

Steady State Analysis of Batch Arrival M/G/1 Retrial Queuing Model with State Dependent Admission, Modified Vacation and Feedback

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Abstract. A batch arrival retrial queue with general retrial times, state dependent admission, modified vacation policy and feed back is analysed in this paper. Arrivals are controlled according to the state of the server. If the orbit is empty, the server takes at most J vacations until at least one customer is received in the orbit when the server returns from the vacation. At the service completion epoch, the test customer may either enter the orbit for another service with probability p or leave the system with probability 1-p. By supplementary variable technique some analytical results for the system size distribution as well as some other performance measures of the system are derived.

Keywords: Retrial queue, state dependent admission control, modified vacation, feedback.

AMS Mathematics Subject Classification (2010): 90B22

1. Introduction

Retrial queues have the feature that arriving customers finding no free servers must leave the service area and repeats his request for service after some random time. Retrial queues play very important roles in the analysis of telephone switching systems, computer and communication networks. In many queuing situations, the customers arrival rate varies according to server state idle, busy, and on vacation. Altman et al. [1] Maden and Abu-Dayyals [5] have studied classical queuing system with restricted admissibility of arriving batches and Bernoulli server vacation. Arivudainambi and Godhandaraman [2] studied retrial queue with two phase service, feedback and K optional vacations. A comprehensive study on recent works for various vacation models can be found in [3,5,6,7,8,9].

2. Model description

In this paper, a batch arrival M/G/1 retrial queuing model with general retrial times, state dependent admission, with modified server vacation is considered.

1. New arrivals arrive in batches according to a compound Poisson process with rate λ . Let X_k denote the number of customers belongs to the kth arrival batch, where

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- $X_k, k = 1, 2, 3, \dots$ Are with a common distribution $P[X_k = n] = C_n, n = 1, 2, \dots$ where $C(z)$ denotes the probability generating function of X and moments $c_n, n \geq 1$.
2. It is assumed that there is no waiting space and therefore, if arriving customers find the server free, one of the arrivals begins his service and others leaves the service area and join the orbit. If the server is busy or on vacation, arriving customers leaves the service area join the orbit. The customers in the orbit try to request his service and inter retrial times have an arbitrary distribution $A(t)$ with corresponding Laplace- Stieljes transforms $\tilde{A}(\theta)$.
 3. There is a single server who provides service to all arriving customers. The service time follows a generally random variable S with distribution function $S(t)$ and Laplace Stiljes transform $\tilde{S}(\theta)$ and moments s_n .
 4. The arriving batches are allowed to join the system with state dependent admission control policy. Let α_1, α_2 and α_3 be the assigned probabilities for an arriving batch to join the system during the period of idle, busy and vacation.
 5. Whenever the orbit is empty, the server leaves for a vacation of random length V . If no customer appears in the orbit when the server returns from a vacation, it leaves again for another vacation with the same length such a pattern continues until it returns from a vacation to find at least one customer is recorded in the orbit or it has already taken J vacations. If the orbit is empty at the end of the J th vacation, the server remains idle for new arrivals in the system. At a vacation completion epoch the orbit is nonempty, the server waits for the customers, if any, in the orbit, or for new customers to arrive. The vacation time V has distribution function $V(t)$ and Laplace /Stieljes transform $\tilde{V}(\theta)$ and moments v_n .
 6. Various stochastic processes involved in the system are independent of each other.

3. System analysis

In this section the steady state difference-differential equations for the retrial system is developed by treating the elapsed retrial time, the elapsed service time and elapsed vacation time as supplementary variables. After that, the probability generating functions for server state and the number of customers in the system/orbit are derived. Steady state equations. In the steady state it is assumed that $A(0) = 0, A(\infty) = 1, S(0) = 0, S(\infty) = 1, V(0) = 1, V(\infty) = 1$ and are continuous at $x = 0$, so that

$$\theta(x)dx = \frac{dA(x)}{1 - A(x)}, \mu(x)dx = \frac{dS(x)}{1 - S(x)}, \text{ and } \omega(x)dx = \frac{dV(x)}{1 - V(x)}, \theta(x)dx \text{ can be interpreted}$$

as the conditional probability density of completion of the retrial time, given that the elapsed time is x . Let $A^0(t), S^0(t)$ and $V_j^0(t)$ be the elapsed retrial time, service time and vacation time respectively at time t . The state of the system at time t can be described by bivariate Markov process $\{C(t), N(t), t \geq 0\}$ where $C(t)$ denotes the server state (0, 1, 2, ...J+1) depending if the server is idle, busy or on 1st vacation...Jth vacation.

