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Annihilator Graph on Union of Two Commutative KS – Semigroups

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Abstract. In this paper, we introduce the concept of Annihilator graph on commutative KS-semigroup and extend the result to union of twocommutative KS-semigroups. We also establish the result that the union of two Annihilator graph of any two commutative KS-semigroups X and Y is equal to the Annihilator graph of the commutative KS- semigroup $X \cup Y$ and also discuss some of its related properties.

Keywords: Annihilator, commutative KS- semigroup, annihilator graph

AMS Mathematics Subject Classification (2010): 06F35, 03G25

1. Introduction

In abstract algebra, mathematical system with one binary operation called group and two binary operations called rings were investigated. In 1966, Imai and Iseki [3] defined a class of algebra called BCK-algebra. A BCK– algebra is named after the combinators B, C and K by Carew Arthur Merideth, an Irish logician. At the same time, Iseki [4] introduced another class of algebra called BCI- algebra, which is a generalization of the class of BCK- algebra and investigated its properties. For the general development of BCI/BCK –algebras, the ideal theory and graph plays an important role. In 2006, Kyung Ho Kim [7] introduced a new class of algebraic structure called KS-semigroup, which also deals with a new class algebras related to BCK-algebra, called a commutative KS-semigroup. In this paper, we introduce the concept of a annihilator graph on AG(X) a commutative KS-semigroup and discussed its properties.

2. Preliminaries

Definition 2.1. [11] A BCK-algebra is a triple $(X,*,0)$ where X is a non empty set, "$*$" is a binary operation on X and $0\in X$ is an element such that the following axioms are satisfied.

i. $x*0 = x$ for all $x \in X$.
ii. $(x*y)*z = (x*z)*y$ for all $x, y, z \in X$.
iii. $x\leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$ for all $x, y, z \in X$.

iv. $(x * z) * (y * z) \le x * y$ for all $x, y, z \in X$.

If X is a BCK-algebra, then the relation $x \le y$ iff $x * y = 0$ is a partial order on X, which is called the natural ordering on X.

Example 2.2. [6] Let X = {0,a,b,c} be a set with $*$-operation given by Table,

$*$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Then $(X,*,0)$ is a BCK-algebra.

3. Commutative KS-semigroup

Definition: 3.1. [9] A semigroup is an ordered pair $(S,*)$, where S is a nonempty set and "$*$" is an associative binary operation on S.

Definition 3.2. [9] A commutative KS-semigroup is a non –empty set X with two binary operations "$*$" and "$\bullet$" and constant 0 satisfying the axioms;

i. $(X,*,0)$ is BCK-algebra.
ii. $(X,\bullet)$ is semigroup.
iii. $x \bullet (y*z) = (x \bullet y)*(x \bullet z)$ and $(x*y) \bullet z = (x \bullet z)*(y \bullet z)$ $\forall x,y,z \in X$.
iv. $x*(x*y) = y*(y*x)$ $\forall x,y \in X$.

Example 3.1. [9] Let X= {0,a,b,c} be a set with the '$*$' and '$\bullet$' operations given by Table,

$*$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

$\bullet$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Clearly, $(X,*,\bullet,0)$ is a commutative KS-semigroup.

Definition 3.3. Let $(X,*_1,\bullet_1,0)$ and $(Y,*_2,\bullet_2,0)$ be any two Commutative KS-semigroups such that $X \cap Y = \{0\}$. Define the binary operations "$*$" and "$\bullet$" on $X \cup Y$ by

$$x*y = \begin{cases} x*_1 y & \text{if } x,y \in X \\ x*_2 y & \text{if } x,y \in Y \\ x & \text{otherwise} \end{cases}$$

$$x \bullet y = \begin{cases} x \bullet_1 y & \text{if } x,y \in X \\ x \bullet_2 y & \text{if } x,y \in Y \\ 0 & \text{otherwise} \end{cases}$$

Theorem 3.1. Let $(X,*_1,\bullet_1,0)$ and $(Y,*_2,\bullet_2,0)$ be a Commutative KS-semigroups such that $X\cap Y=\{0\}$ and "$*$" and "$\bullet$" be the binary operation on $X\cup Y$ defined as follows, for any $x,y \in X\cup Y$,

$$x*y = \begin{cases} x*_1y & \text{if } x,y \in X \\ x*_2y & \text{if } x,y \in Y \\ x & \text{otherwise} \end{cases}$$

$$x\bullet y = \begin{cases} x\bullet_1y & \text{if } x,y \in X \\ x\bullet_2y & \text{if } x,y \in Y \\ 0 & \text{otherwise} \end{cases}$$

Then, $(X\cup Y,*,\bullet)$ is a Commutative KS-semigroup.

