International Journal of
Fuzzy Mathematical
Archive

Non Split Domination on Intuitionistic Fuzzy Graphs

A.NagoorGani[1] and S.Anupriya[2]

[1]PG & Research Department of Mathematics, Jamal Mohamed College (Autonomous)
Tiruchirappalli, Tamil Nadu, India. e-mail: ganijmc@yahoo.co.in
Corresponding Author

[2]Department of Mathematics, J.J. College of Engineering& Technology, Tiruchirappalli
Tamil Nadu, India. e-mail: anupereya_math@yahoo.com

Abstract. A dominating set D of an IFG G = (V, E) is a non-split dominating set, if the induced intuitionistic fuzzy subgraph $\langle V - D \rangle$ is connected. The Non-split domination number $\gamma_{ns}(G)$ of IFG G is the minimum cardinality of all Non-split dominating set. In this paper we study some theorems in Non-split dominating sets of IFG and some results of $\gamma_{ns}(G)$ with other known parameters of IFG G.

1. Introduction
In 1965**,** the notion of fuzzy sets was introduced by Zadeh [21] as a method of representing uncertainty and vagueness. In 1986, Atanassov [1] introduced the concept of IF sets as a generalization of fuzzy sets. The fuzzy relations between fuzzy sets were concluded by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya [3] gave some remarks on fuzzy graphs and some operations on fuzzy graphs were included by Mordeson and Peng [8]. Kulli [7] wrote on theory of domination in graphs. Cockayne [5] introduced the independent domination number in graphs. Somasundaram and Somasundaram [19] presentedmore concepts of independent domination, connected domination in fuzzy graphs. Parvathi and Karunambigai [15] gave a definition of IFG as a special case of IFGS defined by Atanassov and Shannon [18]. Nagoor Gani and Begum [12] gave the definition of order, degree and size in IFG. Later Parvathi and Thamizhendhi [16] introduced dominating set, domination number, independent set, total dominating set and total domination number in IFGs. In this Paper, non-split dominating sets in IFGs are studied.

2. Preliminaries
In this section, some basic definitions relating to IFGS are given. Also the definition of various non-split dominating sets and cardinality of non-split dominating sets in IFGS are studied.

Non Split Domination Pon Intuitionistic Fuzzy Graphs

Definition 2.1. [17] An Intuitionistic Fuzzy Graph IFG is of the form $G = (V, E)$, where
(i) $V = \{v_1, v_2, \dots v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V(i = 1,2,\dots n)$.
(ii) $E \subset V \times V$, where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$. Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non-membership of the vertex relation and $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

Note 1. When $\mu_{2ij} = \gamma_{2ij} = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j .

Definition 2.2. [17] An IFG $H = (V', E')$ is said to be an IF-subgraph (IFSG) of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. That is
$$\mu'_{1i} \le \mu_{1i} \; ; \gamma_{1i}' \ge \gamma_{1i} \text{ and } \mu'_{2ij} \le \mu_{2ij}; \gamma'_{2ij} \ge \gamma_{2ij} \text{ for every } i, j = 1,2,3 \dots n.$$

Definition 2.3. [17] Let $G = (V,E)$ be an IFG. Then the cardinality of G is defined to be
$$|G| = \left| \sum_{v_i \in V} \frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} + \sum_{v_i, v_j \in E} \frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right|$$

Definition 2.4. [17] Let $G = (V, E)$ be an IFG. Then the vertex cardinality of V defined by
$$|V| = \sum_{v_i \in V} \frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \; ; \quad \text{for all } v_i \in V.$$

Definition 2.5. [17] Let $G = (V, E)$ be an IFG. Then the edge cardinality of E defined by
$$|E| = \sum_{v_i, v_j \in E} \frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}; \quad \text{for all } (v_i, v_j) \in E.$$

Definition 2.6. [12] The number of vertices is called the order of an IFG and is denoted by O(G). The number of edges is called size of an IFG and is denoted by S(G).

Definition 2.7. [14] The degree of a vertex v in an IFG, $G = (V, E)$ is defined to be sum of the weights of the strong edges incident at v. It is denoted by $d_G(v)$.
    The minimum degree of G is $\delta(G) = \min\{d_G(v) \mid v \in V\}$.
    The maximum degree of G is $\Delta(G) = \max\{d_G(v) \mid v \in V\}$.