Let $\{t_n ; n = 1, 2, \dots\}$ be the sequence of epochs at which either a service period completion occurs or a vacation time ends. The sequence of random vectors $Z_n = \{N(t_n^+), \delta(t_n^+)\}$ forms a Markov chain which is embedded in the retrial queuing system, where $\delta(t) = A^-(t)$ if $C(t)=0, N(t) > 0, \delta(t)=S^-(t)$ if $C(t)=1, N(t) \geq 0$ and $\delta(t)=V_j^-(t)$ if $C(t)=j+1, N(t) \geq 0, j=1, 2, \dots, J$

Let us define the following probabilities

$$P_0(t) = \Pr\{C(t) = 0, N(t) = 0\}$$

$$P_n(x, t) dx = \Pr\{C(t) = n, \delta(t) = A^-(t) ; x < A^-(t) < x + dx\}, x > 0, n \geq 1,$$

$$\Pi_n(x, t) dx = \Pr\{C(t) = n, \delta(t) = S^-(t) ; x < S^-(t) < x + dx\}, x > 0, n \geq 0$$

$$\Omega_{j,n}(x, t) dx = \Pr\{C(t) = n, \delta(t) = V_j^-(t) ; x < V_j^-(t) < x + dx\}, x > 0, n \geq 0, 1 \leq j \leq J.$$

In steady state, we can set $P_0 = \lim_{t \rightarrow \infty} P_0(t)$ and the limiting densities

$$P_n(x) = \lim_{t \rightarrow \infty} P_n(x, t) \text{ for } x > 0 \text{ and } n \geq 1 \quad \Pi_n(x) = \lim_{t \rightarrow \infty} \Pi_n(x, t) \text{ for } x > 0 \text{ and } n \geq 0 \text{ and}$$

$\Omega_{j,n}(x) = \lim_{t \rightarrow \infty} \Omega_{j,n}(x, t) \text{ for } x > 0 \text{ and } n \geq 0.$ The Kolmogorov forward equations which govern the system under the steady state conditions can be written as

$$\lambda P_0 = \int_0^{\infty} \Omega_{j,0}(x) \omega(x) dx \tag{1}$$

$$\frac{d}{dx} P_n(x) + (\lambda + \theta(x)) P_n(x) = \lambda(1 - \alpha_1) P_n(x) \tag{2}$$

$$\frac{d}{dx} \Pi_0(x) + (\lambda + \mu(x)) \Pi_0(x) = \lambda(1 - \alpha_2) \Pi_0(x) \tag{3}$$

$$\frac{d}{dx} \Pi_n(x) + (\lambda + \mu(x)) \Pi_n(x) = \lambda(1 - \alpha_2) \Pi_n(x) + \lambda \alpha_2 \sum_{k=1}^n C_k \Pi_{n-k}(x) \tag{4}$$

$$\frac{d}{dx} \Omega_{j,0}(x) + (\lambda + \omega(x)) \Omega_{j,0}(x) = \lambda(1 - \alpha_3) \Omega_{j,0}(x) \tag{5}$$

$$\frac{d}{dx} \Omega_{j,n}(x) + (\lambda + \omega(x)) \Omega_{j,n}(x) = \lambda(1 - \alpha_3) \Omega_{j,n}(x) + \lambda \alpha_3 \sum_{k=1}^n C_k \Omega_{j,n-k}(x) \tag{6}$$