Proof: i. $(X,*_1,\bullet_1,0)$ is a BCK-algebra and $(Y,*_2,\bullet_2,0)$ is a BCK-algebra.

ii. $(X,\bullet_1)$ and $(Y,\bullet_2)$ are a semigroup.

iii. The operation $\bullet$ is left and right distributive over the operation $*$

(i.e) (a)$x\bullet(y * z) = (x \bullet y)*(x \bullet z)$ and $(x * y)\bullet z = (x \bullet z)*(y \bullet z)\forall x,y,z \in X$.

(b)$x\bullet(y * z) = (x \bullet y)*(x \bullet z)$ and $(x * y)\bullet z = (x \bullet z)*(y \bullet z)\forall x,y,z \in Y$.

iv. $x*(x * y) = y *(y * x)\forall x,y \in X$.

For any $x,y \in X\cup Y$, Define the $*$ and $\bullet$ operations on $X\cup Y$ as follows,

$$x*y = \begin{cases} x*_1y & \text{if } x,y \in X \\ x*_2y & \text{if } x,y \in Y \\ x & \text{otherwise} \end{cases}$$

$$x\bullet y = \begin{cases} x\bullet_1y & \text{if } x,y \in X \\ x\bullet_2y & \text{if } x,y \in Y \\ 0 & \text{otherwise} \end{cases}$$

To prove that,

i. $(X\cup Y,*,0)$ is a BCK-algebra.

(a) For any $x \in X$, $x*0 =x$.For any $y \in Y$, $y*0 =y$.

(b) $x*y = 0 \Rightarrow$ either $x,y\in X$ or $x,y \in Y$. so, $x*y = 0 \Rightarrow (z*y)*(z*x) = 0$

(c) For any $x \in X,y \in Y$,

Case: (i) Let $z \in X$

$(x*y)*z = (x*z)*y$

$(x*y)*z = x*z$

$(x*z)*y = x*z$

Case: (ii) Let $z\in Y$

$(x*y)*z = (x*z)*y$

$(x*y)*z = x*z = x$

$(x*z)*y = x*y = x$

(d) For any $x \in X,y \in Y$,

Case: (i) Let $z \in X$

$((x*y)*(y*z))*(x*y) = 0$

$((x*z) * (y*z)) * (x*y) = ((x*z) *y) *x = (x*z) *x = 0$

Case: (ii) Let $z \in Y$

$((x*z) * (y*z)) * (x*y) = (x* (y*z)) *x = x*x = 0$

Hence, $(X\cup Y,*,0)$ is a BCK-algebra.

ii. $(X\cup Y, \bullet)$ is a semigroup.If $\forall x,y \in X$ or $x,y \in Y$, then $(X\cup Y, \bullet)$ is a semigroup. Let $x \in X$ and $y \in Y$, then $x\bullet y=0 \in X\cup Y$.

Also, $x\bullet(y\bullet z) = (x\bullet y)\bullet z = 0 \ \forall z \in X$ or $Z \in Y$.

$\therefore (X\cup Y, \bullet)$ is a semigroup.

iii. The operation • is left and right distributive over the operation "∗".

For all $x,y,z \in X$ or $x,y,z \in Y$,

$x\bullet(y*z) = (x\bullet y)*(x\bullet z)$ and $(x*y)\bullet z = (x\bullet z)*(y\bullet z) \forall x \in X$ and $y,z\in Y$.

