g is almost $\hat{\alpha g}$ continuous,

Proof: Let g be almost $\hat{\alpha g}$ continuous. Let x ∈X and V∈ RO(Y) containing f(x).
Then g(x) = (x, f(x))∈RO(XxY). As g is almost $\hat{\alpha g}$ continuous, there exist $\hat{\alpha g}$ open set U of X containing x such that g(U) ⊂ XxY. So,f(U) ⊂ V. Hence f is almost $\hat{\alpha g}$ continuous.
Conversely, let f be almost $\hat{\alpha g}$ continuous. Let x ∈X and W be a regular open set of XxY containing g(x). There exists U_1∈RO(X,τ) and V ∈ RO(Y,σ) such that (x,f(x))∈(U_1xV) ⊂ W. As f is almost $\hat{\alpha g}$ continuous,there exists U_2∈RO(X,τ) such that x∈U_2 and f(U_2) ⊂ V. Let U = U1∩U2. We have x∈U∈ $\hat{\alpha g}$O(X,τ) and g(U) ⊂ (U_1xV) ⊂ W. This implies g is almost $\hat{\alpha g}$ continuous.

5. Connectedness

Definition 5.1. A space X is called $\hat{\alpha g}$ connected if X cannot be written as a disjoint union of two non-empty $\hat{\alpha g}$ open sets.

Theorem 5.2. If f : X → Y is an almost contra $\hat{\alpha g}$ continuous surjection and X is $\hat{\alpha g}$ connected then Y is connected.

Proof: Let Y be not connected. Then Y = U_0∪V_0 such that U_0 and V_0 are disjoint nonempty open sets. Let U = int cl U_0 and V = int cl V_0. Then U and V are disjoint nonempty regular open sets such that Y = U∪V. As f is almost contra $\hat{\alpha g}$ continuous, f^{-1}(U) and f^{-1}(V) are $\hat{\alpha g}$ closed sets of X. We have X = f^{-1}(U)∪f^{-1}(V) such that f^{-1}(U) and f^{-1}(V) are disjoint. Since f is surjective, f^{-1} (U) and f^{-1} (V) are nonempty. This implies X is not $\hat{\alpha g}$ connected. Hence Y is connected.

Theorem 5.3. The almost contra $\hat{\alpha g}$ image of $\hat{\alpha g}$ connected space is connected.

Proof: Let f : X→Y be an almost contra $\hat{\alpha g}$ continuous function of a $\hat{\alpha g}$ connected space X onto a topological space Y. Suppose Y is not a connected space. Then Y = V_1∪V_2, where V_1 and V_2 are disjoint nonempty open sets of Y. So, V_1 and V_2 are clopen in Y. As f is almost contra $\hat{\alpha g}$ continuous, f^{-1} (V_1) and f^{-1}(V_2) are $\hat{\alpha g}$ open in X. Also f^{-1} (V_1) and f^{-1} (V_2) are disjoint nonempty and X = f^{-1} (V_1)∪ f^{-1} (V_2). This contradiction shows Y is connected.

Definition 5.4. A topological space X is said to be $\hat{\alpha g}$ ultra connected if every two non empty $\hat{\alpha g}$ closed subsets of X intersect.

Definition 5.5. A topological space X is said to be hyper connected if every open set is dense.

Theorem 5.6. If X is $\hat{\alpha g}$ ultra connected and f: X → Y is almost contra $\hat{\alpha g}$ continuous surjection, then Y is hyper connected .

Proof: Let Y be not hyper connected, So, there exists an open set V in Y such that V is not dense in Y. So, there exist nonempty regular open set B_1 = int cl V and B_2 = Y – cl V in Y. As f is almost contra $\alpha\hat{g}$ continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint $\alpha\hat{g}$ closed . This contradicts the $\alpha\hat{g}$ ultra connectedness of X. Hence Y is hyperconnected.

6. Separation axioms

Definition 6.1. A topological space X is said to be $\alpha\hat{g}$ T_1 space if for any pair of distinct points x and y, there exist $\alpha\hat{g}$ open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.

Definition 6.2. A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets [23].

Theorem 6.3. If $f : X \to Y$ is an almost contra $\alpha\hat{g}$ continuous injection and Y is weakly Hausdorff, then X is $\alpha\hat{g}$ T_1.

Proof : Let Y be weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$, and $f(y) \in w$ and $f(x) \notin W$. Since f is almost contra $\alpha\hat{g}$ continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $\alpha\hat{g}$ open sets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$ and $y \in f^{-1}(W)$, $x \notin f^{-1}(W)$.
This completes the proof.

Corollary 6.4. If $f : X \to Y$ is contra $\alpha\hat{g}$ continuous injection and Y is weakly Hausdorff, then X is $\alpha\hat{g}$ $T_{1.}$

Definition 6.5. A topological space X is called Ultra Hausdorff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X, containing x *a*nd y respectively.

Definition 6.6. A topological space is said to be $\alpha\hat{g}$ T_2 space if for any pair of distinct points x and y in X, there exist disjoint $\alpha\hat{g}$ open sets G and H such that $x \in G$ and $y \in H$.

Theorem 6.7. If $f : X \to Y$ is an almost contra $\alpha\hat{g}$ continuous injective function from space X into a Ultra Hausdorff space Y, then X is $\alpha\hat{g}$ T_2.