The boundary conditions are

$$P_n(0) = \sum_{j=1}^J \int_0^{\infty} \Omega_{j,n}(x) \omega(x) dx + (1 - p) \int_0^{\infty} \Pi_n(x) \mu(x) dx + p \int_0^{\infty} \Pi_{n-1}(x) \mu(x) dx \tag{7}$$

$$\Pi_0(0) = \lambda C_1 P_0 + \int_0^{\infty} P_1(x) \theta(x) dx \tag{8}$$

$$\Pi_n(0) = \lambda C_{n+1} P_0 + \int_0^{\infty} P_{n+1}(x) \theta(x) dx + \lambda \alpha_1 \int_0^{\infty} C_k P_{n-k+1}(x) dx \tag{9}$$

$$\Omega_{1,0}(x) = \begin{cases} \int_0^{\infty} \Pi_0(x) \mu(x) dx, & n = 0 \\ 0 & n \geq 1 \end{cases} \tag{10}$$

$$\Omega_{j,0}(x) = \begin{cases} \int_0^{\infty} \Omega_{j-1}(x)\omega(x)dx, & n = 0 \\ 0 & n \geq 1 \end{cases} \quad (11)$$

And the normalising condition

$$P_0 + \sum_{n=1}^{\infty} \int_0^{\infty} P_n(x)dx + \sum_{n=0}^{\infty} \int_0^{\infty} \Pi_n(x)dx + \sum_{j=1}^J [\sum_{n=0}^{\infty} \Omega_{j,n}(x)dx] = 1 \quad (12)$$

Next, the probability generating functions are defined as follows

$$P(x,z) = \sum_{n=1}^{\infty} z^n P_n(x) \Big|_{|z| \leq 1}, \Pi(x,z) = \sum_{n=1}^{\infty} z^n \Pi_n(x) \Big|_{|z| \leq 1}, \Omega_{j,n}(x,z) = \sum_{n=1}^{\infty} z^n \Omega_{j,n}(x) \Big|_{|z| \leq 1}$$

Now multiplying Eq (2) b z^n and summing over $n(n=1,2,3,\dots)$

$$\frac{d}{dx} P(x,z) + [\theta(x) + \lambda\alpha_1]P(x,z) = 0 \quad (13)$$

$$\text{Similarly, we get } \frac{d}{dx} \Pi(x,z) + [\mu(x) + \lambda\alpha_2(1 - C(z))]\Pi(x,z) = 0 \quad (14)$$

$$\frac{d}{dx} \Omega_j(x,z) + [\omega(x) + \lambda\alpha_3(1 - C(z))]\Omega_j(x,z) = 0 \quad (15)$$

$$P(0,z) = \left(\sum_{j=1}^J \int_0^{\infty} \Omega_j(x,z)\omega(x)dx - \lambda P_0 - \sum_{j=1}^J \Omega_{j,0}(0) + ((1-p) + pz) \int_0^{\infty} \Pi(x,z)\mu(x)dx \right) \quad (16)$$

$$\Pi(0,z) = \frac{1}{z} \int_0^{\infty} P(z,x)\theta(x)dx + \frac{\lambda C(z)}{z} [P_0 + \alpha_1 \int_0^{\infty} P(z,x)dx] \quad (17)$$

Solving the partial differentials equations (13),(14),(15) and using (16) and (17) we get

$$P(z) = \frac{P_0 \left\{ z \sum_{j=1}^J \frac{[\tilde{V}(\lambda\alpha_3)(1-C(z))-1]}{[\tilde{V}(\lambda\alpha_3)]^{j-1}} - z + ((1-p) + pz)C(z)\tilde{S}(\lambda\alpha_2(1 - C(z)))(1 - \tilde{A}(\lambda\alpha_1)) \right\}}{\alpha_1 \{ z - ((1-p) + pz)(\tilde{A}(\lambda\alpha_1) + C(z)(1 - \tilde{A}(\lambda\alpha_1)))\tilde{S}(\lambda\alpha_2(1 - C(z))) \}} \quad (18)$$

$$\Pi(z) = \frac{\lambda P_0 \left\{ \left[\sum_{j=1}^J \frac{[\tilde{V}(\lambda\alpha_3)(1-C(z))-1]}{[\tilde{V}(\lambda\alpha_3)]^{j-1}} - 1 \right] (\tilde{A}(\lambda\alpha_1) + C(z)(1 - \tilde{A}(\lambda\alpha_1))) + C(z)(1 - \tilde{S}(\lambda\alpha_2(1 - C(z)))) \right\}}{\alpha_1 \{ z - ((1-p) + pz)(\tilde{A}(\lambda\alpha_1) + C(z)(1 - \tilde{A}(\lambda\alpha_1)))\tilde{S}(\lambda\alpha_2(1 - C(z)))(\lambda\alpha_2(1 - C(z))) \}} \quad (19)$$

$$\Omega_j(z) = \frac{P_0(1 - \tilde{V}(\lambda\alpha_3(1 - C(z))))}{[\tilde{V}(\lambda\alpha_3)]^{j-1} \alpha_3(1 - C(z))} \quad (20)$$