Case: (i)

For any $x \in X$, $y \in Y$,

(a) Let $z \in X$

$x \bullet(y * z) = x \bullet y = 0$

$(x\bullet y) * (x\bullet z) = 0* (x \bullet z) = 0$

$\therefore x \bullet(y * z) = (x \bullet y)*(x \bullet z)$

(b) $(x*y) \bullet z = x\bullet z$

$(x\bullet z) * (y\bullet z) = (x \bullet z)* \ 0 = x \bullet z$

$\therefore (x *y) \bullet z = (x \bullet z)*(y \bullet z)$

Case: (ii)

For any $x \in X$, $y \in Y$,

(a) Let $z \in Y$

$x \bullet(y * z) = x \bullet y = 0$

$(x\bullet y) * (x\bullet z) = 0$

$\therefore x \bullet(y * z) = (x \bullet y)*(x \bullet z)$

(b) $(x*y) \bullet z = x\bullet z = 0$

$(x\bullet z) * (y\bullet z) = 0 * (y\bullet z) = 0$

$\therefore (x*y) \bullet z = (x\bullet z) * (y\bullet z)$

iv. $x* (x*y) = y* (y*x) \ \forall x,y \in X$,

$x*(x * y) = y *(y * x)\forall x,y \in Y$,For any $x \in X$ and $y \in Y$,

$x*(x * y) = x *x= 0$; $y *(y * x) = y *y= 0$.

Hence, $(X\cup Y,*,\bullet, 0)$ is a Commutative KS-semigroup.

Example 3.2. Let $X=\{0,a,b,c\}$ and $Y=\{0,1,2,3\}$. Define two operations $*_1$and $\bullet_1$on Xand$*_2$and $\bullet_2$ on Y as follows.

$*_1$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

$\bullet_1$	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	0	c	0	0

Clearly, $(X, *_1, \bullet_1, 0)$ is a Commutative KS-semigroup.

$*_2$	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

$\bullet_2$	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Clearly, $(Y, *_2, \bullet_2, 0)$ is a Commutative KS-semigroup.
Let $X \cup Y = \{0,a,b,c,1,2,3\}$. Define the two operations $*$ and $\bullet$ on $X \cup Y$ as follows,

∗	0	a	b	c	1	2	3
0	0	0	0	0	0	0	0
a	a	0	a	a	a	a	a
b	b	b	0	b	b	b	b
c	c	c	c	0	c	c	c
1	1	1	1	1	0	1	0
2	2	2	2	2	2	0	0
3	3	3	3	3	2	1	0

•	0	a	b	c	1	2	3
0	0	0	0	0	0	0	0
a	0	a	0	0	0	0	0
b	0	0	b	0	0	0	0
c	0	c	0	0	0	0	0
1	0	0	0	0	1	0	1
2	0	0	0	0	0	2	2
3	0	0	0	0	1	2	3

Hence, $(X \cup Y, *, \bullet, 0)$ is a Commutative KS-semigroup.

Definition 3.4. For any $x,y \in X$, X is a Commutative KS-semigroup, denote $x \wedge y = y*(y*x)$. Obviously, $x \wedge y$ is a lower bound of x and y and $x \wedge x = x$, $x \wedge 0 = 0 \wedge x = 0$.

Definition 3.5. Let $(X,*, \bullet, 0)$ be a Commutative KS-semigroup. For any $a \in X$,
Define $ann(a) = \{x \in X / x \wedge a = 0, a \in X\}$ is called the annihilator of a.

Example 3.3. Let $X=\{0,a,b,c\}$ be a set with the $*$ and • operations given by Table,

∗	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

$ann(a) = \{ x \in X / x \wedge a = 0, a \in X\}$
$ann(0) = \{0,a,b,c\}$; $ann(a) = \{0,b\}$; $ann(b) = \{0,a\}$; $ann(c) = \{0\}$.

Definition 3.6. Let $(X,*, \bullet, 0)$ be a commutative KS-semigroup,
Define, $Z(X)=\{a \in X / a.b = 0 \text{ for some } 0 \neq b \in X\}$ as the set of all zero divisors in X.

Remark 3.1. Note that $ann(x) \subseteq X$ and $Z(X) = \bigcup_{0 \neq x} ann(x)$.

Definition 3.7. Let $(X,*, \bullet, 0)$ be a commutative KS-semigroup, then the annihilator graph denoted as $AG(X)$ is defined as, the graph with vertex set $Z^*(X) = Z(X) - \{0\}$ and an edge set $\{xy / x \neq y, x,y \in Z^*(X), ann(x) \cup ann(y) \neq ann(xy)\}$.

