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Triple Layered Fuzzy Graph

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Abstract. In this paper, we defined a new fuzzy graph named Triple Layered Fuzzy Graph (TLFG) which is an extension of DLFG and is illustrated with some examples. Further we introduced some theorems which give the relationship between the TLFG with the parental graph G using order, size and degree of fuzzy graphs.

Keywords. Order, Size, vertex degree, μ - Complement, strong fuzzy graph, triple layered fuzzy graph

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Fuzzy logic has developed into a large and deep subject. Zadeh [18] addresses the terminology and stresses that fuzzy graphs are a generalization of the calculi of crisp graphs. Several other formulations of fuzzy graph problems have appeared in the literature. The first definition of fuzzy graph by Kaufman [16] in 1975 was based on Zadeh's fuzzy relations. But it was Rosenfeld [11] in 1975 who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs. The author introduced fuzzy analogues of several graph theoretic concepts such as subgraphs, paths and connectedness, cliques, bridges and cut nodes, etc. During the same time Yeh and Bang [12] in 1975 also introduced fuzzy graphs independently and studied various connectedness concepts.

The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [6]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [7 - 9]. Nagoorgani and Radha [6] introduced vertex degree in fuzzy graphs. Fuzzy trees and fuzzy hyper graphs are studied in [15] and [17]. Pathinathan and Rosline [1] defined Double Layered Fuzzy graph which gives a 3 – D view to fuzzy graphs. Later they analyzed several properties in DLFG [1-5]. In this paper a new fuzzy graph namely Triple Layered Fuzzy Graph (TLFG) is defined and is illustrated with examples and some properties in the form of theorems. First we go through some of the basic definition if fuzzy graphs.

2. Preliminaries

Definition 2.1. [11] A fuzzy graph G is a pair of functions G:(σ,μ) where σ is a fuzzy subset of a non empty set S and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G:(σ,μ) is denoted by $G^* : (\sigma^*, \mu^*)$

Definition 2.2. [13] Let $G:(\sigma,\mu)$ be a fuzzy graph, the order of G is defined as $O(G)=\sum_{u\in V}\sigma(u).$

Definition 2.3. [13] Let $G:(\sigma,\mu)$ be a fuzzy graph, the size of G is defined as $S(G)=\sum_{u,v\in V}\mu(u,v).$

Definition 2.4. [6] Let $G:(\sigma,\mu)$ be a fuzzy graph, the degree of a vertex u in G is defined as $d(u)=\sum_{\substack{v\neq u\\ v\in V}}\mu(u,v)$ and is denoted as $d_G(u)$.

Definition 2.5. [14] A fuzzy graph $G:(\sigma,\mu)$ is said to be a strong fuzzy graph if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all (u,v) in μ^*.

Definition 2.6. [10] Let G be a fuzzy graph, the μ - complement of G is denoted as $G^\mu:(\sigma^\mu,\mu^\mu)$ where $\sigma^*\cup\mu^*$ and $\mu^\mu(u,v)=\begin{cases}\sigma(u)\wedge\sigma(v)-\mu(u,v) \text{ if } \mu(u,v)\succ 0\\ 0 \text{ if } \mu(u,v)=0\end{cases}$

3. Triple layered fuzzy graph

Let $G:(\sigma,\mu)$ be a fuzzy graph with the underlying crisp graph $G^*:(\sigma^*,\mu^*)$. The pair $TL(G):(\sigma_{TL},\mu_{TL})$ is defined as follows. The node set of $TL(G)$ be $\sigma^*\cup\mu^*\cup\mu^*$. The fuzzy subset σ_{TL} is defined as $\sigma_{TL}=\begin{cases}\sigma(u) \text{ if } u\in\sigma^*\\ 2\mu(uv) \text{ if } uv\in\mu^*\end{cases}$

The fuzzy relation μ_{TL} on $\sigma^*\cup\mu^*$ is defined as

$$\mu_{TL}=\begin{cases}\mu(uv) & \text{if } u,v\in\sigma^*\\ \mu(e_i)\wedge\mu(e_j) & \text{if the edge } e_i \text{ and } e_j \text{ } \textit{have} \text{ a node in common between them}\\ \sigma(u_i)\wedge\mu(e_i) & \text{if } u_i\in\sigma^* \text{ and } e_i\in\mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in clockwise direction.}\\ \sigma(u_i)\wedge\mu(e_i) & \text{if } u_i\in\sigma^* \text{ and } e_i\in\mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in anticlockwise direction.}\\ 0 & \text{Otherwise}\end{cases}$$

By definition, $\mu_{TL}(u,v)\leq\sigma_{TL}(u)\wedge\sigma_{TL}(v)$ for all u,v in $\sigma^*\cup\mu^*$. Here μ_{TL} is a fuzzy relation on the fuzzy subset σ_{TL}. Hence the pair $TL(G):(\sigma_{TL},\mu_{TL})$ is defined as Triple Layered Fuzzy Graph (TLFG).

