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Ordering Policy for Deteriorating Items with Time Dependent Demand

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Abstract. This paper deals with an inventory model to determine the optimal ordering quantity and optimal cycle time. It is assumed that the annual demand is a decreasing function of time and deteriorating units are replaced during the cycle time. Numerical examples are given to illustrate our results.

Keywords: Inventory model, deteriorating units, demand

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

In real world, the seasonal goods fashion foods have been attended where they have been ordered at a period and have been sold at a given duration. The objective of inventory management deals with minimization of the inventory carrying cost. Thus, it is very important to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand. Many mathematical models have been developed for controlling inventory. The majority of the earlier inventory models consider that the demand of item is constant. This is a future of static environment, while in today's dynamic environment nothing is fixed or constant. In most of the cases the demand of items increases with time. Most of the companies are working towards increasing their demand of items with time. The deteriorating inventory system has been studying considerably in the recent years, for example [5] Ghare and Schrader(1963) initially worked in this field and it was extended by [7] Haris) EOQ model with deterioration and shortages. [6] Goyal and Giri gave a survey on recent trends in the inventory models of deteriorating items. [10] In 1978, Philp relaxed the assumptions of constant deterioration rate by considering a two parameter Weibull distribution and assuming that the average carrying cost can be estimated is half of the replenishment size. [12] Shah and Jaiswal extended this model to allow for backlogging. [4] Dave and Patel studied firstly the inventory model of deteriorating items with linear increasing demand when shortages were not allowed.[15] Mirsha introduced an Inventory model for time dependent holding cost and deteriorating with salvage value and shortages. [13] Singh and Pattnayak gave An EOQ Model for Deteriorating items with Linear Demand, Variable Deterioration and partial Backlogging. In this proposed study, an inventory model has been developed for deteriorating items. In this model deteriorating items are replaced under some conditions. The deteriorating items are constant and shortages are allowed in this model.

2. Notations and assumptions

Notations

$D(t)$ - The demand rate per unit time.
θ - The deteriorating items per unit time.
H - The holding cost per unit.
C - The purchasing cost per unit time.
Q - The ordering quantity.
A - The ordering cost per unit.
T - The length of the cycle time.
C_1 - The inventory shortage cost per unit time.
T_r - The replacement time for deteriorating items.
TC - The total present value of the cost over the cycle time.

Assumptions

1. The replenishment is uniform.
2. The shortage cost is a linear function of time $D(t)=a(1-bt)$ If $a>0$ and $0<b>1$..
3. The holding cost is constant.
4. Shortages are allowed.
5.The deteriorating items are replaced when an inventory level reaches Q/2.
6. The purchasing cost of items used for replacement is C/2.

3. Mathematical formulation

If the inventory model with above described assumption and notation is depicted in fig 1, we consider I(t) as total amount of inventory at the beginning of the each period. The variation of inventory level I(t) with respect to time t due to combined effect of demand and deterioration. But we have to replace the deteriorating items at T_r. Then inventory level goes to zero at $T_1$and shortages occur during time period(T_1, T).The differential equation in which on hand inventory I(t) satisfying in three different part of cycle time T are given by

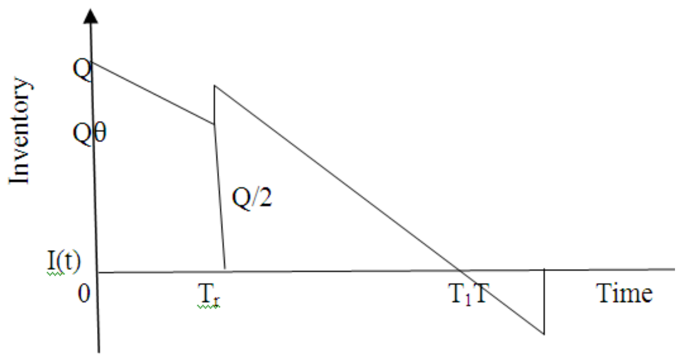

Figure 1:

$$\frac{dI(t)}{dt} + \theta I(t) = -a(1 - bt) \qquad 0 \le t \le T_r \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -a(1 - bt) T_r \le t \le T_1 \tag{2}$$

$$\frac{dI(t)}{dt} = -a(1 - bt) T_1 \le t \le T \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -a(1 - bt) \qquad 0 \le t \le T_r$$

$$I(t)e^{\theta t} = -a \int (1 - bt)e^{\theta t} dt$$

$$I(t)e^{\theta t} = -a \left[\frac{(1 - bt)e^{\theta t}}{\theta} + \frac{be^{\theta t}}{\theta^2}\right] + c$$

$I(t)e^{\theta t} = \frac{-a}{\theta^2}[\theta(1 - bt) + b]e^{\theta t} + c$, where c is the constant of integration.

