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On Qlick Transformation Graphs

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585 106, India
e-mail: vrkulli@gmail.com

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Abstract. Let G be a graph with vertex set V, edge set E and block set B and let x, y, z be three variables each taking value + or –. The qlick transformation graph G^{xyz} is the graph whose vertex set is the union of the set of edges and the set of blocks of G. For any two vertices u and v in G^{xyz}, we define x, y, z as follows:

(i) Let u, v $\in$ E. x = + if u and v are adjacent in G. x = – if u and v are not adjacent in G.
(ii) Let u,v $\in$ B. y= + if u and v are adjacent in G. y = – if u and v are not adjacent in G.
(iii) Let u $\in$ E and v $\in$ B, z = + if u and v are incident with each other in G. z = – if u and v are not incident with each other in G.

In this paper, we initiate a study of qlick transformation graphs.

Keywords: qlick graph, plick graph, line block graph, block line forest, qlick transformation graph.

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1. Introduction

By a graph we mean a finite, undirected graph without loops or multiple edges. All definitions and notations not given in this paper may be found in Kulli [1].

If $b = \{u_1, u_2, \ldots, u_r; r\geq 2\}$ is a block of a graph G, then we say that vertex u_1 and block b are incident with each other, as are u_2 and b and so on. If $b = \{e_1, e_2, \ldots, e_s; s\geq 1\}$ is a block of G, then we say that edge e_1 and block b are incident with each other, as are e_2 and b and so on. If two distinct blocks b_1 and b_2 are adjacent with a common cutvertex, then they are adjacent blocks. This idea was introduced by Kulli in [2]. The vertices, edges and blocks of a graph are called its members.

The qlick graph $Q(G)$ of G is the graph whose vertex set is the set of edges and blocks of G and two vertices are adjacent if the corresponding edges and blocks are adjacent or the corresponding members are incident. The plick graph $P(G)$ of G is the graph whose vertex set is the set of edges and blocks of G and two vertices are adjacent if the corresponding edges are adjacent or the corresponding members are incident. These concepts were introduced by Kulli in [3] and were studied, for example, in [4, 5, 6, 7].

The line block graph $L_b(G)$ of G is the graph whose vertex set is the set of edges and blocks of G in which two vertices are adjacent if the corresponding blocks are adjacent or the corresponding members are incident. This concept was introduced in [8] and was studied, for example, in [9].

The block line forest $B_f(G)$ of G is the graph whose vertex set is the set of edges and blocks of G in which two vertices are adjacent if the corresponding members are incident. This concept was introduced in [10]. Many other graph valued functions in graph theory were studied, for example, in [11, 12, 13, 14, 15, 16, 17, 18, 19].

The block graph $B(G)$ of a graph G is the graph whose vertex set is the set of blocks of G and two vertices are adjacent if the corresponding blocks are adjacent. This concept was studied by Harary in [20] and further this was studied, for example, in [21, 22, 23]. The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. This concept was studied, for example, in [24, 25, 26, 27, 28, 29].

Some transformation graphs were studied, for example, in [30, 31].

Let $\overline{G}$ denote the complement of G.

2. Qlick transformation graphs

The qlick graph inspired us to introduce qlick transformation graphs. We now define qlick transformation graphs G^{xyz} when x or y or z is either + or –.

Definition 1. Let G be a graph with vertex set V, edge set E and block set B and let x, y, z be three variables each taking value + or –. The qlick transformation graph G^{xyz} is the graph whose vertex set is the union of the set of edges and the set of blocks of G. For any two vertices u and v in G^{xyz}, we define x, y, z as follows.

(i) Let $u, v \in E$. $x = +$ if u and v are adjacent in G. $x = -$ if u and v are not adjacent in G.

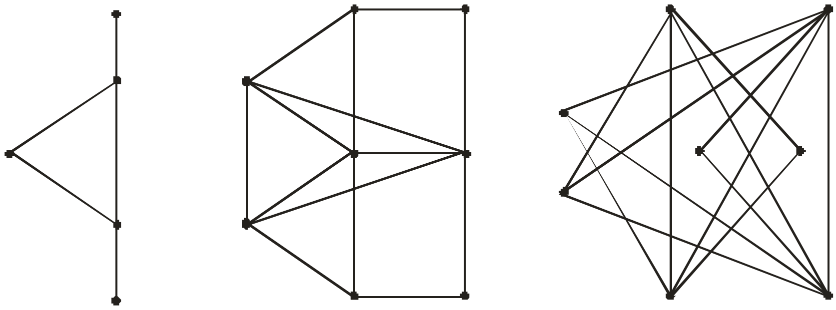
(ii) Let $u, v \in B$. $y = +$ if u and v are adjacent in G. $y = -$ if u and v are not adjacent in G.