Example 3.1. Consider a fuzzy graph $G:(\sigma,\mu)$ with n = 3 vertices whose crisp graph is a cycle.

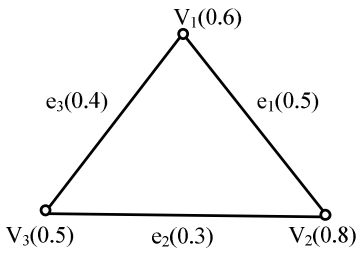

Figure 1: Fuzzy graph $G:(\sigma,\mu)$

Then the triple layered fuzzy graph is given by

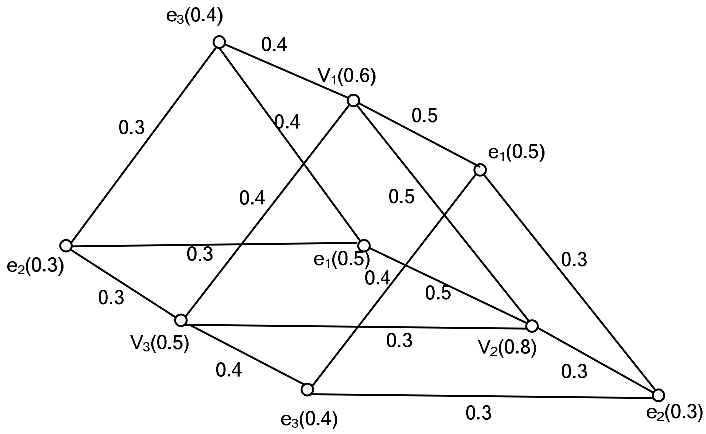

Figure 2: Triple layered fuzzy graph $TL(G):(\sigma_{TL},\mu_{TL})$

Remark 3.1. For each value of n we can get different TLFG.

4. Theoretical concepts of TLFG

Theorem 4.1. Order TL(G) = Order(G) + 2 Size(G), where G is a fuzzy graph.

Proof: As the node set of TL(G) is $\sigma^* \cup \mu^* \cup \mu^*$ and the fuzzy subset σ_{TL} on $\sigma^* \cup \mu^* \cup \mu^*$ is defined as $\sigma_{TL} = \begin{cases} \sigma(u) \text{ if } u \in \sigma^* \\ 2\mu(uv) \text{ if } uv \in \mu^* \end{cases}$

Order TL(G) = $\sum_{u \in VUE \cup E} \sigma_{TL}(u)$ (by definition 2.2)

$$= \sum_{u \in V} \sigma_{TL}(u) + \sum_{u \in E} \sigma_{TL}(u)$$

$$= \sum_{u \in V} \sigma(u) + 2\sum_{u \in E} \mu(u) \text{ (by definition of } \sigma_{TL}(u)\text{)}$$
= Order (G) + 2 Size (G).

Theorem 4.2. *Size* $\mathrm{T}L(G)\ =\ 3\ \mathit{Size}\ (G)\ +\ 2\sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j)$, where *G* is a fuzzy graph and *i, j* ∈ *N*.

Proof: $\mathit{Size}\ \mathrm{T}L(G)\ =\ \sum_{u,v \in VUEUE} \mu_{TL}(u,v)$ (by definition 2.3)

$$= \sum_{u,v \in V} \mu_{TL}(u,v) + 2\sum_{e_i, e_j \in E} \mu_{TL}(e_i, e_j) + \sum_{u_i \in V, e_i \in E} \mu_{TL}(u_i, e_i) + \sum_{u_j \in V, e_i \in E} \mu_{TL}(u_j, e_i)$$

(u_i and u_j are the end node of e_i both in clockwise and anti clockwise direction in the third and fourth summation respectively)

$$= \text{size (G)} + 2\sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j) + \sum_{u_i \in V, e_i \in E} \sigma(u_i) \wedge \mu(e_i) + \sum_{u_j \in V, e_i \in E} \sigma(u_j) \wedge \mu(e_i)$$

$$= \text{size (G)} + 2\sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j) + \sum_{e \in E} \mu(e) + \sum_{e \in E} \mu(e).$$

Since in the third and fourth summation, we are considering only one vertex in each edge either clockwise and anticlockwise direction, its membership value is less than the value of the vertices.

$$\mathit{Size}\ \mathrm{T}L(G) = \text{size (G)} + 2\sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j) + 2\mathit{size}\ \text{(G)}$$
$$= \text{3size (G)} + 2\sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j)$$

Theorem 4.3. $|E_{TL}(G)| = 3|E(G)| + 2|E(L(G))|$.

Proof: Each edge in G is replaced by a two new vertex in TL(G). The pair of adjacent edges in G contributes two new edges in TL(G) and each edge in G is neighbourhood of two vertices both in clockwise and anticlockwise direction. Also the vertex which are adjacent in G is also adjacent in TL(G).