The solution of equation (1), with boundary condition I(0)=Q, t=0

$$Q = \frac{-a}{\theta^2}[\theta + b] + c$$

$$Q + \frac{a}{\theta^2}[\theta + b] = c$$

$$I(t)e^{\theta t} = \frac{-a}{\theta^2}[\theta(1 - bt) + b]e^{\theta t} + Q + \frac{a}{\theta^2}[\theta + b]$$

$$I(t) = \frac{-a}{\theta^2}[\theta(1 - bt) + b] + Q + \left[\frac{a}{\theta^2}(\theta + b)\right] e^{-\theta t}$$

$$I(t) = Q - \theta Q t - at. \tag{4}$$

Thus the replacement time is obtained by putting the boundary condition t=T_r , I(T_r)=$\frac{Q}{2}$in equation (1),

$$\frac{Q}{2} = Q - \theta Q T_r - aT_r$$

$$-\frac{Q}{2} = -\theta Q T_r - aT_r$$

$$\frac{Q}{2(\theta Q + a)} = T_r \tag{5}$$

During the period (T_r,T_1) the inventory depletes due to the detoured or demand. Hence the inventory level at any time during (T_r,T_1) is described by differential equation.

$$\frac{dI(t)}{dt} + \theta I(t) = -a(1 - bt) T_r \le t \le T_1$$

$$I(t)e^{\theta t} = -a \left[\frac{(1 - bt)e^{\theta t}}{\theta} + \frac{be^{\theta t}}{\theta^2}\right] + c$$

$$I(t) = -a \left[\frac{(1 - bt)}{\theta} + \frac{b}{\theta^2}\right] + ce^{-\theta t}$$

With the boundary condition when t=T_r , I(T_r)=$\frac{Q}{2} + \theta Q$, we have

$$\frac{Q(1 + 2\theta)}{2} = -a \left[\frac{(1 - bT_r)}{\theta} + \frac{b}{\theta^2}\right] + ce^{-\theta T_r}$$

$$ce^{-\theta T_r} = a \left[\frac{(1 - bT_r)}{\theta} + \frac{b}{\theta^2}\right] + \frac{Q(1 + 2\theta)}{2}$$

$$c = e^{\theta T_r} \left[a\left(\frac{(1 - bT_r)}{\theta} + \frac{b}{\theta^2}\right) + \frac{Q(1 + 2\theta)}{2}\right]$$

$$I(t) = -a\left[\frac{(1-bt)}{\theta} + \frac{b}{\theta^2}\right] + \left[a\left(\frac{(1-bT_r)}{\theta} + \frac{b}{\theta^2}\right) + \frac{Q(1+2\theta)}{2}\right] e^{\theta(T_r - t)}$$

$$I(t) = \frac{Q(1+2\theta)}{2} + \left[a - abT_r + \frac{\theta Q(1+2\theta)}{2}\right](T_r - t) \qquad (6)$$

With the boundary condition $t=T_1$,$I(T_1)=0$

$$T_1 = \frac{\frac{Q(1+2\theta)}{2}}{\left[a - abT_r + \frac{\theta Q(1+2\theta)}{2}\right]} + T_r$$

$$T_1 = \frac{Q(1+2\theta)}{[2a(1-bT_r)+\theta Q(1+2\theta)]} + T_r \qquad (7)$$

Thus the initial order quantity is obtained using the boundary conditions t=0 I(t)=Q

$$Q = -\frac{a}{\theta} - \frac{ab}{\theta^2} + \frac{Q(1+2\theta)}{2} + \frac{a}{\theta} - \frac{ab}{\theta}T_r + \frac{ab}{\theta^2} + \frac{Q(1+2\theta)\theta T_r}{2} + aT_r - abT_r^{\,2} + \frac{ab}{\theta}T_r$$

$$Q - \frac{Q(1+2\theta)}{2} - \frac{Q(1+2\theta)\theta T_r}{2} = aT_r - abT_r^{\,2}$$

$$Q\left[1 - \frac{(1+2\theta)}{2} - \frac{(1+2\theta)\theta T_r}{2}\right] = aT_r(1-bT_r)$$

$$Q = \frac{aT_r(1-bT_r)}{\left[1 - \frac{(1+2\theta)}{2} - \frac{(1+2\theta)\theta T_r}{2}\right]} \qquad (8)$$

