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On Lexicographic Products of Two Fuzzy Graphs

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Abstract. In this paper, lexicographic products of two fuzzy graphs namely, lexicographic min-product and lexicographic max-product which are analogous to the concept lexicographic product in crisp graph theory are defined. It is illustrated that the operations lexicographic products are not commutative. The connected, effective and complete properties of the operations lexicographic products are studied. The degree of a vertex in the lexicographic products of two fuzzy graphs is obtained. A relationship between the lexicographic min-product and lexicographic max-product is also obtained.

Keywords: Connected fuzzy graph, effective fuzzy graph, regular fuzzy graph, lexicographic min-product and lexicographic max-product

AMS Mathematics Subject Classification (2010): 03E72, 05C07

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Later on, Bhattacharya [1] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson and Peng [3]. We defined the direct sum of two fuzzy graphs and studied the properties of that operation [6]. Also we defined the strong product of two fuzzy graphs and studied its properties [8]. In this paper, we have introduced the concept of lexicographic products of two fuzzy graphs namely, lexicographic min-product and lexicographic max-product which are analogous to the concept lexicographic product in crisp graph theory. We have illustrated that these operations are not commutative and studied the connected, effective and complete properties of these operations. We have obtained the degree of a vertex in the lexicographic products of two fuzzy graphs and obtained a relationship between the lexicographic min-product and lexicographic max-product. First let us recall some preliminary definitions that can be found in [1]-[9]. A fuzzy graph G is a pair of functions (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G:(\sigma, \mu)$ is denoted by $G^*:(V, E)$ where $E \subseteq V\times V$. $G:(\sigma, \mu)$ is called a connected fuzzy graph if for all $u, v \in V$ there exists at least one non-zero path between u and v. $G:(\sigma, \mu)$ is called effective if $\mu(u\,v) = \sigma(u) \wedge \sigma(v)$ for all $u\,v \in E$ and complete if $\mu(u\,v) = \sigma(u) \wedge \sigma(v)$ for all u, v∈V. The degree of a vertex u of G:(σ,μ) is defined as $d_G(u)=\sum_{u\neq v}\mu(uv)=\sum_{uv\in E}\mu(uv)$. G′:(σ′, μ′) is called a spanning fuzzy sub graph of G:(σ, μ) if σ = σ′ and μ′ ⊆ μ, that is, if σ(u)=σ′(u) for every u∈V and μ′(e)≤ μ(e) for every e∈E. The lexicographic product of $G_1:(V_1,E_1)$ with $G_2:(V_2,E_2)$ is defined as $G_1[G_2]:(V, E)$ where $V=V_1\times V_2$ and $E=\{(u_1, v_1)(u_2, v_2)/u_1u_2\in E_1$ or $u_1= u_2$ and $v_1v_2\in E_2\}$.

2. Lexicographic min-product

Definition 2.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ denote two fuzzy graphs. Define G:(σ, μ) with underlying crisp graph G*:(V,E) where $V=V_1\times V_2, E=\{(u_1,v_1)(u_2,v_2)/u_1u_2\in E_1$ or $u_1=u_2$ and $v_1v_2\in E_2\}$, by, $\sigma(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$, for all $(u_1, v_1) \in V_1 \times V_2$ and

$$\mu\big((u_1, v_1)(u_2, v_2)\big) = \begin{cases}\mu_1(u_1u_2) & ,\text{if } u_1u_2\in E_1\\ \sigma_1(u_1)\wedge\mu_2(v_1v_2) & ,\text{if } u_1=u_2, v_1v_2\in E_2.\end{cases}$$

If $u_1u_2\in E_1$, $\mu_1(u_1u_2) = \sigma_1(u_1)\wedge\sigma_1(u_2) \le [\sigma_1(u_1)\vee\sigma_2(v_1)]\wedge[\sigma_1(u_2)\vee\sigma_2(v_2)]=\sigma(u_1,v_1)\wedge\sigma(u_2, v_2)$. If $u_1= u_2$, $v_1v_2\in E_2$, $\sigma_1(u_1)\wedge\mu_2(v_1v_2)\le \sigma_1(u_1) \wedge [\sigma_2(v_1) \wedge \sigma_2(v_2)]= [\sigma_1(u_1)\wedge\sigma_2(v_1)] \wedge [\sigma_1(u_2)\wedge\sigma_2(v_2)] \le [\sigma_1(u_1)\vee\sigma_2(v_1)] \wedge [\sigma_1(u_2)\vee\sigma_2(v_2)]= \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$.
Hence $\mu((u_1, v_1)(u_2, v_2)) \le \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$. Therefore G:(σ, μ) is a fuzzy graph. This is called the lexicographic min-product of G_1 with G_2 and is denoted by $G_1[G_2]_{min}:(\sigma, \mu)$.