Theorem 3.2. Let $(X,*_1, \bullet_1,0)$ and $(Y,*_2, \bullet_2,0)$ be any two commutative KS-Semigroup and $Z(X)$ and $Z(Y)$ be the set of all zero divisors of X and Y respectively, If $Z^*(X) = X - \{0\}$ and $Z^*(Y) = Y - \{0\}$. Then $AG(X) \cup AG(Y) = AG(X \cup Y)$.
Proof: We have to prove that $AG(X \cup Y) = AG(X) \cup AG(Y)$.
Let, $Z^*(X) = X - \{0\}$ and $Z^*(Y) = Y - \{0\}$ then $Z^*(X \cup Y) = X \cup Y - \{0\}$
$\therefore V(AG(X \cup Y)) = V(AG(X)) \cup V(AG(Y))$.

To prove: $E(AG(X\cup Y)) = E(AG(X)) \cup E(AG(Y))$.
First let us prove that there does not exist any edge $xy\in E(AG(X\cup Y)$ such that $x\in V(AG(X))$ and $y \in V(AG(Y))$. Suppose if possible let $xy\in E(AG(X\cup Y)$ such that $x\in V(AG(X))$ and $y\in V(AG(Y))$.In $(X\cup Y), *,\bullet,0)$, we have , $ann(x) = Y \cup ann(x)$ in X and $ann(y) = X \cup ann(y)$ in Y.
$ann(x)\cup ann(y) = (Y \cup ann(x)$ in X$) \cup (X \cup ann(y)$ in Y$)= X \cup Y = ann(0)$ in $X \cup Y$.
$ann(x) \cup ann(y) = ann(xy)$ in $X \cup Y$. but $xy\in E(AG(X\cup Y)$, which is a contradiction.
Hence, that there does not exist any edge $xy\in E(AG(X\cup Y)$ such that $x \in V(AG(X))$ and $y \in V(AG(Y))$.
Now, let $xy \in E(AG(X))$ and hence, $ann(x) \cup ann(y) \neq ann(xy)$ in X.
In $X \cup Y$,$ann(x) = Y \cup \{ann(x) / x \in X\}$
$ann(y) = Y \cup \{ann(y) / y \in X\}$
$ann(xy) = Y \cup \{ann(xy) / x.y\in X\}$
$ann(x) \cup ann(y) = Y \cup \{ann(x) \cup ann(y) / x.y\in X\}$
$ann(x) \cup ann(y) \neq Y \cup \{ann(xy) / x.y\in X\}$
$ann(x) \cup ann(y) \neq ann(xy)$ in $X \cup Y$. Therefore, $xy \in E(AG(X\cup Y))$.
Let $xy \in E(AG(Y))$
$\therefore ann(x) \cup ann(y) \neq ann(xy)$ in Y.
In $X \cup Y$,$ann(x) = X \cup \{ann(x) / x \in Y\}$
$ann(y) = X \cup \{ann(y) / y \in Y\}$
$ann(xy) = X \cup \{ann(xy) / x.y\in Y\}$
$ann(x) \cup ann(y) = X \cup \{ann(x) \cup ann(y) / x.y\in Y\}$
$ann(x) \cup ann(y) \neq X \cup \{ann(xy) / x.y\in Y\}$
$ann(x) \cup ann(y) \neq ann(xy)$ in $X \cup Y$.Therefore, $xy \in E(AG(X\cup Y))$.
Let $xy \in E(AG(X\cup Y))$, then $ann(x) \cup ann(y) \neq ann(xy)$ in $X\cup Y$.
In $X\cup Y$,$ann(x) – (Y-\{0\}) = ann(x)$ in X.
$ann(x) = ann(x) \cup (Y-\{0\})$ in X.
$ann(x) = ann(x) \cup Y$ in X.
$[ann(x) \cup Y] \cup [ann(y) \cup Y] \neq [ann(xy) \cup Y]$ if $x,y\in X$ (or)
$[ann(x) \cup X] \cup [ann(y) \cup X] \neq [ann(xy) \cup X]$ if $x,y\in Y$
$[ann(x) \cup ann(y)]\cup Y \neq [ann(xy) \cup Y]$ in X (or) $[ann(x)\cup ann(y)]\cup X \neq [ann(xy)\cup X]$ in Y
$ann(x) \cup ann(y) \neq ann(xy)$ in X or $ann(x) \cup ann(y) \neq ann(xy)$ in Y
Hence, either $xy \in E(AG(X))$ or $xy \in E (AG(Y))$.That is, $xy \in E(AG(X)) \cup E(AG(Y))$.
Hence, $AG(X\cup Y) = AG(X) \cup AG(Y)$.