Definition 2.8. [14] Two vertices v_i and v_j are said to be neighbours in IFG, if either one of the following conditions hold
(i). $\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) > 0$,
(ii). $\mu_2(v_i, v_j) = 0, \gamma_2(v_i, v_j) > 0$,

(iii). $\mu_2(v_{i,}v_j) > 0, \gamma_2(v_i, v_j) = 0, v_i, v_j \epsilon V$.

Definition 2.9. [14] A path in an IFG is a sequence of distinct vertices $v_1, v_2, \dots \dots v_n$ such that either one of the following conditions is satisfied:

(i). $\mu_2(v_{i,}v_j) > 0, \gamma_2(v_i, v_j) > 0$, forsome i and j,

(ii). $\mu_2(v_{i,}v_j) = 0, \gamma_2(v_i, v_j) > 0$, forsomeiandj,

(iii). $\mu_2(v_{i,}v_j) > 0, \gamma_2(v_i, v_j) = 0$, for some i and j.

The length of a path $P = v_1, v_2, \dots \dots v_{n+1}(n > 0)$ is n.

Definition 2.10. [14] Two vertices that are joined by a path is called connected.

Definition 2.11. [16] If v_i, v_j are vertices in $G = (V, E)$ and if they are connected by means of a path then the strength of that path is defined as $(\min_{i,j} \mu_{2\,ij}, \max_{i,j} \gamma_{2\,ij})$, where $\min_{i,j} \mu_{2\,ij}$ is the $\mu-$strength of the weakest arc and $\max_{i,j} \gamma_{2\,ij}$ is the $\gamma-$strength of the strongest arc.

Definition 2.12. [16] If $v_i, v_j \epsilon V \subseteq G$, then $\mu-$ strength of connectedness between v_i and v_j is $\mu_2^\infty(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) / k = 1,2..n\}$ γ–strength of connectedness between v_i and v_j is $\gamma_2^\infty(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) / k = 1,2..n\}$. If u, v are connected by means of path of length k then $\mu_2^k(u, v)$ is defined as

$\sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \mu_2(v_2, v_3) \dots \wedge \mu_2(v_{k-1}, v)/(u_1v_1, v_2, \dots v_{k-1}, v\epsilon V)\}$ and $\gamma_2^k(u, v)$ is defined as

$\inf\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2) \vee \gamma_2(v_2, v_3) \dots \vee \gamma_2(v_{k-1}, v) / (u_1v_1, v_2, \dots v_{k-1}, v\epsilon V)\}$

Definition 2.13. [16] An IFG, G = (V,E) is said to be complete IFG if $\mu_{2ij} = \min\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = \max\{\gamma_{1i}, \gamma_{1j}\}$ for every $v_i, v_j \in V$.

Definition 2.14. [21] The complement of an IFG, G = (V, E) is an IFG, $\overline{G} = (\overline{V}, \overline{E})$, where

1) $\overline{V} = V$,

2) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$, for all i=1,2....n

3) $\overline{\mu_{2ij}} = \min(\mu_{1i}, \mu_{1i,}) - \mu_{2ij}$, and $\overline{\gamma_{2ij}} = \max(\gamma_{1i}, \gamma_{1i,}) - \gamma_{2ij}$, for all i=1,2..n

Definition 2.15. [14] An IFG, G = (V,E) is said to bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that

    i) $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ if $v_i, v_j \in V_1$ (or) $v_i, v_j \in V_2$

    ii) $\mu_2(v_i, v_j) > 0$ and $\gamma_2(v_i, v_j) > 0$ if $v_i, \in V_1$ and $v_j \in V_2$ for some i and j (or) $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) > 0$ if $v_i, \in V_1$ and $v_j \in V_2$ for some i and j (or) $\mu_2(v_i, v_j) > 0$ and $\gamma_2(v_i, v_j) = 0$ if $v_i, \in V_1$ and $v_j \in V_2$ for some i and j

Non Split Domination Pon Intuitionistic Fuzzy Graphs

Definition 2.16. [16] A bipartite IFG $G = (V, E)$ is said to be complete if $\mu_2(v_i, v_j) = \min\left(\mu_1(v_i), \mu_1(v_j)\right)$ and $\gamma_2(v_i, v_j) = \max\left(\gamma_1(v_i), \gamma_1(v_j)\right)$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{v_{1i}, v_{2i}}$.