Then P_0 can be calculated using normalising condition and is given by

$$P_0 = \frac{\alpha_1 \alpha_3 c_1 (1 - p - c_1(1 - \tilde{A}(\lambda\alpha_1)) - s_1 \lambda \alpha_2 c_1)}{\alpha_3 c_1 (1 - \tilde{A}(\lambda\alpha_1))(N1 - 1 + p + c_1 + s_1 \lambda \alpha_1 c_1) + \alpha_1 \alpha_3 c_1 s_1 (c_1 A(\lambda\alpha_1) + N1) + N1 \alpha_1 (1 - p - c_1(1 - A(\lambda\alpha_1)) - s_1 \lambda \alpha_2 c_1)} \quad (21)$$

3.1. Performance measures

In this section, we obtain some interesting system performance measures like, the mean number of customers in the orbit (L_q), the average number of customer in the system (L_s).

From (21), it follows that the stability condition is $1 - p - c_1 (1 - \tilde{A}(\lambda\alpha_1)) - s_1 \lambda \alpha_2 c_1 > 0$

The probability generating function of mean number of customer in the queue is

$$\phi_q(z) = P_0 + P(z) + \Pi(z) + \sum_{j=1}^J \Omega_j(z)$$

Mean number of customers in the queue is given by $L_q = \lim_{z \rightarrow 1} \phi_q'(z)$

$$L_q = (ab - cd) / 3b^2$$

a =

$$\begin{aligned} P0 & \left(-3N2a_1a_2 - 3a_2a_3c_2 + 3a_1a_2a_3c_2 + 3a_2a_3c_2A - 6a_2a_3P \right. \\ & c_1^2A + 3a_2a_3N2c_1 + 3a_2a_3NIc_2 + 6a_2a_3Pc_1^2 - 3N2a_2a_1P \\ & + 3a_2a_3Pc_2 - 3N2a_1a_2c_1 - 3NIa_1a_2c_2 + 6a_2a_3c_1c_2 \\ & - 6a_1a_2a_3Pc_1^2 - 3a_1a_2a_3Pc_2 - 3a_2a_3PAc_2 + 6a_1a_2NIc_1 \\ & - 3a_2a_3AN2c_1 - 3a_2a_3ANIc_2 - 6a_2a_3ANIc_1 \\ & + 6NIa_1a_2Pc_1A + 6a_1a_2a_3c_1Ac_2 + 3(-a_1a_2a_3c_1 \\ & + a_1a_3NI - a_1a_2NI + 2a_2a_3c_1)s_2l^2a_2^2c_1^2 + 3(-a_1a_2a_3c_1 \\ & - a_2a_3c_1A + a_1a_3N2 + a_1a_3c_1A + a_2a_3c_1 - a_1a_2NI) \\ & s_1la_2c_2 + 3(-N2a_1a_2 + a_2a_3c_2 + 2a_1a_3N2 - 8a_1a_2a_3c_1P \\ & + 2a_1a_3NIc_1 - a_1a_2a_3c_2 - a_2a_3c_2A - a_1a_3N2A \\ & + 2a_1a_2a_3c_1^2A - 2a_1a_2a_3c_1^2 - 2a_2a_3c_1^2A + a_1a_3c_2A \\ & - 2a_1a_2NIc_1 + 2a_1a_2NIc_1A + 2a_2a_3Pc_1 - 2NIa_1a_2P \\ & - 2a_1a_3NIc_1A - 2a_2a_3Pc_1A - 2a_1a_2a_3c_1P + 2a_2a_3c_1^2) \\ & s_1la_2c_1 + 6a_1a_2a_3Pc_1^2A - 6a_1a_2a_3c_1c_2 - 6a_2a_3c_1Ac_2 \\ & \left. + 3N2a_1a_2c_1A + 3NIa_1a_2c_2A - 6a_1a_3N2c_1A \right) \end{aligned}$$