Example 3.4. Let X={0,a,b,c} and Y={0,1,2,3}. Define two operations $*_1$ and $\bullet_1$ on Xand $*_2$, and $\bullet_2$ on Y respectively. If $Z^*(X) =Z(X) – \{0\}$ and $Z^*(Y) =Z(Y) – \{0\}$, then $AG(X\cup Y) = AG(X) \cup AG(Y)$. Let X = {0,a,b,c} be a set with the $*$ and $\bullet$ operations given by Table,

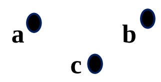
$*_1$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

$\bullet_1$	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	0	c	0	0

Clearly, $(X, *_1, \bullet_1, 0)$ is a Commutative KS-Semigroup.
The $\mathrm{ann}(a) = \{ x \in X / x \wedge a = 0, a \in X\}$.
$\mathrm{ann}(0) = \{0,a,b,c\}$; $\mathrm{ann}(a) = \{0,b,c\}$; $\mathrm{ann}(b) = \{0,a,c\}$; $\mathrm{ann}(c) = \{0,a,b\}$.
The set of all zero divisors of X is $Z(X) = \{0,a,b,c\}$.
The vertex set $Z^*(X) = Z(X) - \{0\} = \{a, b, c\}$.
The edge set $= \{xy/x \neq y, x,y \in Z^*(X), \mathrm{ann}(x) \cup \mathrm{ann}(y) \neq \mathrm{ann}(xy)\}$.

$\mathrm{ann}(a) \cup \mathrm{ann}(b) = \{0,a,b,c\} = \mathrm{ann}(ab) = \mathrm{ann}(0)$.
$\mathrm{ann}(a) \cup \mathrm{ann}(c) = \{0,a,b,c\} = \mathrm{ann}(ac) = \mathrm{ann}(0)$.
$\mathrm{ann}(b) \cup \mathrm{ann}(c) = \{0,a,b,c\} = \mathrm{ann}(bc) = \mathrm{ann}(0)$.

The annihilator graph AG(X) of X is given by Figure 3.1



Let $Y = \{0,a,b,c\}$ be a set with the $*$ and $\bullet$ operations given by Table,

$*_2$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	2
3	3	3	3	0

$\bullet_2$	0	1	2	3
0	0	0	0	0
1	0	0	0	1
2	0	0	0	2
3	0	0	0	3

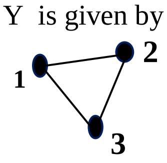
Clearly, $(Y, *_2, \bullet_2, 0)$ is a Commutative KS-Semigroup.
The $\mathrm{ann}(a) = \{ x \in X / x \wedge a = 0, a \in X\}$.

$\mathrm{ann}(0) = \{0,1,2,3\}$; $\mathrm{ann}(1) = \{0,3\}$; $\mathrm{ann}(2) = \{0,1,3\}$; $\mathrm{ann}(3) = \{0,1,2\}$.

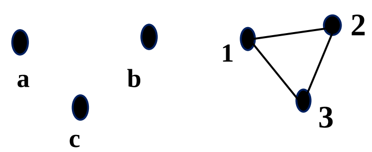
The set of all zero divisors of Y is $Z(Y) = \{0,1,2,3\}$.
The vertex set $Z^*(Y) = Z(Y) - \{0\} = \{1, 2, 3\}$.
The edge set $= \{xy/x \neq y, x,y \in Z^*(Y), \mathrm{ann}(x) \cup \mathrm{ann}(y) \neq \mathrm{ann}(xy)\}$.