Definition 2.17. [17] Let u be a vertex in an IFG $G = (V, E)$, then $N(u) = \{v \in V / (u, v)$ is a strong arc$\}$ is called neighbourhood of u.

Definition 2.18. [17] A vertex $u \in V$ of an IFG $G = (V, E)$ is said to be an isolated vertex if $\mu_2(u, v) = 0$ and $\gamma_2(u, v) = 0$ for all $v \in V$. That is $N(u) = \varphi$. Thus, an isolated vertex does not dominate any other vertex in G.

Definition 2.19. [17] An arc (u, v) is said to be a strong arc, if $\mu_2(u, v) \geq \mu_2^\infty(u, v)$ and $\gamma_2(u, v) \geq \gamma_2^\infty(u, v)$

Definition 2.20. [17] Let $G = (V, E)$ be an IFG on V. Let $u, v \in V$, we say that u dominates v in G, if there exists a strong arc between them.

Note 2. If $\mu_2(u, v) < \mu_2^\infty(u, v)$ and $\gamma_2(u, v) < \gamma_2^\infty(u, v)$ for all $u, v \in V$, then the only dominating set of G is V.

Definition 2.21. [17] A subset S of V is called a dominating set in G if for every $v \in V–S$, there exists $u \in S$ such that u dominates v.

Definition 2.22. [17] A dominating set S of an IFG is said to be minimal dominating set if no proper subset of S is a dominating set.

Definition 2.23. [17] Minimum cardinality among all minimal dominating set is called lower-domination number of G, and is denoted by d(G). Maximum cardinality among all minimal dominating set is called upper-domination numberofG, and is denoted by D(G).

Definition 2.24. [17] Two vertices in an IFG, $G = (V,E)$ are said to be independent if there is no strong arc between them.

Definition 2.25. [17] A subset S of V is said to be independent set of G.
If $\mu_2(u, v) < \mu_2^\infty(u, v)$ and $\gamma_2(u, v) < \gamma_2^\infty(u, v)$ for all $u, v \in S$

Definition 2.26. [17] An independent set S of G in an IFG is said to be maximal independent, if for everyvertexv $\in V - S$, the set $S \cup \{v\}$ is not independent.

Definition 2.27. [17] The minimum cardinality among all maximal independent set is called lower independence number of G, and it is denoted by i(G).

Definition 2.28. [17] The maximum cardinality among all maximal independent set is called upper independence number of G, and it is denoted by I(G).

Definition 2.29. [7] A dominating set D of a graph $G = (V, E)$ is a non-split dominating set, if the induced sub graph$\langle V - D \rangle$ is connected. The non-split domination number $\gamma_{ns}(G)$ of graph G is the minimum cardinality of a non-split dominating set.

Definition 2.30. [7] A dominating set D of a connected graph G is a path non-split dominating set if the induced sub graph $\langle V - D \rangle$ is a path in G. The path non-split domination number $\gamma_{pns}(G)$ of G is the minimum cardinality of a path non-split dominating set.

Definition 2.31. [7] A dominating set D of a connected graph G is a cycle non-split dominating set if the induced sub graph $\langle V - D \rangle$ is a cycle in G. The cycle non-split domination number $\gamma_{cns}(G)$ of G is the minimum cardinality of a cycle non-split dominating set.

Definition 2.32. [7] A dominating set D of a graph$G = (V, E)$ is a strong non-split dominating set if the induced sub graph $\langle V - D \rangle$ is complete. The strong non-split domination number $\gamma_{sns}(G)$ of G is the minimum cardinality of a strong non-split dominating set.

Definition 2.33. [7] A dominating set D of a connected graph G is a global non-split dominating if D is a non-split dominating set of both$G$ and $\overline{G}$. The global non-split domination number $\gamma_{gns}(G)$ of G is the minimum cardinality of a global non-split dominating set of G.

Definition 2.34. [6] A vertex $v \in G$is said to be end-vertex of IFG, if it has at most one strong neighbour in G.

