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On Entire Dominating Transformation Graphs and Fuzzy Transformation Graphs

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Abstract. Let G=(V, E) be a graph. Let S be the set of all minimal dominating sets of G. Let x, y, z be three variables each taking value + or –. The entire transformation graph G^{xyz} is the graph having V∪S as the vertex set and for any two vertices u and v in V ∪ S, u and v are adjacent in G^{xyz} if and only if one of the following conditions holds: (i) u, v ∈ V. x = + if u, v ∈ D where D is a minimal dominating set of G. x = – if u, v ∉ D where D is a minimal dominating set of G. (ii) u, v ∈ S. y = + if u ∩ v ≠ ϕ . y= – if u ∩ v = ϕ (iii) u ∈ V and v ∈ S. z = + if u ∈ v. z = – if u ∉ v. In this paper, we initiate a study of entire dominating transformation graphs in domination theory. Also we introduce some fuzzy transformation graphs.

***Keywords*:** dominating graph, semientire dominating graph, entire dominating graph, fuzzy entire dominating graph, transformation

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1. Introduction

The graphs considered in this paper are finite, undirected without loops and multiple edges. Any undefined term here may be found in [1].

Let $G=(V, E)$ be a graph. A set $D \subseteq V$ is a dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. Recently several domination parameters are given in the books by Kulli in [2, 3, 4].

A dominating set D of G is minimal if every $v \in D$, $D - \{v\}$ is not a dominating set of G.

Let S be the set of all minimal dominating sets of G.

The entire dominating graph $ED(G)$ of G is the graph with the vertex set $V \cup S$ in which two vertices u, v are adjacent if $u, v \in D$, where D is a minimal dominating set in G or $u, v \in S$ and $u \cap v \neq \phi$ or $u \in V$ and v is a minimal dominating set in G containing u. This concept was introduced by Kulli in [5]. Many other graph valued functions in domination theory were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] and also graph valued functions in graph theory were studied, for example, in [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

The semientire dominating graph $Ed(G)$ of G is the graph with the vertex set $V \cup S$ in which two vertices u, v are adjacent if $u, v \in D$, where D is a minimal dominating set in G or $u \in V$ and v is a minimal dominating set in G containing u. This concept was introduced by Kulli in [33].

The dominating graph $D(G)$ of G is the graph with vertex set $V \cup S$ in which two vertices u, v are adjacent in $D(G)$ if $u \in V$ and v is a minimal dominating set of G containing u. This concept was introduced in [34].

The middle dominating graph $M_d(G)$ of G is the graph with vertex set $V \cup S$ in which two vertices u, v are adjacent in $M_d(G)$ if $u \cap v \neq \phi$ where $u, v \in S$ or $u \in V$ and v is a minimal dominating set of G containing u. This concept was introduced in [35].

The common minimal dominating graph $CD(G)$ of G in the graph having the same vertex set as G with two vertices in $CD(G)$ adjacent if $u, v \in D$ where D is a minimal dominating set in G. This concept was introduced in [36].

The minimal dominating graph $MD(G)$ of G is the graph with minimal dominating sets as its vertices in which two vertices u, v are adjacent in $MD(G)$ if $u \cap v \neq \phi$. This concept was introduced in [37].

Let $\overline{G}$ denote the complement of G.

2. Entire dominating transformation graphs

Inspired by the definition of the entire dominating graph of a graph, we introduce the following transformation graphs.

Definition 1. Let $G = (V, E)$ be a graph and let S be the set of all minimal dominating sets of G. Let x, y, z be three variables each taking value + and –. The entire dominating transformation graph G^{xyz} is the graph having $V \cup S$ as the vertex set and for any two vertices u and v in $V \cup S$, u and v are adjacent if and only if one of the following conditions holds:

i) $u, v \in V$. $x = +$ if $u, v \in D$ where D is a minimal dominating set of G. $x = -$ if $u, v \notin D$ where D is a minimal dominating set of G.

ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = -$ if $u \cap v = \phi$.

iii) $u \in V$ and $v \in S$. $z = +$ if $u \in v$. $z = -$ if $u \notin v$.