$\mathrm{ann}(1) \cup \mathrm{ann}(2) = \{0,1,3\} \neq \mathrm{ann}(12) = \mathrm{ann}(0) = \{0,1,2,3\}$
$\mathrm{ann}(1) \cup \mathrm{ann}(3) = \{0,1,2,3\} \neq \mathrm{ann}(13) = \mathrm{ann}(1) = \{0,3\}$
$\mathrm{ann}(2) \cup \mathrm{ann}(3) = \{0,1,2,3\} \neq \mathrm{ann}(23) = \mathrm{ann}(2) = \{0,1,3\}$

The annihilator graph AG(Y) of Y is given by Figure 3.2



The annihilator graph of$AG(X) \cup AG(Y)$ is given as Figure 3.3



Let $(X\cup Y,*,\bullet,0)$ be a set with the $*$ and $\bullet$ operations given by **Table**,

$*$	**0**	**a**	**b**	**c**	**1**	**2**	**3**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
a	**a**	**0**	**a**	**a**	**a**	**a**	**a**
b	**b**	**b**	**0**	**b**	**b**	**b**	**b**
c	**c**	**c**	**c**	**0**	**c**	**c**	**c**
1	**1**	**1**	**1**	**1**	**0**	**0**	**1**
2	**2**	**2**	**2**	**2**	**2**	**0**	**2**
3	**3**	**3**	**3**	**3**	**3**	**3**	**0**

$\bullet$	**0**	**a**	**b**	**c**	**1**	**2**	**3**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
a	**0**	**a**	**0**	**0**	**0**	**0**	**0**
b	**0**	**0**	**b**	**0**	**0**	**0**	**0**
c	**0**	**c**	**0**	**0**	**0**	**0**	**0**
1	**0**	**0**	**0**	**0**	**0**	**0**	**1**
2	**0**	**0**	**0**	**0**	**0**	**0**	**2**
3	**0**	**0**	**0**	**0**	**0**	**0**	**3**

Clearly, $(X\cup Y,*,\bullet,0)$ is a Commutative KS-Semigroup.

$\therefore$ ann(0) = {0,a,b,c,1,2,3}; ann(a) = {0,b,c,1,2,3}; ann (b) = {0,a,c,1,2,3}

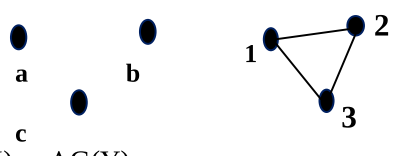
ann(c) = {0,a,b,1,2,3}; ann(1) = {0,3,a,b,c};ann(2) = {0,1,3,a,b,c};ann(3) = {0,1,2,a,b,c}

The vertex set $Z^*(X\cup Y) = Z(X\cup Y) - \{0\} = \{a,b,c,1,2,3\}$.

The edge set = $\{xy/x \neq y,\ x,y \in Z^*(X\cup Y),\ ann(x) \cup ann(y) \neq ann(xy)\}$.

ann(a) $\cup$ann(b) = {0,a,b,c,1,2,3} = ann(ab) = ann(0) = {0,a,b,c,1,2,3}
ann(a) $\cup$ann(c) = {0,a,b,c,1,2,3} = ann(ac) = ann(0) = {0,a,b,c,1,2,3}
ann(b) $\cup$ann(c) = {0,a,b,c,1,2,3} = ann(bc) = ann(0) = {0,a,b,c,1,2,3}
ann(1) $\cup$ann(2) = {0,1,3,a,b,c} $\neq$ ann(12) = ann(0) = {0,a,b,c,1,2,3}
ann(1) $\cup$ann(3) = {0,1,2,3,a,b,c} $\neq$ ann(13) = ann(1) = {0,3,a,b,c}
ann(2) $\cup$ann(3) = {0,1,2,3,a,b,c} $\neq$ ann(23) = ann(2) = {0,1,3,a,b,c}
ann(a) $\cup$ann(1) = {0,a,b,c,1,2,3} = ann(a1) = ann(0) = {0,a,b,c,1,2,3}
ann(a) $\cup$ann(2) = {0,a,b,c,1,2,3} = ann(a2) = ann(0) = {0,a,b,c,1,2,3}
ann(a) $\cup$ann(3) = {0,a,b,c,1,2,3} = ann(a3) = ann(0) = {0,a,b,c,1,2,3}
ann(b) $\cup$ann(1) = {0,a,b,c,1,2,3} = ann(b1) = ann(0) = {0,a,b,c,1,2,3}
ann(b) $\cup$ann(2) = {0,a,b,c,1,2,3} = ann(b2) = ann(0) = {0,a,b,c,1,2,3}
ann(b) $\cup$ann(3) = {0,a,b,c,1,2,3} = ann(b3) = ann(0) = {0,a,b,c,1,2,3}
ann(c) $\cup$ann(1) = {0,a,b,c,1,2,3} = ann(c1) = ann(0) = {0,a,b,c,1,2,3}
ann(c) $\cup$ann(2) = {0,a,b,c,1,2,3} = ann(c2) = ann(0) = {0,a,b,c,1,2,3}
ann(c) $\cup$ann(3) = {0,a,b,c,1,2,3} = ann(c3) = ann(0) = {0,a,b,c,1,2,3}