Definition 2.35. [6] An edge (v_i, v_j) is said to be a bridge in IFG G, if either $\mu_{2xy}'^{\infty} < \mu_{2xy}^{\infty}$ and $\gamma_{2xy}'^{\infty} \geq \mu_{2xy}^{\infty}$ (or)$\mu_{2xy}'^{\infty} \leq \mu_{2xy}^{\infty}$ and $\gamma_{2xy}'^{\infty} > \mu_{2xy}^{\infty}$, for some $v_x, v_y \in V$.

Definition 2.36. [6] A vertex v_i is said to be a cutvertex in IFG G if deleting a vertex v_i reduces the strength of the connectedness between some pair of vertices.

3. Non-split dominating set
Definition 3.1. A dominating set D of an intuitionistic fuzzy graph $G = (V, E)$ is a non-split dominating set, if the induced intuitionistic fuzzy sub $\langle V - D \rangle$is connected. The non-split domination number $\gamma_{ns}(G)$ of intuitionistic fuzzy graph G is the minimum cardinality of all non-split domination set.

Example 3.1. Consider the Figure 1

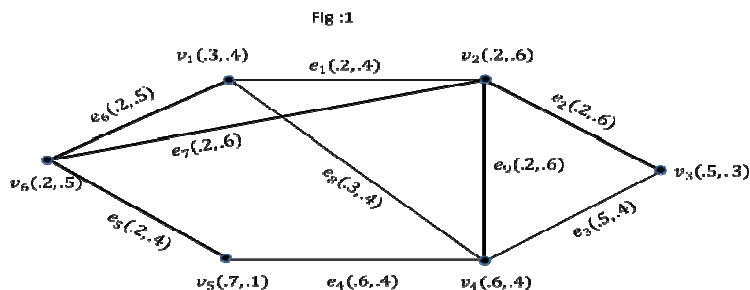Non Split Domination Pon Intuitionistic Fuzzy Graphs

Fig :1

Figure 1:

Here strong arcs are e_2, e_5, e_6, e_7 and e_9. Let $D = \{v_2, v_6\}$ and $V - D = \{v_1, v_3, v_4, v_5\}$ is connected. Here $\gamma_{ns}(G) = v_2 + v_6 = 0.65$.

Theorem 3.1. A non-split dominating set D of IFG G is minimal if and only if for each vertex $v \in D$, one of the following conditions is satisfied.

  i)   There exists a vertex $u \in V - D$ such that $N(u) \cap D = \{v\}$.
  ii)   v is an isolated vertex in $\langle D \rangle$
  iii) $N(v) \cap (V - D) \neq \varphi$

Proof: Let D be a minimal dominating set of IFG G. Suppose for each node $v \in D$, the set $D' = D - \{v\}$ is not a dominating set. Thus there is a node $u \in V - D'$ which is not dominated by any node in D'. If $u \in V - D$ and u is not dominated by D', but is dominated by D then v is the strong neighbour of u. Hence $N(u) \cap D = \{v\}$. Also v is an isolated vertex in D and neighbourhood of each vertex in D is a strong neighbour in $V - D$. This implies $N(v) \cap (V - D) \neq \varphi$.

Conversely, Let D be an non-split dominating set and each node $v \in D$, one of the following condition holds. Let us prove that D is minimal. Suppose D is not a minimal non-split dominating set. Then there exists a node $v \in D$ such that $D - \{v\}$ is a dominating set. Thus v is a strong neighbor to atleast one node in V-D, which implies v is not a strong neighbor of any node in D. Hence there is a node $u \in V - D$ such that $N(u) \cap D \neq v$. This implies one of the condition does not hold, which is a contradiction to our assumption. Hence D is minimal. Hence the theorem.

Theorem 3.2. For any Intuitionistic fuzzy graph G, $\gamma_{ns}(G) \leq \frac{O(G)}{\Delta_\mu(G)}$

where $\Delta_\mu(G)$ is the maximum; $\mu - $ degree of G.