(iii) Let $u \in E$ and $v \in B$. $z = +$ if u and v are incident with each other in G. $z = -$ if u and v are not incident with each other in G.

Using the above qlick transformation, we obtain eight distinct qlick transformation graphs: G^{---}, G^{--+}, G^{-+-}, G^{+--}, G^{-++}, G^{++-}, G^{+-+}, G^{+++}.

By definition, any vertex of G is not vertex of qlick transformation graph, so that we consider only graphs without isolated vertices.

Example 2. In Figure 1, a graph G, its qlick transformation graphs G^{+++} and G^{---} are shown.



G $\qquad$ G^{+++} $\qquad$ G^{---}

Figure 1:

Proposition 3. If G is a nontrivial connected graph, then

(1) $\overline{G^{+++}} = G^{---}$ $\qquad$ (2) $\overline{G^{++-}} = G^{--+}$

(3) $\overline{G^{+-+}} = G^{-+-}$ $\qquad$ (4) $\overline{G^{+--}} = G^{-++}$.

Proof: Each follows from the definition of G^{xyz} and $\overline{G}$.

Proposition 4. If C_p is a cycle with $p \geq 3$ vertices, then

(1) $C_p^{+++} = C_p^{+-+} = W_{p+1}$ $\qquad$ (2) $C_p^{++-} = C_p^{+--} = K_1 \cup C_p$

(3) $C_p^{-++} = C_p^{--+}$ $\qquad$ (4) $C_p^{---} = C_p^{-+-} = \overline{K}_{p+1}$.

3. The qlick transformation graph G^{+++}

Among qlick transformation graphs one is the qlick graph $Q(G)$. It is easy to see that

Proposition 5. For any graph G without isolated vertices, $\qquad Q(G) = G^{+++}$.

Remark 6. For any graph G without isolated vertices, $L(G)$ and $B(G)$ are vertex and also edge disjoint induced subgraphs of G^{+++}.

4. The qlick transformation graph G^{-++}

Proposition 7. The line block graph $L_b(G)$ is a spanning subgraph of the qlick transformation graph G^{-++}.

Proof: This follows from definitions of $L_b(G)$ and G^{-++}.

Theorem 8. For any graph G without isolated vertices,

$$G^{-++} = L_b(G) \cup \overline{L(G)}.$$

We now characterize graphs G for which $G^{-++} = L_b(G)$.

Theorem 9. For a nontrivial connected graph G,

$$L_b(G) \subseteq G^{-++}.$$

Furthermore, $L_b(G) = G^{-++}$ if and only if every pair of edges in G are adjacent.

Proof: By Proposition 7, $L_b(G) \subseteq G^{-++}$.

Suppose $L_b(G) = G^{-++}$. We now prove that every pair of edges in G are adjacent. On the contrary, assume G has two edges e_1 and e_2 such that they are not adjacent. Then the corresponding vertices of e_1 and e_2 are adjacent in G^{-++}, but they are not adjacent in $L_b(G)$. Thus $L_b(G) \neq G^{-++}$, which is a contradiction. This proves that every pair of edges in G are adjacent.

Conversely suppose every pair of edges of G are adjacent. Then the corresponding vertices of edges of G are not mutually adjacent in G^{-++}. Thus $G^{-++} \subseteq L_b(G)$ and since $L_b(G) \subseteq G^{-++}$, this implies that $L_b(G) = G^{-++}$.

Iterated qlick-transformation graphs G^{-++} are defined by $(G^n)^{-++} = G(G^{n-1})^{-++}$ for n ≥ 2 where $(G^1)^{-++} = G^{-++}$.

Theorem 10. Let G be a nontrivial connected graph. The graphs G and G^{-++} are isomorphic if and only if $G = K_2$.

Proof: Suppose $G = G^{-++}$. We now prove that $G = K_2$. On the contrary, assume G is a connected graph with $p \geq 3$ vertices. We consider the following two cases.