The annihilator graph $AG(X\cup Y)$ of $X\cup Y$is given in Figure 3.4.



Hence, $AG(X\cup Y) = AG(X) \cup AG(Y)$.

Example 3.5. Let X={0,a,b,c} and Y={0,1,2,3}. Define two operations$*_1$and $\bullet_1$ on X and$*_2$and $\bullet_2$on Y respectively,

Let $Z^*(X) = X - \{0\}$ and $Z^*(Y) \neq Y - \{0\}$, then $AG(X\cup Y) \neq AG(X)\cup AG(Y)$.

Let X = {0,a,b,c} be a set with the $*$ and • operations given by Table,

$*_1$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

$\bullet_1$	0	a	b	C
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	0	c	0	0

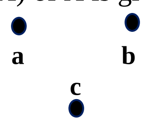
Clearly, $(X, *_1, \bullet_1, 0)$ is a Commutative KS-Semigroup.
The ann(a) = $\{ x \in X / x \wedge a = 0, a \in X\}$.
ann(0) = {0,a,b,c}; ann(a) = {0,b,c}; ann(b) = {0,a,c} ; ann(c) = {0,a,b}.
The set of all zero divisors Z(X) = {0,a,b,c}.The vertex set $Z^*(X) = X - \{0\} = \{a, b, c\}$.
The edge set = $\{xy/x \neq y,\ x,y \in Z^*(X),\ ann(x) \cup ann(y) \neq ann(xy)\}$.

ann(a) $\cup$ann(b) = {0,a,b,c} = ann(ab) = ann(0) = {0,a,b,c}.

ann(a) $\cup$ann(c) = {0,a,b,c} = ann(ac) = ann(0) = {0,a,b,c}.

ann(b) $\cup$ann(c) = {0,a,b,c} = ann(bc) = ann(0) = {0,a,b,c}.

The annihilator graph ofAG(X) of X is given by Figure 3.5 .



Let Y = {0,1,2,3} be a set with the $*$ and • operations given by Table,

$*_2$	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

$\bullet_2$	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Clearly, $(Y, *_2, \bullet_2, 0)$ is a Commutative KS-Semigroup.
The ann(a) = $\{ x \in X / x \wedge a = 0, a \in X\}$.

ann(0) = {0,1,2,3}; ann(1) = {0,2}; ann(2) = {0,1}; ann(3) = {0}

The set of all zero divisors Z(Y) = {0,1,2}.The vertex set $Z^*(Y) \neq Y - \{0\}$.
$Z^*(Y) = \{1, 2\}$.
The edge set = $\{xy/x \neq y,\ x,y \in Z^*(Y),\ ann(x) \cup ann(y) \neq ann(xy)\}$.

ann(1) $\cup$ann(2) = {0,1,2} $\neq$ ann(12) = ann(0) = {0,1,2,3}

The annihilator graph AG(Y) of Y is given by Figure 3.6

The annihilator graph of AG(X) $\cup$ AG(Y) is given as Figure 3.7

Let $(X\cup Y,*,\bullet,0)$ be a set with the $*$ and $\bullet$ operations given by Table,