Proof: Let D be an non-split dominating set of IFG G with $|D| = \gamma_{ns}(G)$. Since every vertex in $V - D$ is adjacent to some vertices in D, we have

$$|V - D| \leq \sum_{i=1}^{n} d(v_i) \leq \gamma_{ns}(G).\Delta_\mu(G)$$
$$\Rightarrow |V - D| + |D| \geq \gamma_{ns}(G).\Delta_\mu(G)$$
$$\Rightarrow O(G) - \gamma_{ns}(G) + \gamma_{ns}(G) \geq \gamma_{ns}(G).\Delta_\mu(G)$$
$$\Rightarrow O(G) \geq \gamma_{ns}(G).\Delta_\mu(G)$$

Thus, $\gamma_{ns}(G) \leq \frac{O(G)}{\Delta_\mu(G)}$. Hence the result.
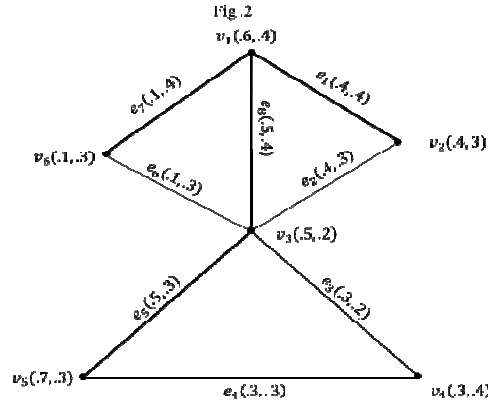
Example 3.2. Consider the figure 2.

56

Figure 2:

Here strong arcs are e_1, e_4, e_5, e_7 and e_8

Let $D = \{v_1, v_5\}$ and $V - D = \{v_2, v_3, v_4, v_6\}, O(G) = 6$.

Non-split domination number $\gamma_{ns}(G) = 1.3$

$$\Delta_\mu(G) = V\{d_\mu(v_1)/\ v_i \epsilon V\}$$
$$= V\{d_\mu(v_1), d_\mu(v_2), d_\mu(v_3), d_\mu(v_4), d_\mu(v_5), d_\mu(v_6)\}$$
$$= 1.0$$
$$1.3 \leq \frac{6}{1.0} \Rightarrow 1.3 \leq 6.0$$
$$\gamma_{ns}(G) \leq \frac{O(G)}{\Delta_\mu(G)}.$$

Definition 3.2. Let D be a minimum dominating set in IFG G. If the induced subgraph $\langle V - D \rangle$ is a path in IFG G, then D is called path non-split dominating set. The path non-split domination number$\gamma_{ns}(G)$ of G is the minimum cardinality of all path non-split dominating set.
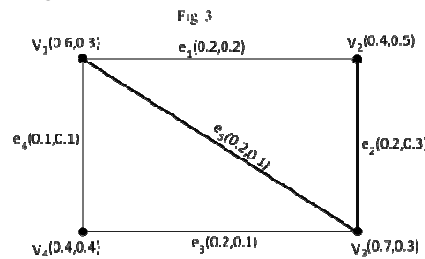
Example 3.3. Consider the figure 3.

Figure 3:

Here strong arcs are $e_2, e_3,$ and e_5

Let $D = \{v_3\}$ and $V - D = \{v_1, v_2, v_4\}$

Path non-split domination number $\gamma_{pns}(G) = 0.7$.

Theorem 3.3. Let G be an Intuitionistic fuzzy graph. If D is a path non-split dominating set, then there exists at least one adjacent vertex of each vertex $v \in D$ in $\langle V - D \rangle$.

Proof: Let G be an Intuitionistic fuzzy graph. Let D be a path non-split dominating set. Suppose there is no adjacent vertex of each $v \in D$ occur in $\langle V - D \rangle$. Then these vertices occur in dominating set D. This implies for some $u \in V - D$, there is no $v \in D$. Also path non-split dominating set is not minimum. Which contradicts the definition of path non-split dominating set. Hence there should be at least one adjacent vertex of each node $v \in D$ in $\langle V - D \rangle$. Hence the proof.
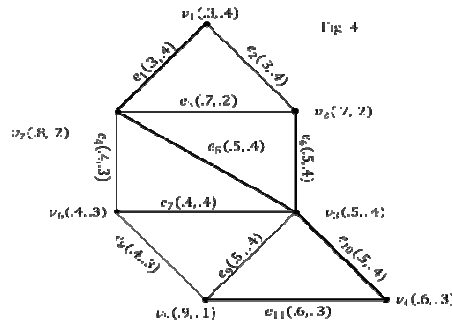
Example 3.4. Consider the figure 4.