Remark 2.1. The operation lexicographic min-product of two fuzzy graphs is not commutative. That is $G_1[G_2]_{min}:(\sigma, \mu)$ is different from $G_2[G_1]_{min}:(\sigma', \mu')$. This is illustrated through the following Figure-1.

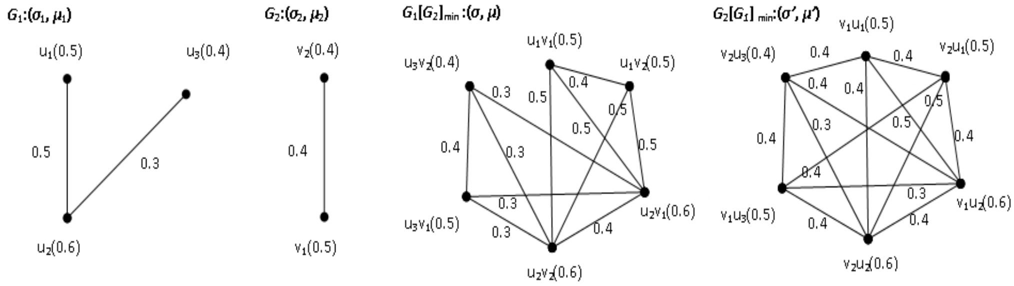

Figure 1:

Remark 2.2. The lexicographic min-product of two effective fuzzy graphs need not be effective. Also the lexicographic min-product of two complete fuzzy graphs need not be complete. The following Figure-2 illustrates this remark.

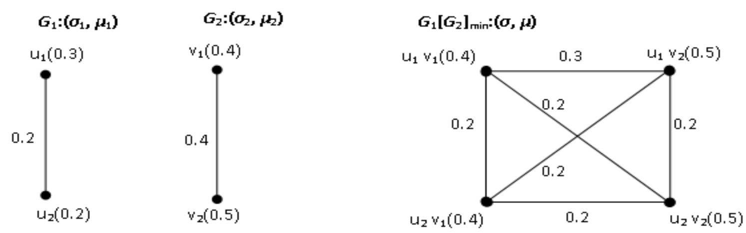

Figure 2:

Notation: The relation $\sigma_1 \le \sigma_2$ means that $\sigma_1(u) \le \sigma_2(v)$ for every $u\in V_1$ and for every $v\in V_2$ where σ_i is a fuzzy subset of V_i, i =1,2.

Theorem 2.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two effective fuzzy graphs with underlying crisp graphs $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ respectively such that $\sigma_1 \geq \sigma_2$ and μ_1 and μ_2 are constant functions of same value, then the lexicographic min-product of $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is an effective fuzzy graph.

Proof: Proceeding as in the definition, if $u_1u_2 \in E_1$, $\mu((u_1, v_1)(u_2, v_2)) = \mu_1(u_1u_2) = \sigma_1(u_1) \wedge \sigma_1(u_2) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$. If $u_1=u_2$ and $v_1v_2 \in E_2$, $\mu((u_1,v_1)(u_2,v_2)) = \mu_2(v_1v_2) = \mu_1(u_1u_2) = \sigma_1(u_1) \wedge \sigma_1(u_2) = \sigma(u_1,v_1) \wedge \sigma(u_2,v_2)$. Thus $\mu((u_1,v_1)(u_2,v_2)) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$ for all $((u_1, v_1)(u_2, v_2)) \in E$. Hence $G_1[G_2]_{min}:(\sigma, \mu)$ is an effective fuzzy graph.

Remark 2.3. In theorem 2.1 if we replace "$\sigma_1 \geq \sigma_2$" by "$\sigma_1 \leq \sigma_2$" then $G_1[G_2]$ will be replaced by $G_2[G_1]$. Also theorem 2.1 is true for the complete fuzzy graphs.

Theorem 2.2. The lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of two connected fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ is a connected fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ is connected.

Proof: From the definition, $G_1[G_2]_{min}:(\sigma, \mu)$ has $|V_2|$ copies of G_1. That is for each vertex in G_2 there is a copy of G_1 in $G_1[G_2]_{min}:(\sigma, \mu)$. Also G_1 is connected. Hence $G_1[G_2]_{min}:(\sigma, \mu)$ is connected.