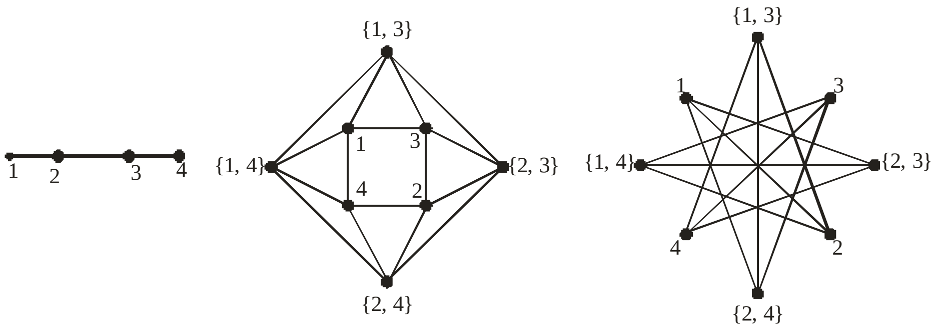
Using the above entire transformation, we obtain eight distinct entire transformation graphs: G^{+++}, G^{+-+}, G^{++-}, G^{-++}, G^{+--}, G^{-+-}, G^{--+}, G^{---}.

Example 2. In Figure 1, a graph G, its entire transformation graphs G^{+++} and G^{---} are shown.

Proposition 3. For any graph G,

i) $\overline{G^{+++}} = G^{---}$ ii) $\overline{G^{++-}} = G^{--+}$

iii) $\overline{G^{+-+}} = G^{-+-}$ iv) $\overline{G^{+--}} = G^{-++}$



G $\qquad$ G^{+++} $\qquad$ G^{---}

Figure 1

3. The entire transformation graph G^{+++}

Among entire transformation graphs one is the entire dominating graph G^{+++}. Therefore we have

Proposition 4. For any graph G, $ED(G) = G^{+++}$.

Remark 5. For any graph G, $Ed(G)$ is a spanning subgraph of G^{+++}.

Remark 6. For any graph G, $D(G)$ is a spanning subgraph of G^{+++}.

Remark 7. For any graph G, $M_d(G)$ is a spanning subgraph of G^{+++}.

Remark 8. For any graph G, $MD(G)$ and $CD(G)$ are vertex and also edge disjoint induced subgraphs of G^{+++}.

Theorem A[5]. For any graph G, $ED(G)$ is complete if and only if G is totally disconnected.

Theorem 9. For any graph G, G^{+++} is complete if and only if G is totally disconnected.
Proof: This follows from Proposition 4 and Theorem A.

Theorem B[5]. For any graph G, $Ed(G)= ED(G)$ if and only if one of the following conditions holds.

i) G has exactly one minimal dominating set containing all vertices of G.
ii) Every pair of minimal dominating sets of G are disjoint.

Theorem 10. For any graph G, $G^{+++} = Ed(G)$ if and only if one of the following conditions holds.

i) G has exactly one minimal dominating set containing all vertices of G.
ii) Every pair of minimal dominating sets of G are disjoint.

Proof: This follows from Proposition 4 and Theorem B.

4. The entire transformation graph G^{+-+}

We start with some simple observations.

Remark 11. For any graph G, $Ed(G)$ is a spanning subgraph of G^{+-+}.

Remark 12. For any graph G, $CD(G)$ is a spanning subgraph of G^{+-+}.

Remark 13. For any graph G, $CD(G)$ and $D(G)$ are edge disjoint subgraphs of G^{+-+}.

We characterize graphs whose transformation graphs G^{+-+} are complete.

Theorem 14. The transformation graph G^{+-+} is complete if and only if G is totally disconnected.

Proof: Suppose G is totally disconnected. Then G has exactly one minimal dominating set D containing all vertices of G. Let u be the corresponding vertex of D in G^{+-+}. Thus the vertex set of G^{+-+} is $V \cup \{u\}$. Since D contains all vertices of G and D is the only minimal dominating set in G, every two vertices are adjacent in G^{+-+}. Thus G^{+-+} is complete.

Conversely suppose G^{+-+} is complete. We now prove that G is totally disconnected. On the contrary, assume G is not totally disconnected. Then there exist minimal dominating sets D_1 in D_2 in G. We consider the following two cases.

Case 1. Suppose $D_1 \cap D_2 \neq \phi$. Then corresponding vertices of D_1 and D_2 are not adjacent in G^{+-+}, a contradiction.

Case 2. Suppose $D_1 \cap D_2 = \phi$. Let $D_1=\{u_1, u_2, ..., u_m, m \geq 1\}$ and $D_2=\{v_1, v_2, ..., v_n, n \geq 1\}$. Then there exist vertices u_i in D_1 and v_j in D_2 such that u_i and v_j are not adjacent in G^{+-+}, which is a contradiction.

From Case 1 and Case 2, we conclude that G has exactly one minimal dominating set which contains all vertices of G. This implies that G is totally disconnected.

Theorem 15. If G is not a nontrivial complete graph, then G^{+-+} contains a triangle.