The state of inventory during (T_1,T) can be represented by the differential equation.

$$\frac{dI(t)}{dt} = -a(1-bt) T_1 \le t \le T$$

$$I(t) = -a\int (1-bt)dt$$

$$I(t) = -a\left(t - \frac{bt^2}{2}\right) + c$$

With the boundary condition $t=T_1 I(T_1)=0$

$$0 = -a\left(T_1 - \frac{bT_1^{\,2}}{2}\right) + c$$

$$c = a\left(T_1 - \frac{bT_1^{\,2}}{2}\right)$$

The solution of the equation is

$$I(t) = -a\left(t - \frac{bt^2}{2}\right) + a\left(T_1 - \frac{bT_1^{\,2}}{2}\right)$$

$$I(t) = a\left[\left(T_1 - \frac{bT_1^{\,2}}{2}\right) - \left(t - \frac{bt^2}{2}\right)\right]$$

$$I(t) = a\left[(T_1 - t) - \frac{b}{2}\left(T_1^{\,2} - t^2\right)\right] \qquad (9)$$

The ordering cost is A. The total inventory holding cost during [0,T] is

$$HC = h\left[\int_0^{T_r} I(t)dt + \int_{T_r}^{T_1} I(t)dt\right]$$

$$HC = h\left[\int_0^{T_r}(Q-\theta Qt-at)dt + \int_{T_r}^{T_1}\left[\frac{Q(1+2\theta)}{2}+\left[a-abT_r+\frac{\theta Q(1+2\theta)}{2}\right](T_r-t)\right]dt\right]$$

$$HC = h\left\{\left[QT_r-(\theta Q-a)\frac{{T_r}^2}{2}\right]+\frac{Q(1+2\theta)}{2}(T_1-T_r) + \left(a-abT_r+\frac{\theta Q(1+2\theta)}{2}\right)\left(T_r(T_1-T_r)-\left(\frac{{T_1}^2}{2}-\frac{{T_r}^2}{2}\right)\right)\right\}$$

$$HC = h\left\{\left[QT_r-(\theta Q-a)\frac{{T_r}^2}{2}\right]+\frac{Q(1+2\theta)}{2}(T_1-T_r)+\left(a-abT_r+\frac{\theta Q(1+2\theta)}{2}\right)\left(T_1T_r-\frac{{T_1}^2}{2}-\frac{{T_r}^2}{2}\right)\right\} \quad (10)$$

The shortage cost during [T_1,T]

$$SC = C_1\left[\int_{T_1}^{T}I(t)dt\right]$$

$$SC = C_1a\left[\int_{T_1}^{T}\left[(T_1-t)-\frac{b}{2}\left({T_1}^2-t^2\right)\right]dt\right]$$

$$SC = C_1a\left[\left(T_1(T-T_1)-\left(\frac{T^2-{T_1}^2}{2}\right)\right)-\frac{b}{2}\left({T_1}^2(T-T_1)-\left(\frac{T^3-{T_1}^3}{3}\right)\right)\right]$$

$$SC = C_1a\left[T_1T-\frac{{T_1}^2}{2}-\frac{T^2}{2}-\frac{b}{2}\left({T_1}^2T-\frac{2T^3}{3}-\frac{{T_1}^3}{3}\right)\right]$$

$$SC = C_1a\left[T_1T-\frac{({T_1}^2+T^2)}{2}-\frac{b}{2}\left({T_1}^2T-\frac{(2T^3-{T_1}^3)}{3}\right)\right] \quad (11)$$

The purchasing cost during [0, T_1]

$$PC = CQ+\theta Q\frac{C}{2}$$

$$PC = \frac{QC}{2}(2+\theta) \quad (12)$$

The total cost per unit time TC (T) is

$$TC(T) = \frac{1}{T}[OC+SC+PC+HC]$$

Now TC (T) will be minimum when $\frac{\partial TC(T)}{\partial T}=0$ and $\frac{\partial^2 TC(T)}{\partial T^2}>0$, The optimum values of T for the minimum average total cost TC(T) is the solution of equation