Conversely, assume that $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $G_1[G_2]_{min}:(\sigma, \mu)$ is connected. To prove: $G_1:(\sigma_1,\mu_1)$ is connected.

Suppose that $G_1:(\sigma_1,\mu_1)$ is not connected. Then there exists at least two different vertices u_1, u_2 in V_1 such that there is no path between them. But since $G_1[G_2]_{min}:(\sigma, \mu)$ is connected, for any two vertices of the form (u_1, v_i) and $(u_2, v_j) \in V_1 \times V_2$ there is at least one path between them. This implies that there must be at least one path between the vertices u_1, u_2. This is a contradiction. Hence $G_1:(\sigma_1,\mu_1)$ is connected.

Example 2.1. Consider the two fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ where $G_1:(\sigma_1,\mu_1)$ is not a connected fuzzy graph and $G_2:(\sigma_2,\mu_2)$ is a connected fuzzy graph. The lexicographic min-product of $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is not a connected fuzzy graph.

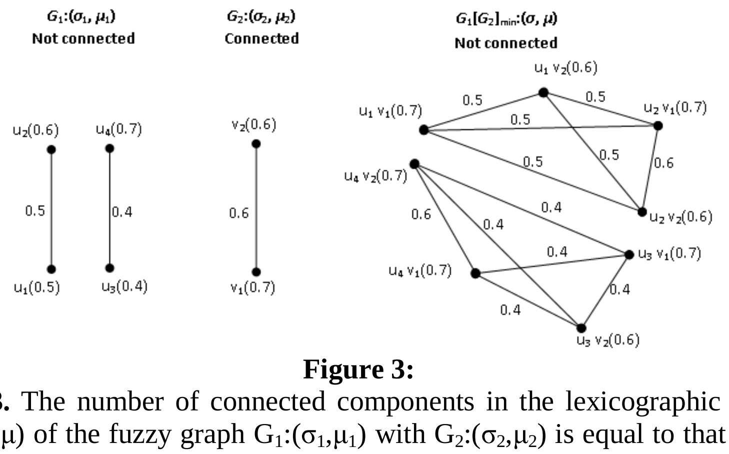

Figure 3:

Theorem 2.3. The number of connected components in the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is equal to that of the fuzzy graph $G_1:(\sigma_1,\mu_1)$.

Proof: Let $G_1:(\sigma_1,\mu_1)$ be a connected fuzzy graph and $G_2:(\sigma_2,\mu_2)$ be a fuzzy graph.Then by theorem 2.2 the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is connected. This implies that both G_1 and $G_1[G_2]_{min}:(\sigma, \mu)$ are connected and hence the theorem. Suppose that the fuzzy graph $G_1:(\sigma_1,\mu_1)$ is not connected and has 'm' disjoint connected components(say). Then we can rename the vertices of G_1 in such a way that $\{u_1, u_2,\ldots,u_{k1}\}$, $\{u_{k1+1}, u_{k1+2},\ldots,u_{k2}\},\ldots\ldots,\{u_{km+1}, u_{km+2},\ldots,u_{km+n}\}$ are the vertex sets of the 'm' disjoint connected components of G_1. If $\{v_1, v_2,\ldots,v_n\}$ is the vertex set of G_2 then for each vertex v_i in G_2, there is a copy of each connected component of G_1 in the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$. There is no edge between these components. For, if there is an edge between u_1v_i, $u_{k1+1}v_i$, then there must be an edge between u_1, u_{k1+1} in G_1 which is a contradiction. Thus each connected component in the lexicographic min-product $G1[G_2]_{min}:(\sigma, \mu)$ is disjoint from every other component and hence the theorem.

Example 2.2. The following Figure 4 gives an example of the lexicographic min-product of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ where $G_1:(\sigma_1,\mu_1)$ is not a connected fuzzy graph with three disjoint connected components; $G_2:(\sigma_2,\mu_2)$ is a connected fuzzy graph and $G_1[G_2]_{min}$ is a non-connected fuzzy graph with three disjoint connected components.