Proof: Suppose $G \neq K_p$, $p \geq 2$. Then G has at least one minimal dominating set D containing two or more vertices. Let $u_1, u_2, ..., u_n \in D$, $n \geq 2$. Then the corresponding vertices of $u_1, u_2, ..., u_n$ and D in G^{+-+} are mutually adjacent. Hence G^{+-+} contains a triangle.

Theorem 16. $G^{+-+} = K^+_p$ if and only if $G = K_p$.

Proof: Suppose $G = K_p$. Then each vertex v_i of K_p forms a minimal dominating set $\{v_i\}$. Thus v_i and $\{v_i\}$ are adjacent vertices in G^{+-+}. Since $\{v_i\} \cap \{v_j\} = \phi$, for $1 \leq i, j \leq p$, it implies that every pair of minimal dominating sets are adjacent in G^{+-+}. Also since each minimal dominating set $\{v_i\}$ contains only one vertex, it follows that no two corresponding vertices of V are adjacent in G^{+-+}. Thus $G^{+-+} = K^+_p$.

Conversely suppose $G^{+-+} = K^+_p$. We now prove that $G = K_p$. Assume $G \neq K_p$. By Theorem 15, at least two corresponding vertices of G lie in a triangle. Thus $G^{+-+} \neq K^+_p$, which is a contradiction. Thus $G = K_p$.

The above theorem may be written as

Theorem 17. $G^{+++} = K^{+}_{p}$ if and only if each minimal dominating set of G contains exactly one vertex.

5. Fuzzy transformation graphs

In this section, we present some fuzzy transformation graphs in fuzzy domination theory.

A fuzzy graph $G = (V, \sigma, \mu)$ is a nonempty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

A subset D of V is said to be a fuzzy dominating set of a fuzzy graph G if for every $v \in V - D$, there exists $u \in D$ such that (u, v) is a strong arc. The fuzzy domination number $\gamma(G)$ of a fuzzy graph G is the minimum cardinality of a fuzzy dominating set of G. This concept was introduced by Nagoor Gani and et.al. in [38]. A fuzzy dominating set D of a fuzzy graph G is called a minimal fuzzy dominating set of G if for every node $v \in D$, $D - \{v\}$ is not a fuzzy dominating set.

Let $G=(V, \sigma, \mu)$ be a fuzzy graph. Let S be the set of all minimal fuzzy dominating sets of G.

The fuzzy dominating graph $F_d(G)$ of a fuzzy graph G is the fuzzy graph with a nonempty set $V \cup S$ and for any two nodes u, v in $V \cup S$, (u, v) is a strong arc if $u \in V$ and v is a minimal fuzzy dominating set of G containing u.

The fuzzy minimal dominating graph $FM_d(G)$ of a fuzzy graph G is the fuzzy graph with a nonempty set S and for any two nodes u, v in S, (u, v) is a strong arc if $u \cap v \neq \phi$.

The fuzzy common minimal dominating graph $FC_d(G)$ of a fuzzy graph G is the fuzzy graph with the same nonempty set V as G and for any two nodes u, v in V, (u, v) is a strong arc if $u, v \in D$, where D is a minimal fuzzy dominating set in G.

The fuzzy semientire dominating graph $FS_d(G)$ of a fuzzy graph G is the fuzzy graph with a nonempty set $V \cup S$ and for any two nodes u, v in $V \cup S$, (u, v) is a strong arc if $u, v \in D$, where D is a minimal fuzzy dominating set in G or $u \in V$ and v is a minimal fuzzy dominating set of G containing u.

The fuzzy entire dominating graph $FE_d(G)$ of a fuzzy graph G is the fuzzy graph with a nonempty set $V \cup S$ and for any two nodes u, v in $V \cup S$, (u, v) is a strong arc if $u, v \in D$, where D is a minimal fuzzy dominating set in G or $u, v \in S$ and $u \cap v \neq \phi$ or $u \in V$ and v is a minimal fuzzy dominating set of G containing u.

We now define fuzzy entire dominating transformation graphs.

Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Let S be the set of all minimal fuzzy dominating sets of G. Let x, y, z be three variables each taking value + or –. The fuzzy entire dominating transformation graph G^{xyz} is the fuzzy graph with a nonempty set $V \cup S$ and for any two nodes u, v in $V \cup S$, (u, v) is a strong arc if one of the following conditions holds:

i) $u, v \in V$. $x = +$ if $u, v \in D$ where D is a minimal fuzzy dominating set of G. $x = -$ if $u, v \notin D$ where D is a minimal fuzzy dominating set of G.

ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = -$ if $u \cap v = \phi$.

iii) $u \in V$ and $v \in S$. $z = +$ if $u \in v$. $z = -$ if $u \notin v$.

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