Proof: Let x and y be distinct points in X. As f is injective $f(x) \neq f(y)$. As Y is Ultra Hausdorff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and f(y) respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$are disjoint $\alpha\hat{g}$ open sets in X. Hence the assertion.

Definition 6.8. A topological space is called Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition 6.9. A topological space X is said to be $\alpha\hat{g}$ normal if each pair of disjoint closed sets can be separated by disjoint $\alpha\hat{g}$ open sets.

Theorem 6.10. If $f: X \to Y$ is an almost contra $\alpha\hat{g}$ continuous closed injection and Y is Ultra normal, then X is $\alpha\hat{g}$ normal.

Proof: Let E and F be disjoint closed subsets of X. As f is closed and injective f(E) and f(F) are disjoint closed sets in Y. Since f is Ultra normal, there exist disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. As f is almost contra $\hat{\alpha g}$ continuous, f^{-1} (U) and f^{-1} (V) are disjoint $\hat{\alpha g}$ open sets in X. This completes the proof.

Lemma 6.11. f : X →Y is almost $\hat{\alpha g}$ continuous implies for each $x \in X$ and for every regular open set V of Y containing f(x), there exists $\hat{\alpha g}$ open set U in X containing x such that f(U)⊂V.

Proof : Let f : X → Y be almost $\hat{\alpha g}$ continuous. Let V be regular open in Y containing f(x). f^{-1} (V) is $\hat{\alpha g}$ open in X containing x. Let U = f^{-1} (V). This implies U is $\hat{\alpha g}$ open in X containing x and f(U) = f (f^{-1} (V)) ⊂ V.

Theorem 6.12. If f : X→ Y is almost $\hat{\alpha g}$ continuous and Y is semiregular, then f is $\hat{\alpha g}$ continuous

Proof : Let $x \in$ X and V be be an open set of Y containing f(x). By the definition of semi regularity of Y, there exists a regular open set G of Y such that f(x) ∈ G ⊂ V. Since f is almost $\hat{\alpha g}$ continuous, there exists U ∈ $\hat{\alpha g}$ – O(X,x) such that f(U) ⊂ G. Hence we have, f(U)⊂G⊂V. This shows f is $\hat{\alpha g}$ continuous.

7. Compactness

Definition 7.1. A space X is said to be

1) $\hat{\alpha g}$ compact if every $\hat{\alpha g}$ open cover of X has a finite subcover.
2) $\hat{\alpha g}$ closed compact if every $\hat{\alpha g}$ closed cover of X has a finite subcover.
3) Nearly compact if every regular open cover of X has a finite subcover.
4) Countably $\hat{\alpha g}$ compact if every countable cover of X by $\hat{\alpha g}$ open sets has a finite subcover.
5) Countably $\hat{\alpha g}$ closed compact if every countable cover of X by $\hat{\alpha g}$ closed sets has a finite subcover.
6) Nearly countable compact if every countable cover of X by regular open sets has a finite subcover.
7) $\hat{\alpha g}$ Lindelof if every $\hat{\alpha g}$ open cover of X has a countable subcover.
8) $\hat{\alpha g}$ closed Lindelof if every $\hat{\alpha g}$ closed cover of X has a countable subcover.
9) Nearly Lindlof if every regular open cover of X has a countable subcover.
10) S- Lindelof if every cover of X by regular closed sets has a countable subcover.
11) Countably S – closed if every countable cover of X by regular closed sets has a finite subcover.
12) S - closed if every regular closed cover of X has a finite subcover.

Theorem 7.2. Let f : X → Y be an almost contra $\hat{\alpha g}$ continuous surjection. Then the following properties hold:

1) If X is $\hat{\alpha g}$ closed compact, then Y is nearly compact.
2) If X is countably $\hat{\alpha g}$ closed compact, then Y is nearly countably compact.
3) If X is $\hat{\alpha g}$ closed Lindelof, then Y is nearly Lindelof.

Proof :

(1) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. As f is almost contra $\hat{\alpha g}$ continuous, $\{ f^{-1} (V\alpha) : \alpha \in I\}$ is $\hat{\alpha g}$ closed cover of X.

Since X is $\hat{\alpha g}$ closed compact, there exists a finite subset I_0 of I such that $X = \cup \{ f^{-1} (V\alpha) : \alpha \in I_0\}$.

As f is surjective, Y = ∪ {Vα : α∈ I_0}, which is a finite subcover of Y. Hence Y is nearly compact.
The proof of (2) and (3) are similar.

Theorem 7.3. Let f : X →Y be an almost contra $\hat{\alpha g}$ continuous surjection. Then the following hold:
1)If X is $\hat{\alpha g}$ compact then Y is S-closed.
2)If X is countably $\hat{\alpha g}$ compact, then Y is countably S - closed.
3)If X is $\hat{\alpha g}$ Lindelof, then Y is S - Lindelof.
Proof : 1) Let {Vα : α∈ I}. be any regular closed cover of Y. As f is almost contra $\hat{\alpha g}$ continuous, { f^{-1} (Vα) : α ∈ I } is $\hat{\alpha g}$ open cover of X. Since X is $\hat{\alpha g}$ compact, there exist a finite subset I_0 of I such that X =∪{ f^{-1} (Vα) ; α∈ I_0}. As f is surjective. Y = ∪ {Vα :α∈ I_0} is a finite subcover for Y. This shows Y is S- closed.
The proof of (2) and (3) are similar.

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