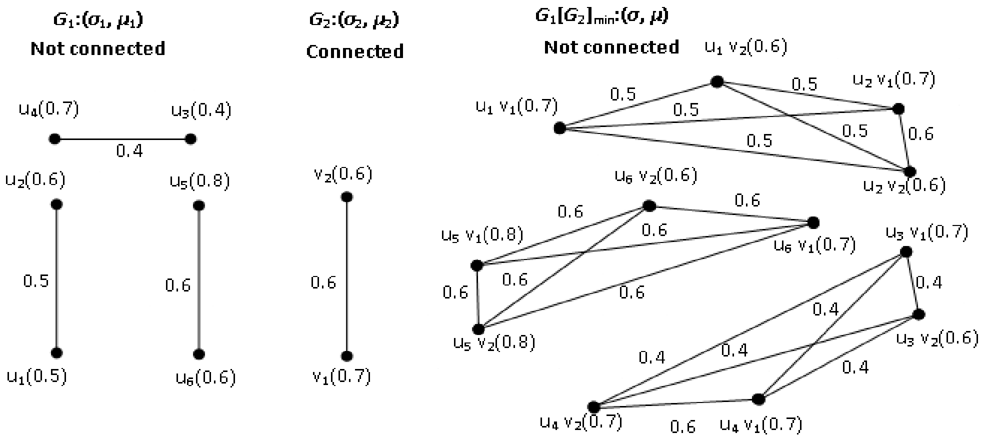

Figure 4:

3. Degree of a vertex in the lexicographic min-product

The degree of any vertex in the lexicographic min-product$G_1[G_2]_{min}$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is given by,

$$d_{G_1[G_2]_{min}}(u_i, v_j) = \sum_{u_iu_k \in E_1, v_\ell \in V_2} \mu_1(u_iu_k) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \wedge \mu_2(v_jv_\ell) .$$

Theorem 3.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \geq \mu_2$, then the degree of a vertex in the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is given by, $d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j).$

Proof: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$. This implies that $\sigma_1 \wedge \mu_2 = \mu_2$. Then the degree of any vertex $(u_i, v_j) \in V_1 \times V_2$ is given by,

$$d_{G_1[G_2]_{min}}(u_i, v_j) = \sum_{u_iu_k \in E_1, v_\ell \in V_2} \mu_1(u_iu_k) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \wedge \mu_2(v_jv_\ell)$$

$$= |V_2| \sum_{u_iu_k \in E_1} \mu_1(u_iu_k) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \mu_2(v_jv_\ell) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$$

Corollary 3.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \geq \mu_2$and μ_2 is a constant function of value 'c', then the degree of a vertex in the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the two fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ is given by, $d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c$.

Theorem 3.2. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$ then the degree of a vertex in the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is given by, $d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i)$.

Proof: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$. This implies that $\sigma_1 \wedge \mu_2 = \sigma_1$. Then, the degree of any vertex $(u_i, v_j) \in V_1 \times V_2$ is given by,

$$d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \wedge \mu_2(v_jv_\ell)$$

$$= |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i)$$

Corollary 3.2. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$ and σ_1 is a constant function of value 'c', then the degree of a vertex in the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is given by, $d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c$.

Example 3.1. Consider the two fuzzy graphs $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ such that $\sigma_1 \geq \mu_2$. Their lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is given in Figure-1 of Remark 2.1. Now, $d_{G_1[G_2]_{min}}(u_2, v_2) = 2.0$ and $|V_2|\, d_{G_1}(u_2) + d_{G_2}(v_2) = 2(0.3 + 0.5) + 0.4 = 2.0$

Consider the two fuzzy graphs $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ such that $\sigma_1 \leq \mu_2$. Their lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is given in Figure - 2 of Remark 2.2. Now, $d_{G_1[G_2]_{min}}(u_1, v_1) = 0.7$ and $|V_2|\, d_{G_1}(u_1) + d_{G_2^*}(v_1)\sigma_1(u_1) = 2 \times 0.2 + 1 \times 0.3 = 0.7$

3.1. Lexicographic max-product

Definition 3.1.1. Define $G:(\sigma, \mu)$ with underlying crisp graph $G^*:(V, E)$ where $V=V_1 \times V_2$, $E = \{(u_1, v_1)(u_2, v_2) / u_1 u_2 \in E_1$ or $u_1 = u_2$ and $v_1 v_2 \in E_2\}$ by, $\sigma(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$, for all $(u_1, v_1) \in V_1 \times V_2$ and

$$\mu\big((u_1, v_1)(u_2, v_2)\big) = \begin{cases} \mu_1(u_1 u_2) & ,\text{if } u_1u_2 \in E_1 \\ \sigma_1(u_1) \vee \mu_2(v_1v_2) & ,\text{if } u_1 = u_2, v_1v_2 \in E_2. \end{cases}$$