Thus, we have $|E_{TL}(G)| = 3|E(G)| +\ 2$ (no of pairwise adjacent edges in G^*)

$$= 3|E(G)| +\ 2|E(L(G))|.$$

Theorem 4.4. If G is a strong fuzzy graph then TL(G) is also a strong fuzzy graph.

Proof: By definition 2.5, we have $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all (u,v) in μ^*.

Assume G is a strong fuzzy graph. We need to prove TL(G) is a strong graph. Consider an edge (u,v) in TL(G). Then

$$\mu_{TL} = \begin{cases} \mu(uv) & \text{if } u,v \in \sigma^* \\ \mu(e_i) \wedge \mu(e_j) & \text{if the edge } e_i \text{ and } e_j \textit{ have} \text{ a node in common between them} \\ \sigma(u_i) \wedge \mu(e_i) & \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in clockwise direction.} \\ \sigma(u_i) \wedge \mu(e_i) & \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in anticlockwise direction.} \end{cases}$$

Case i: It is trivial from our assumption that G is a strong graph. Thus $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all (u,v) in μ_{TL}^*.

Case ii: If $\mu_{TL}(u,v) = \mu(e_i) \wedge \mu(e_j)$ and if $u = e_i,\ v = e_j \in \mu^*$ are adjacent in G^* then $\mu_{TL}(u,v) = \sigma_{TL}(e_i) \wedge \sigma_{TL}(e_j)$ (by the definition of σ_{TL})

Case iii: If $\mu_{TL}(u,v) = \sigma(u_i) \wedge \mu(e_i)$ and each e_i is incident to u_i in clockwise direction in G^*. Then

$$\mu_{TL}(u,v) = \sigma_{TL}(u_i) \wedge \sigma_{TL}(e_j) \text{ (by the definition of } \sigma_{TL})$$

Case iv: If $\mu_{TL}(u,v) = \sigma(u_i) \wedge \mu(e_i)$ and each e_i is incident to single u_i in anticlockwise direction in G^*. Then

$$\mu_{TL}(u,v) = \sigma_{TL}(u_i) \wedge \sigma_{TL}(e_j) \text{ (by the definition of } \sigma_{TL})$$

Hence if G is a strong fuzzy graph, by case i, ii, iii and iv; we have $\mu_{TL}(u,v) = \sigma_{TL}(u) \wedge \sigma_{TL}(v)$ for all (u,v) in μ_{TL}^*.

Theorem 4.5. Let G be a fuzzy graph then

$$d_{TL(G)}(u) = \begin{cases} d_G(u) + (\sigma(\mathrm{u}_i) \wedge \mu(e_i)) + (\sigma(\mathrm{u}_j) \wedge \mu(e_i)) & \text{if } u \in \sigma^* \\ \sum_{e_i \in \mu^*} \mu(e_i) \wedge \mu(e_j) + (\sigma(u_i) \wedge \mu(e_i)) & \text{if } \mathrm{u} \in \mu^* \end{cases}$$

Proof: By definition 2.4, we have $d_G(u) = \sum_{\substack{v \neq u \\ v \in V}} \mu(u,v)$

Case i: Let $u \in \sigma^*$, then

$d_{TL(G)}(u) = \sum_{v \in \sigma^*} \mu_{TL}(u,v) + \mu_{TL}(u_i, e_i) + \mu_{TL}(u_j, e_i)$ where u_i and u_j are the end nodes of e_i

$$= \sum_{v \in \sigma^*} \mu(u,v) + (\sigma(\mathrm{u}_i) \wedge \mu(e_i)) + (\sigma(\mathrm{u}_j) \wedge \mu(e_i))$$

($\because$ in the first summation the vertices which are adjacent in G is also adjacent in TLFG)

$$= d_G(u) + (\sigma(\mathrm{u}_i) \wedge \mu(e_i)) + (\sigma(\mathrm{u}_j) \wedge \mu(e_i))$$

Case ii: Let $u \in \mu^*$, then

$$d_{TL(G)}(u) = \sum_{e_i, e_j \in \mu^*} \mu_{TL}(e_i, e_j) + \mu_{TL}(u_i, e_i) = \sum_{e_i, e_j \in \mu^*} \mu(e_i) \wedge \mu(e_j) + \sigma(\mathrm{u}_i) \wedge \mu(e_i)$$

Remark 4.1. If G is a strong fuzzy graph then the μ - complement of TL(G) is isolated vertices. Thus $d_{TL}(u) = 0$ for all u in $\sigma^* \mathrm{U} \mu^*$.

5. Conclusion

The triple layered fuzzy graph (TLFG) is defined as an extension of DLFG. In this paper the TLFG is defined and some of its relationship with the parental graph whose crisp graph is a cycle is studied. Further work can be done to develop multi layered fuzzy graph and its application in networking.

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