$*$	0	a	b	c	1	2	3
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
a	**a**	**0**	**a**	**a**	**a**	**a**	**a**
b	**b**	**b**	**0**	**b**	**b**	**b**	**b**
c	**c**	**c**	**c**	**0**	**c**	**c**	**c**
1	**1**	**1**	**1**	**1**	**0**	**1**	**0**
2	**2**	**2**	**2**	**2**	**2**	**0**	**0**
3	**3**	**3**	**3**	**3**	**2**	**1**	**0**

$\bullet$	0	a	b	c	1	2	3
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
a	**0**	**a**	**0**	**0**	**0**	**0**	**0**
b	**0**	**0**	**b**	**0**	**0**	**0**	**0**
c	**0**	**c**	**0**	**0**	**0**	**0**	**0**
1	**0**	**0**	**0**	**0**	**1**	**0**	**1**
2	**0**	**0**	**0**	**0**	**0**	**2**	**2**
3	**0**	**0**	**0**	**0**	**1**	**2**	**3**

Clearly, $(X\cup Y,*,\bullet,0)$ is a Commutative KS-Semigroup.

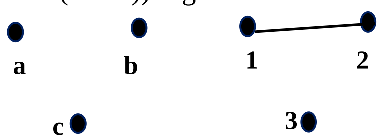
ann(0) = {0,a,b,c,1,2,3}; ann(a) = {0,b,c,1,2,3}; ann (b) = {0,a,c,1,2,3}
ann(c) = {0,a,b,1,2,3};ann(1) = {0,2,a,b,c}; ann(2) = {0,1,a,b,c}; ann(3) = {0,a,b,c}.

The vertex set $Z^*(X\cup Y) = Z(X\cup Y) - \{0\} = \{a,b,c,1,2,3\}$.

The edge set = $\{xy/x \neq y,\ x,y \in Z^*(X\cup Y),\ ann(x) \cup ann(y) \neq ann(xy)\}$.

ann(a) ∪ann(b) = {0,a,b,c,1,2,3} = ann(ab) = ann(0) = {0,a,b,c,1,2,3}
ann(a) ∪ann(c) = {0,a,b,c,1,2,3} = ann(ac) = ann(0) = {0,a,b,c,1,2,3}
ann(b) ∪ann(c) = {0,a,b,c,1,2,3} = ann(bc) = ann(0) = {0,a,b,c,1,2,3}
ann(1) ∪ann(2) = {0,1,2,a,b,c} ≠ ann(12) = ann(0) = {0,a,b,c,1,2,3}
ann(1) ∪ann(3) = {0,2,a,b,c} = ann(13) = ann(1) = {0,2,a,b,c}
ann(2) ∪ann(3) = {0,1,a,b,c} = ann(23) = ann(2) = {0,1,a,b,c}
ann(a) ∪ann(1) = {0,a,b,c,1,2,3} = ann(a1) = ann(0) = {0,a,b,c,1,2,3}
ann(a) ∪ann(2) = {0,a,b,c,1,2,3} = ann(a2) = ann(0) = {0,a,b,c,1,2,3}
ann(a) ∪ann(3) = {0,a,b,c,1,2,3} = ann(a3) = ann(0) = {0,a,b,c,1,2,3}
ann(b) ∪ann(1) = {0,a,b,c,1,2,3} = ann(b1) = ann(0) = {0,a,b,c,1,2,3}
ann(b) ∪ann(2) = {0,a,b,c,1,2,3} = ann(b2) = ann(0) = {0,a,b,c,1,2,3}
ann(b) ∪ann(3) = {0,a,b,c,1,2,3} = ann(b3) = ann(0) = {0,a,b,c,1,2,3}
ann(c) ∪ann(1) = {0,a,b,c,1,2,3} = ann(c1) = ann(0) = {0,a,b,c,1,2,3}
ann(c) ∪ann(2) = {0,a,b,c,1,2,3} = ann(c2) = ann(0) = {0,a,b,c,1,2,3}
ann(c) ∪ann(3) = {0,a,b,c,1,2,3} = ann(c3) = ann(0) = {0,a,b,c,1,2,3}

The annihilator graph of AG(X∪Y)) Figure 3.8



Hence, $AG(X\cup Y) \neq AG(X) \cup AG(Y)$

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