If $u_1u_2 \in E_1$, $\mu_1(u_1 u_2) = \sigma_1(u_1) \wedge \sigma_1(u_2) \leq [\sigma_1(u_1) \vee \sigma_2(v_1)] \wedge [\sigma_1(u_2) \vee \sigma_2(v_2)] = \sigma(u_1,v_1) \wedge \sigma(u_2, v_2)$.
If $u_1=u_2, v_1v_2 \in E_2$, $\sigma_1(u_1) \vee \mu_2(v_1 v_2) \leq \sigma_1(u_1) \vee [\sigma_2(v_1) \wedge \sigma_2(v_2)] = [\sigma_1(u_1) \vee \sigma_2(v_1)] \wedge [\sigma_1(u_2) \vee \sigma_2(v_2)]$

$= \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$.

Hence $\mu((u_1, v_1)(u_2, v_2)) \leq \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$. Therefore $G:(\sigma, \mu)$ is a fuzzy graph. This is called the lexicographic max-product of G_1 with G_2 and is denoted by $G_1[G_2]_{max}:(\sigma, \mu)$.

Remark 3.1.1. The operation lexicographic max-product of two fuzzy graphs is not commutative. That is, $G_1[G_2]_{max}:(\sigma, \mu) \neq G_2[G_1]_{max}:(\sigma', \mu')$. Also the lexicographic max-product of two effective fuzzy graphs need not be effective and the lexicographic max-product of two complete fuzzy graphs need not be a complete. It is illustrated through the following Figure-5.

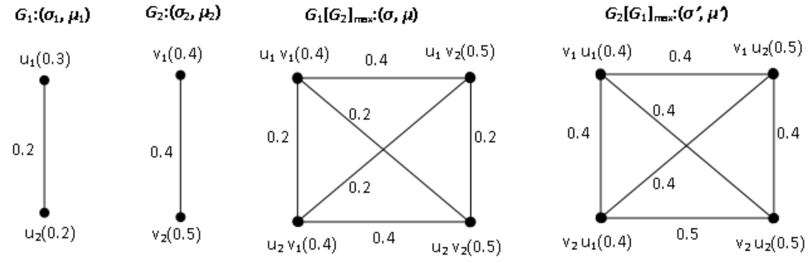

Figure 5:

Theorem 3.1.1. The lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ of two connected fuzzy graphs G_1 and G_2 is a connected fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ is connected.
(Proof of this theorem is similar to the proof of theorem 2.2.)

Theorem 3.1.2. The number of connected components in the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ of the fuzzy graph G_1 with G_2 is equal to that of the fuzzy graph $G_1:(\sigma_1,\mu_1)$.(Proof of this theorem is similar to the proof of theorem 2.3.)

3.2. Degree of a vertex in the lexicographic max-product

The degree of any vertex in the lexicographic max-product$G_1[G_2]$of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = \sum_{u_iu_k \in E_1, v_\ell \in V_2} \mu_1(u_iu_k) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \vee \mu_2(v_jv_\ell) \ .$$

Theorem 3.2.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$, then the degree of any vertex in $G_1[G_2]_{max}:(\sigma, \mu)$ is given by,

$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2| \, d_{G_1}(u_i) + d_{G_2}(v_j)$.

Proof: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$. This implies that $\sigma_1 \vee \mu_2 = \mu_2$. Then the degree of any vertex $(u_i, v_j) \in V_1 \times V_2$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = \sum_{u_iu_k \in E_1, v_\ell \in V_2} \mu_1(u_iu_k) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \vee \mu_2(v_jv_\ell)$$

$$= |V_2| \sum_{u_iu_k \in E_1} \mu_1(u_iu_k) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \mu_2(v_jv_\ell) = |V_2| \, d_{G_1}(u_i) + d_{G_2}(v_j)$$

Corollary 3.2.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$, and μ_2 is a constant function of value c, then for any vertex (u_i, v_j) in $G_1[G_2]_{max}:(\sigma,\mu)$ is given by, $d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c.$

Theorem 3.2.2. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ then the degree of any vertex in $G_1[G_2]_{max}:(\sigma, \mu)$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i).$$