$$b = 2a_1a_2a_3c_1(-c_1 + c_1A - s_1la_2c_1 + 1)$$

$$\begin{aligned} c = & 3a_1a_2a_3c_1(c_2(A-1) + 2c_1P(A-1) + 2s_1la_2c_1(c_1(A-1) \\ & - P) - s_2l^2a_2^2c_1^2 - s_1la_2c_2) + 3a_1a_2a_3c_2(c_1(A-1) - P \\ & - s_1la_2c_1 + 1) \end{aligned}$$

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d =

$$\begin{aligned}
 P0 \left(-3 N2 a_1 a_2 - a_2 a_3 N2 + a_2 a_3 c_2 + 2 a_1 a_2 N1 - 2 a_1 a_3 c_1 \right. \\
 + 2 a_1 a_3 N2 + 2 a_1 a_2 a_3 c_1 + 2 a_2 a_3 c_1 A + 2 a_1 a_3 N1 c_1 \\
 - a_1 a_2 a_3 c_2 - a_2 a_3 c_2 A - a_1 a_3 N2 A + 4 a_1 a_2 a_3 c_1^2 A \\
 - 4 a_1 a_2 a_3 c_1^2 - 4 a_2 a_3 c_1^2 A + 2 a_1 a_3 c_2 A - 4 a_1 a_2 N1 c_1 + \\
 a_2^2 A N2 + 6 a_1 a_2 N1 c_1 A - 2 a_2 a_3 A N1 c_1 + 2 (a_1 a_2 a_3 \\
 - a_1 a_2 a_3 P - a_1 a_2 a_3 c_1 - a_2 a_3 c_1 A + a_1 a_3 N1 + a_1 a_3 c_1 A \\
 + a_2 a_3 c_1 - a_1 a_2 N1 - a_1 a_2 a_3 P A) s_1 l a_2 c_1 + 4 a_2 a_3 P c_1 \\
 - 4 N1 a_1 a_2 P - 4 a_1 a_3 N1 c_1 A - 4 a_2 a_3 P c_1 A - 4 a_1 a_2 a_3 c_1 P \\
 \left. + 4 a_2 a_3 c_1^2 \right)
 \end{aligned}$$

The probability generating function of mean number of customer in the queue is

$$\phi_s(z) = P_0 + P(z) + z\Pi(z) + \sum_{j=1}^J \Omega_j(z)$$

Mean number of customers in the system is given by $L_s = \lim_{z \rightarrow 1} \phi_s(z)$