$$\frac{\partial TC(T)}{\partial T}=0$$

$$\frac{\partial TC(T)}{\partial T} = -\frac{A}{T^2} - \frac{h}{T^2}\left\{\left[QT_r - (\theta Q - a)\frac{{T_r}^2}{2}\right] + \frac{Q(1+2\theta)}{2}(T_1 - T_r) + \left(a - abT_r + \frac{\theta Q(1+2\theta)}{2}\right)\left(T_1T_r - \frac{{T_1}^2}{2} - \frac{{T_r}^2}{2}\right)\right\} - \frac{QC}{T^2}\frac{(2+\theta)}{2} + C_1a\left(\frac{{T_1}^2}{2T^2} - \frac{1}{2} - \frac{b}{2}\left(\frac{2{T_1}^3}{3} - \frac{2T}{3}\right)\right) = 0$$

$$-\frac{A}{T^2} - \frac{h}{T^2}\left\{\left[QT_r - (\theta Q - a)\frac{{T_r}^2}{2}\right] + \frac{Q(1+2\theta)}{2}(T_1 - T_r) + \left(a - abT_r + \frac{\theta Q(1+2\theta)}{2}\right)\left(T_1T_r - \frac{{T_1}^2}{2} - \frac{{T_r}^2}{2}\right)\right\} - \frac{QC}{T^2}\frac{(2+\theta)}{2} + \frac{C_1a}{T^2}\left(\frac{{T_1}^2}{2} - \frac{2{T_1}^3}{3}\right) + C_1a\left(\frac{bT}{3} - \frac{1}{2}\right) = 0$$

$$T^2C_1a\left(\frac{bT}{3} - \frac{1}{2}\right) = A + h\left\{\left[QT_r - (\theta Q - a)\frac{{T_r}^2}{2}\right] + \frac{Q(1+2\theta)}{2}(T_1 - T_r) + \left(a - abT_r + \frac{\theta Q(1+2\theta)}{2}\right)\left(T_1T_r - \frac{{T_1}^2}{2} - \frac{{T_r}^2}{2}\right)\right\} + QC\frac{(2+\theta)}{2} - C_1a\left(\frac{{T_1}^2}{2} - \frac{2{T_1}^3}{3}\right)$$

$$(2bT^3 - 3T^2) = \frac{6}{C_1a}\left\{A + h\left\{\left[QT_r - (\theta Q - a)\frac{{T_r}^2}{2}\right] + \frac{Q(1+2\theta)}{2}(T_1 - T_r) + \left(a - abT_r + \frac{\theta Q(1+2\theta)}{2}\right)\left(T_1T_r - \frac{{T_1}^2}{2} - \frac{{T_r}^2}{2}\right)\right\} + QC\frac{(2+\theta)}{2} - C_1a\left(\frac{{T_1}^2}{2} - \frac{2{T_1}^3}{3}\right)\right\}$$

Provided that it satisfies the following conditions

$$\frac{\partial^2 TC(T)}{\partial T^2} = \frac{2A}{T^3} + \frac{2h}{T^3}\left\{\left[QT_r + \theta Q(a-1)\frac{{T_r}^2}{2}\right] + \frac{Q(1+2\theta)}{2}(T_1 - T_r) + \left(a(1 - bT_r) + \frac{\theta Q(1+2\theta)}{2}\right)\left(\frac{(T_1 - T_r)^2}{2}\right)\right\} + \frac{2QC}{T^3}\frac{(2+\theta)}{2} + \frac{2C_1a{T_1}^2}{T^3}\left(\frac{3 - 2bT_1}{6}\right) + C_1a\left(\frac{b}{3}\right) > 0$$

$\frac{\partial^2 \mathrm{TC(T)}}{\partial \mathrm{T}^2} > 0$, $[\mathrm{T}_1 > \mathrm{T}_r, 3 > 2b\mathrm{T}_1, \mathrm{a} > 1$,(Therefore its terms are positive)]

3.1. Numerical examples

Example 3.1.1. Let a=100 units/year b=0.2, A=100 per order, C=8 per unit, C_1=100 per unit, h=60 unit/annum, θ=0.2 by the help of mathematical calculations, we obtain the optimum solutions are Q*= 150 and T*= 1.5 putting Q* and T* we get the optimum average cost TC (T)*=3297.

Example 3.1.2. Let a=400 units/year=0.2, A=100 per order, C=8 per unit, C_1=100 per unit, h=60 unit/annum, θ=0.2 by the help of mathematical calculations, we obtain the optimum solutions are Q*=603 and T*=1.9 putting Q* and T* we get the optimum average cost TC (T)*=10226.

5. Conclusion

In this paper, an inventory model is developed in which annual demand is decreasing function of time and deteriorating items are replaced during the cycle time. This model is solved analytically by minimizing the total cost finally; the proposed model is verified by the numerical example.

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