Figure 4:

Here strong arcs are $e_1, e_5, e_6, e_7, e_{10}$ and e_{11}
Let $D = \{v_1, v_3, v_4\}$ and $V - D = \{v_2, v_5, v_6, v_7\}$
v_1- adjacent to v_2 and v_1
v_3- adjacent to $v_2, v_5, v_4, v_6,$ and v_7
v_4- adjacent to v_3 and v_5, one of the adjacent vertices of v_1, v_3 & v_4 occur in $< V - D >$.

Theorem 3.4. If an intuitionistic fuzzy graph G with independent vertices then every independent vertex of G must be in path non-split dominating set D.

Proof: Let G be an Intuitionistic fuzzy graph with independent vertices. Let D be an non-split dominating set. Suppose an independent vertex $v \in V - D$. Then there is no strong arc from v to any $u \in D$ which contradicts the definition of dominating set. This leads D is not a path non-split dominating set. Therefore every independent vertex of IFG G should be in D. Hence the proof.
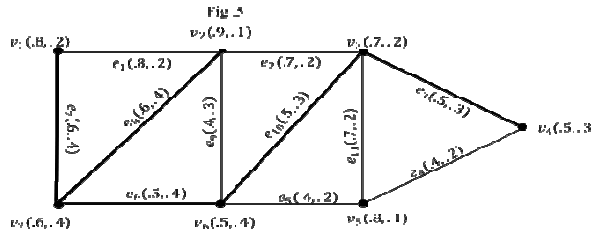
Example 3.5. Consider the figure 5.

Figure 5:

Here strong arcs are e_3, e_6, e_7, e_8 and e_{10}
Let $D = \{v_4, v_5, v_7\}$ and $V - D = \{v_1, v_2, v_3, v_6\}$

58

Here v_5 is an independent vertex which lies in D.

Theorem 3.5. Let G be an Intuitionistic fuzzy graph with end nodes. If D is a path non – split dominating set then maximum number of these end nodes occur in $\langle V - D \rangle$

Proof: Let G be an Intuitionistic fuzzy graph with end nodes. Let D be a path non-split domination set. Suppose D contains maximum number of end nodes. Then D is not minimum. Also for each $v \in V - D$, there exists $u \in D$. This results an non-split dominating set exists, which contradicts the assumption that D is a path non-split dominating set. Hence maximum number of end nodes should occur only in $\langle V - D \rangle$. Hence the theorem.

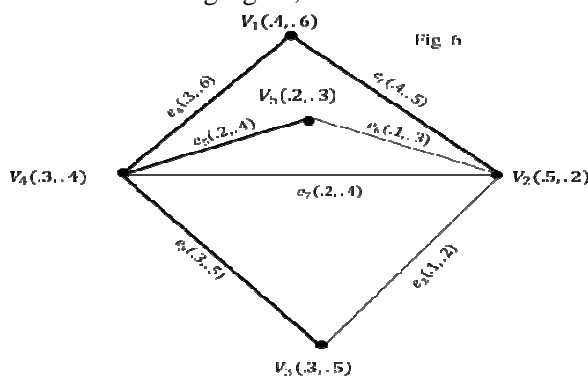**Example 3.6.** Consider the following figure,

Figure 6:

Here strong arcs are e_1, e_2, e_4 and e_5
Let $D = \{v_1, v_4\}$ and $V - D = \{v_2, v_3, v_5\}$
The end nodes are $v_2, v_3,$ and v_5 . These end nodes lie in $\langle V - D \rangle$.

Definition 3.3. Let D be a minimum dominating set in IFG G. If the induced sub graph $\langle V - D \rangle$ is a cycle in IFG G, then D is called cycle non-split dominating set. The cycle non-split domination number $\gamma_{cns}(G)$ of G is the minimum cardinality of all cycle non-split dominating set.
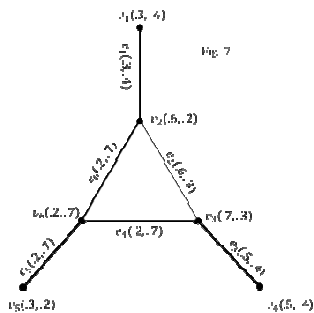
Example 3.7. Consider a figure,

Figure 7:

59

Non Split Domination Pon Intuitionistic Fuzzy Graphs

Let $D = \{v_1, v_4, v_5\}$ and $V - D = \{v_2, v_3, v_6\}$
Here D is called cycle non-split dominating set. The cycle non-split domination number$\gamma_{cns}(G) = 1.7$.