Proof: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \ge \mu_2$. This implies that $\sigma_1 \vee \mu_2 = \sigma_1$. Then, the degree of any vertex $(u_i, v_j) \in V_1 \times V_2$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k,\, v_j v_\ell \in E_2} \sigma_1(u_i) \vee \mu_2(v_j v_\ell)$$

$$= |V_2|\, d_{G_1}(u_i) + \sum_{u_i = u_k,\, v_j v_\ell \in E_2} \sigma_1(u_i) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i)$$

Corollary 3.2.2. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ and σ_1 is a constant function of value c, then the degree of any vertex in $G_1[G_2]_{max}:(\sigma, \mu)$ is given by, $d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c.$

Example 3.2.1. Consider the two fuzzy graphs $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2,\mu_2)$ such that $\sigma_1 \le \mu_2$. Their lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is given in Figure-5 of Remark 3.1.1. Now, $d_{G_1[G_2]_{max}}(u_1, v_1) = 0.8$ and $|V_2|\, d_{G_1}(u_1) + d_{G_2}(v_1) = 2 \times 0.2 + 0.4 = 0.8$

Consider the two fuzzy graphs $G_2:(\sigma_2, \mu_2)$and $G_1:(\sigma_1, \mu_1)$ such that $\sigma_2 \ge \mu_1$. Their lexicographic max-product $G_2[G_1]_{max}: (\sigma', \mu')$ is given in Figure-5 of Remark 3.1.1. Now, $d_{G_2[G_1]_{max}}(v_2, u_1) = 1.3$ and $|V_1|\, d_{G_2}(v_2) + d_{G_1^*}(u_1)\sigma_2(v_2) = 2 \times 0.4 + 1 \times 0.5 = 1.3$

3.3. Relationship between the lexicographic products

Theorem 3.3.1. The lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph G_1 with G_2 is a spanning fuzzy sub graph of the lexicographic max-product $G_1[G_2]_{max}:(\tau, \nu)$.

Proof: Consider the lexicographic products $G_1[G_2]_{max}:(\tau, \nu)$ and $G_1[G_2]_{min}:(\sigma, \mu)$ of G_1 with G_2 defined on $G^*:(V,E)$ where $V=V_1\times V_2$, $E=\{(u_1,v_1)(u_2,v_2) \,/\, u_1u_2\in E_1$ or $u_1=u_2$ and $v_1 v_2 \in E_2\}$.From the definitions of the lexicographic max-product, and the lexicographic min-product, it is clear that $\tau(u_1, v_1) = \sigma(u_1, v_1)$ for all $(u_1, v_1) \in V$ and

$$\nu\big((u_1, v_1)(u_2, v_2)\big) \ge \mu\big((u_1, v_1)(u_2, v_2)\big) \text{ for all } (u_1, v_1)(u_2, v_2) \in E.$$

Thus $\tau = \sigma$ and $\mu \subseteq \nu$. Hence the lexicographic min-product is a spanning fuzzy sub graph of the lexicographic max-product.

Example 3.3.1. Consider the following Figure-6. Here the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is a spanning fuzzy sub graph of the lexicographic max-product $G_1[G_2]_{max}:(\tau, \nu)$.

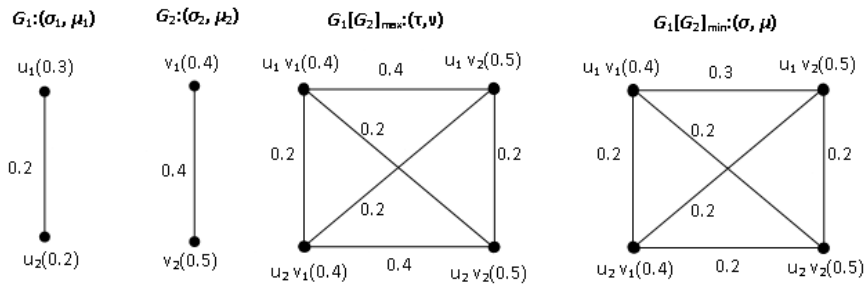

Figure 6:

7. Conclusion

In this paper, we have introduced the concept of lexicographic products of two fuzzy graphs namely, lexicographic min-product and lexicographic max-product which are analogous to the concept of lexicographic product in crisp graph theory. We have illustrated that the operations lexicographic products are not commutative and studied the connected, effective and complete properties of these operations. We have obtained the degree of a vertex in the lexicographic products of two fuzzy graphs. Also we have obtained a relationship between the lexicographic min-product and lexicographic max-product. In addition to the existing operations these operations and properties will also be helpful to study large fuzzy graph as a combination of small fuzzy graphs and to derive its properties from those of the small ones.

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