Case 1. Suppose G is not a tree. Then G has at least p edges and has at least one block. Thus G^{-++} has at least p+1 vertices. Therefore the number vertices of G is less than that in G^{-++}. Hence $G \neq G^{-++}$, a contradiction.

Case 2. Suppose G is a tree. Then G has $p - 1$ edges and $p - 1$ blocks. Then G^{-++} has $2p-2$ vertices. Hence the number of vertices of G is less than that in G^{-++}. Thus $G \neq G^{-++}$, a contradiction.

From Case 1 and Case 2, we conclude that $G = K_2$.

Conversely suppose $G = K_2$. Then clearly $G = G^{-++}$.

The following results are immediate.

Corollary 11. Let G be a nontrivial connected graph. Then $G = (G^n)^{-++}$, $n \geq 1$ if and only if $G = K_2$.

Corollary 12. Let G be a graph without isolated vertices. Then $G = (G^n)^{-++}$, $n \geq 1$ if and only if $G = mK_2$, $m \geq 1$.

5. The qlick transformation graph G^{+-+}.

Proposition 13. The plick graph $P(G)$ is a spanning subgraph of the qlick transformation graph G^{+-+}.

Proof: This follows form definitions of $P(G)$ and G^{+-+}.

Theorem 14. For a nontrivial connected graph G, $P(G) \subseteq G^{+-+}$.

Furthermore, $P(G) = G^{+-+}$ if and only if G has at most one cutvertex.

Proof: By Proposition 13, $P(G) \subseteq G^{+-+}$.

Suppose $P(G) = G^{+-+}$. We now prove that G has at most one cutvertex. On the contrary, assume G has at least two cutvertices. Then G has two blocks b_1 and b_2 such that they are not adjacent. Then the corresponding vertices of b_1 and b_2 are adjacent in

G^{+-+}, but they are not adjacent in $P(G)$. Thus $P(G) \neq G^{+-+}$, a contradiction. This proves that G has at most one cutvertex.

Conversely suppose G has at most one cutvertex. We now consider the following two cases.

Case 1. Suppose G has no cutvertex. Then clearly $P(G) = G^{+-+}$.

Case 2. Suppose G has exactly one cutvertex. Then every pair of blocks in G are adjacent. Then the corresponding vertices of blocks of G are not mutually adjacent in G^{+-+}. Thus $G^{+-+} \subseteq P(G)$ and since $P(G) \subseteq G^{+-+}$, it implies that $P(G) = G^{+-+}$.

From Case 1 and Case 2, we conclude that $P(G) = G^{+-+}$.

6. The qlick transformation graph G^{--+}

Proposition 15. The block line forest $B_f(G)$ is a spanning subgraph of a qlick transformation graph G^{--+}.

Proof: This follows from definitions of $B_f(G)$ and G^{--+}.

Theorem 16. For a nontrivial connected graph G,

$$G^{--+} = B_f(G) \cup \overline{L(G)} \cup \overline{B(G)}.$$

We now characterize graphs G for which $G^{--+} = B_f(G)$.

Theorem 17. For a nontrivial connected graph G,

$B_f(G) \subseteq G^{--+}$.

Furthermore, $B_f(G) = G^{--+}$ if and only if G satisfies the following conditions.

(i) every pair of edges are adjacent.

(ii) every pair of blocks are adjacent.

Proof: By Proposition 15, $B_f(G) \subseteq G^{--+}$.

Suppose $B_f(G) = G^{--+}$. Let G be a non trivial connected graph. We prove (i). On the contrary, assume a pair of edges are not adjacent in G. Then the corresponding vertices of edges in G^{--+} are adjacent, but they are not adjacent in $B_f(G)$. Thus $B_f(G) \neq G^{--+}$, a contradiction. Hence G satisfies (i). We now prove (ii). On the contrary, assume a pair of blocks are not adjacent in G. Then corresponding vertices of blocks in G^{--+} are adjacent; but they are not adjacent in $B_f(G)$. Thus $B_f(G) \neq G^{--+}$, a contradiction. Hence G satisfies (ii).

Conversely, suppose every pair of edges are adjacent in G. Then the corresponding vertices of edges of G are not mutually adjacent in G^{--+}. Also suppose every pair of blocks are adjacent in G. Then the corresponding vertices of blocks of G are not mutually adjacent in G^{--+}. Thus $G^{--+} \subseteq B_f(G)$ and since $B_f(G) \subseteq G^{--+}$, it implies that $B_f(G) = G^{--+}$.

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