Theorem 3.6. If G is an Intuitionistic fuzzy graph with degree of each node is equal. Then there exists a cycle non-split dominating set.
Proof: Let G be an Intuitionistic fuzzy graph with all nodes of equal degree. Here atleast one node has all its edges strong. Also adjacent vertexes of this node have maximum number of strong arcs. This implies the IFG G contains more cycles. Also for each$u \in V - D$ there exists $v \in D$and $\langle V - D \rangle$ is a connected cycle. By definition of domination, D is a dominating set. Since$\langle V - D \rangle$ is a cycle, D is cycle non-split dominating set.Hence the proof.

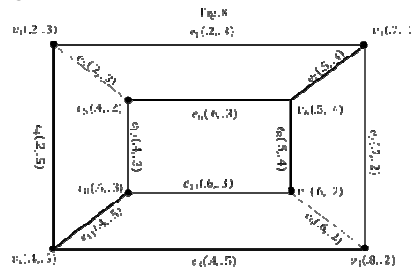**Example 3.8.** Consider a figure,

Figure 8:

Here strong arcs are e_3, e_4, e_6, e_7 and e_8
Let $D = \{v_4, v_6\}$ and $V - D = \{v_1, v_2, v_3, v_5, v_7, v_8\}$
Here $\langle V - D \rangle$ is a cycle and D is cycle non-split dominating set.

Theorem 3.7. Let G be an Intuitionistic fuzzy graph. If D is a cycle non –split dominating set, then there exists at least one strong arc in D.
Proof:
Case (i): Let G be an Intuitionistic fuzzy graph with degree of each node is not equal. Let D be a dominating set and the vertices of D is isolated. Also $\langle V - D \rangle$ is connected and form a cycle. Therefore D is a cycle non-split dominating set.
Case (ii): Suppose G is an IFG with degree of each node is equal. Then by theorem 3.15, D is a cycle non-split dominating set and is connected. Since one adjacent vertex of each $v \in D$ occur in $\langle V - D \rangle$, there exists at least one strong arc in D. Hence the theorem.

Definition 3.4. A dominating set D of IFG G is a strong Non-split dominating set if the induced subgraph $\langle V - D \rangle$ is complete.

Theorem 3.8. Every complete intuitionistic fuzzy graph with vertices $P > 3$ contains strong non-split dominating set.
Proof: Let G be complete IFG with vertices $p \leq 3$. Then for each $u \in V - D$ there exists a $v \in D$ such that D is a path non-split dominating set or isolated vertex. This is a contradiction to the definition of strong non-split dominating set. Therefore, consider P>3 Since G is complete, degree of each node is equal.

60

Also by definition, $\mu_{2ij} = \min\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = \max\{\gamma_{1i}, \gamma_{1j}\}$ for every $v_i, v_j \in V$. Hence there exists at least one strong arc in each node. This implies for each $u \in V - D$ there exists $v \in D$ and <V-D> is complete. Hence D is strong non-split dominating set.
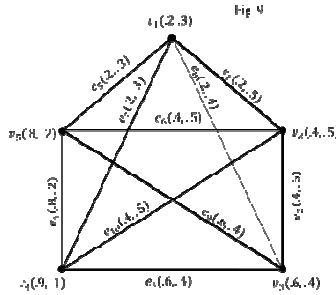
Example 3.9.

Figure 9:

Here strong arcs are $e_1, e_2, e_3, e_5, e_6, e_7, e_9$ and e_{10}.
Let D= $\{v_3, v_4\}$ and $V - D = \{v_1, v_2, v_5\}$ is complete. Therefore D is a strong non-split dominating set.

Definition 3.5. A dominating set D of a connected IFG G is global non-split dominating set, if D is non-split dominating set of both G and $\overline{G}$.