$$L_s = (ab - cd) / 3b^2$$

where a =

$$\begin{aligned}
 P0 \left(-6 a_2 a_3 A N1 c_1 + 3 a_1 a_2 a_3 c_2 + 3 a_2 a_3 c_2 A + 3 a_1 a_3 N2 \right. \\
 - 3 a_1 a_3 N2 A + 6 a_2 a_3 N1 c_1 - 3 a_2 a_3 A N1 c_2 + 3 a_1 a_3 N1 A \\
 - 3 a_1 a_3 N1 c_2 + 3 a_2 a_3 N2 c_1 - 3 a_2 a_3 A N2 c_1 + 3 a_2 a_3 N1 c_2 \\
 - 3 N2 a_1 a_2 P + 3 a_2 a_3 P c_2 - 3 N2 a_1 a_2 c_1 - 6 a_2 a_3 c_1 A c_2 \\
 - 3 a_2 a_3 P c_2 A + 3 N2 a_1 a_2 c_1 A + 3 N1 a_1 a_2 c_2 A \\
 - 3 a_1 a_2 a_3 P c_2 - 6 a_1 a_2 a_3 c_1 c_2 + 6 a_2 a_3 c_1 c_2 + 6 a_2 a_3 c_1^2 P \\
 - 6 a_1 a_2 a_3 c_1^2 P - 6 a_2 a_3 P c_1^2 A + 6 a_1 a_2 a_3 P c_1^2 A \\
 - 6 N1 a_1 a_2 P c_1 + 6 a_1 a_2 a_3 c_1 A c_2 + 6 N1 a_1 a_2 P c_1 A + 3 (\\
 - 3 a_2 a_3 c_1 - a_1 a_2 a_3 c_1 - a_2 a_3 c_1 A - N1 a_1 a_2 + a_1 a_3 N1 \\
 + a_1 a_3 c_1 A) s_2 l^2 a_2^2 c_1^2 + 3 (a_2 a_3 c_1 - a_1 a_2 a_3 c_1 - a_2 a_3 c_1 A \\
 - N1 a_1 a_2 + a_1 a_3 N1 + a_1 a_3 c_1 A) s_1 l a_2 c_2 + 3 (-2 a_1 a_2 a_3 c_1^2 \\
 - 2 a_1 a_3 N1 c_1 A + 2 a_2 a_3 c_1^2 - 2 a_1 a_2 a_3 P c_1 - 2 a_2 a_3 P c_1 A \\
 + 2 a_1 a_2 a_3 c_1^2 A + 2 N1 a_1 a_2 c_1 A - 2 N1 a_1 a_2 c_1 - 2 a_3 a_2 c_1^2 A \\
 - a_1 a_2 a_3 c_2 - a_2 a_3 c_2 A + a_1 a_3 c_2 A + a_1 a_3 N2 + 2 a_1 a_3 c_1 A \\
 + a_1 a_3 N2 A - 2 N1 a_1 a_2 P - a_1 a_3 N1 A + 2 a_1 a_3 N1 c_1 \\
 + 2 a_2 a_3 c_1 P + 2 a_1 a_3 N1 + a_2 a_3 c_2 - N2 a_1 a_2) s_1 l a_2 c_1 \\
 \left. - 3 a_2 a_3 c_2 \right)
 \end{aligned}$$

$$b = 2 a_1 a_2 a_3 c_1 (-c_1 + c_1 A - s_1 l a_2 c_1 + 1)$$

$$c = 3 a_1 a_2 a_3 c_1 (-c_2 + c_2 A - 2 P c_1 + 2 P c_1 A + 2 s_1 l a_2 c_1 (-c_1 + c_1 A) - s_2 l^2 a_2^2 c_1^2 - s_1 l a_2 c_2) + 3 a_1 a_2 a_3 c_2 (-c_1 + c_1 A - s_1 l a_2 c_1 + 1)$$

$$d = P0 (-2 a_1 a_2 a_3 c_1^2 - 2 a_2 a_3 A N1 c_1 + 2 a_2 a_3 c_1^2 - 2 a_1 a_2 a_3 P c_1 - 2 a_2 a_3 P c_1 A + 2 a_1 a_2 a_3 c_1^2 A + 2 N1 a_1 a_2 c_1 A - 2 N1 a_1 a_2 c_1 - 2 a_3 a_2 c_1^2 A - a_1 a_3 N2 + 2 a_2 a_3 N1 c_1 + 2 (a_2 a_3 c_1 - a_1 a_2 a_3 c_1 - a_2 a_3 c_1 A - N1 a_1 a_2 + a_1 a_3 N1 A + a_1 a_3 c_1 A) s_1 l a_2 c_1 - 2 N1 a_1 a_2 P + 2 a_1 a_2 a_3 c_1 + 2 a_2 a_3 c_1 A + 2 a_2 a_3 c_1 A - 2 a_2 a_3 c_1 + 2 N1 a_1 a_2 + a_1 a_3 N1 - 4 a_2 a_3 c_2)$$

where l is λ and $N1 = \lambda \alpha_3 v_1 \frac{1 - [\tilde{V}(\lambda \alpha_3)]^J}{[\tilde{V}(\lambda \alpha_3)]^J (\tilde{V}(\lambda \alpha_3) - 1)}$

$$N2 = \lambda \alpha_3 v_2 \frac{1 - [\tilde{V}(\lambda \alpha_3)]^J}{[\tilde{V}(\lambda \alpha_3)]^J (\tilde{V}(\lambda \alpha_3) - 1)}$$

5. Conclusion

In this paper, a queue with general retrial times and state dependent admission feed back and modified vacation policy is discussed. The explicit results for the average number of customers in the system/orbit are also derived. The analyses and results presented in this paper may be useful for network system designers and software system designers.

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