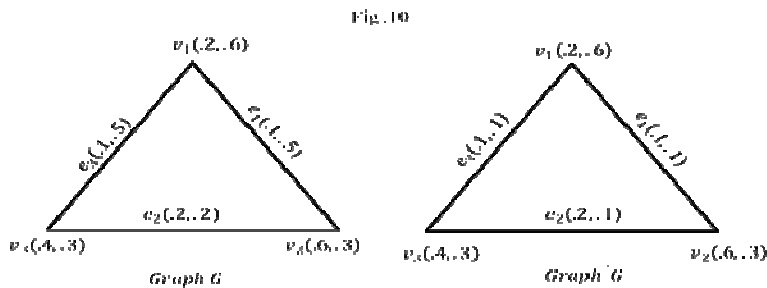**Example: 3.10.**

Figure 10:

Here strong arcs are e_1, e_3.   Here strong arcs are e_1, e_2, e_3.
Let D = $\{v_1\}$, $V - D = \{v_2, v_3\}$. Let D = $\{v_1\}$, $V - D = \{v_2, v_3\}$.
Hence D is a global dominating set.

4. Conclusion
This paper identifies non-split domination number on intuitionistic fuzzy graph G. We have defined different types of non-split dominating sets such as path non-split domination, cycle non-split domination, strong non-split domination and global non-split domination.Using these definitions we explore more theorems in future work.

REFERENCES

1. K.T.Atanassov, Intuitionistic fuzzy sets: theory and applications,Physica-Verlag, New York, 1999.

Non Split Domination Pon Intuitionistic Fuzzy Graphs

2. K.Atanassov, Operations on Intuitionistic Fuzzy Graphs, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 1396-1401, August 2009.
3. P.Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recognition Letters,* 6 (1987) 297-302.
4. K.R.Bhutani and A.Rosenfeld, Strong arcs in fuzzy graphs, *Information Sciences*, 152 (2003) 319 – 322.
5. E.J.Cockayne, and S.T.Hedetneme, Towards a theory of domination in graphs, *Networks,* 7(1977) 247-261.
6. M.G.Karunambigai and R.Parvathi, Intuitionistic Fuzzy Graphs, Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence.
7. V.R.Kulli, Cycle Nonsplit domination and path nonsplit domination in graphs,*Acta Ciencia Indica*, XXXIII M (2) (2007) 641.
8. J.N.Mordeson and C.S.Peng, Operation on fuzzy graphs, *Information Sciences,* 79 (1994) 159-170.
9. A.NagoorGani and V.T.Chandrasekaran, A First Look at Fuzzy Graph Theory, Allied Publishers Pvt.Ltd.,   2010
10. A.NagoorGani and V.T.Chandrasekaran, Domination in fuzzy graphs, *Adv. in Fuzzy Sets and Systems*, 1 (1) (2006) 17-26.
11. A.NagoorGani and J.Malarvizhi, Isomorphism on fuzzy graphs, *World Academy of Science, Engineering and Technology*, 2 (2008) 11-28.
12. A.NagoorGani and S.Shajitha Begum, Degree, order and size in intuitionistic fuzzy graphs, *International Journal of Algorithms, Computing and Mathematics*, 3(3) (2010) 11-16.
13. A.NagoorGani and S.Shajitha Begum, Perfect intuitionistic fuzzy graphs, *Bulletin of Pure and Applied Sciences*, 30 (2011) 145-152.
14. A.NagoorGani, S.Yahya Mohamed and R.Jahir Hussain, Semi global dominating set in fuzzy graphs, *Intern. J. Fuzzy Mathematical Archive*, 4(2) (2014) 104-111.
15. R.Parvathi, M.G.Karunambigai and K.Atanassov, Operations on intuitionistic fuzzy graphs*, Proceedings of IEEE International Conference on Fuzzy Systems*, 1396-1401, August 2009.
16. R.Parvathi and G.Tamizhendhi, Domination in intuitionistic fuzzy graphs, *NIFS*, 16(2) (2010) 39-49.
17. A.Rosenfeld, Fuzzy graphs. In L.A.Zadeh,K.S.Fu, M.Shimura (eds.) Fuzzy sets and their applications, Academic, New York, 1975
18. A.Shannon and K.Atanassov, On a generalization of intuitionistic fuzzy graphs, *NIFS,*12(1) (2006) 24-29.
19. A.Somasundaram and S.Somsudaram, Domination in fuzzy graphs-I, *Pattern Recognition Letters*, 19 (1998) 787-791.
20. M.S.Sunitha and A.Vijayakumar, Complement of fuzzy graph, *Indian Journal of Pure and Applied Mathematics*, 33(9) (2002) 1451-1464
21. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338 – 353.