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Point Set Domination of Intuitionistic Fuzzy Graphs

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Abstract. Let G = (V,E) be the IFG and D be set of vertices, then D is said to be intuitionistic point set dominating set of G if for every S$\subseteq$ V- D there exists a node d $\in$ D such that the intuitionistic fuzzy sub graph $< S \cup \{d\} >$ induced by S$\cup$\{d\} is a connected intuitionistic fuzzy graph. In this paper, we study some results on Point set dominating set and its number of IFGs and semi complete IFG. Also we establish the bounds with other domination parameters of IFGs.

1. Introduction

Atanassov [1] introduced the concept of in intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. Parvathy and Karunambigai's paper [6] introduced the concept of IFG and analyzed its components. Nagoor Gani and Begum [3] defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. The concept of domination in fuzzy graphs is introduced by Somasundaram and Somasundaram [12] in the year 1998. Parvathi and Thamizhendhi [7] introduced the concepts of domination number in intuitionistic fuzzy graphs. Domination is active subject in fuzzy graph and intuitionistic fuzzy graphs, and has numerous applications to distributed computing, the web graph and adhoc networks. Study on domination concepts in intuitionistic fuzzy graphs is more accurate than fuzzy graphs, which is useful in the traffic density and telecommunication systems. Point-set domination number of a graph is analyzed by Sampathkumar and Pushpa Latha [9] in 1993.

In this paper, we introduce intuitionistic fuzzy point set domination and its number of IFGs and discussed some bounds with other domination parameters. Some properties of intuitionistic fuzzy point set domination and intuitionistic fuzzy set domination in IFGs are derived. Also the intuitionistic fuzzy psd-sets of purely semi-complete intuitionistic fuzzy graph are established.

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2. Preliminaries

Definition 2.1. An intuitionistic fuzzy graph is of the form G = (V, E) where

(i) V={v1,v2,….,vn} such that μ_1: V$\rightarrow$[0,1]and γ_1: V $\rightarrow$[0,1] denote the degree of membership and non-membership of the element vi $\in$V, respectively, and $0 \le \mu_1$ (vi) + γ_1 (vi) ≤ 1 for every vi $\in$V, (i = 1,2, ……. n),

(ii) E $\subseteq$ V x V where μ_2: V x V $\rightarrow$[0,1] and γ_2: V x V $\rightarrow$[0,1] are such that μ_2(vi , vj) $\le$ min [μ_1(vi), μ_1(vj)] and γ_2 (vi , vj) $\le$ max [γ_1(vi), γ_1(vj)] and $0 \le \mu_2$ (vi, vj) + γ_2 (vi, vj) ≤ 1 for every (vi ,vj) $\in$ E, (i, j = 1,2, …,n).

Definition 2.2. An IFG H = < V', E' > is said to be an intuitionistic fuzzy sub graph (IFSG) of the IFG, G = < V, E > if V' $\subseteq$V and E' $\subseteq$ E. In other words, if μ_{1i}' $\le \mu_{1i}$; γ_{1i}' $\ge \gamma_{1i}$ and μ_{2ij}' $\le \mu_{2ij}$; γ_{2ij}' $\ge \gamma_{2ij}$ for every i, j = 1,2………n.

Definition 2.3. Let G = (V,E) be a IFG. Then the cardinality of G is defined as
$$|G| = \left| \sum_{vi \in V} \frac{1+\mu 1(vi)-\gamma 1 (vi)}{2} + \sum_{vi,vj \in E} \frac{1+\mu 2(vi,vj)-\gamma 2(vi,vj)}{2} \right|$$

Definition 2.4. The vertex cardinality of IFG G is defined by
$|V| = \left| \sum_{vi \in V} \frac{1+\mu 1(vi)-\gamma 1 (vi)}{2} \right|$ = p and the edge cardinality of IFG G is defined by
$|E| = \left| \sum_{vi,vj \in E} \frac{1+\mu 2(vi,vj)-\gamma 2(vi,vj)}{2} \right|$ = q.
The vertex cardinality of IFG is called the order of G and denoted by O(G). The edge cardinality of G is called the size of G, denoted by S(G).

Definition 2.5. An edge e = (x, y) of an IFG G = (V, E) is called an effective edge if
$$\mu_2(x, y) = \mu_1(x) \wedge \mu_1 (y) \text{ and } \gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y).$$

Definition 2.6. The effective degree of a vertex v in a IFG G = (V, E) is defined to be sum of the effective edges incident at v, and denoted by d_E(v). The minimum effective degree of G is δ_E(G) = $\wedge$ {d_E (v)/v$\in$ V}

Definition 2.7. An intuitionistic fuzzy graph is complete if μ_{2ij} = min (μ_{1i}, μ_{1j}) and γ_{2ij} = max (γ_{2i} ,γ_{2j}) for all (vi , vj) $\in$ V.

Definition 2.8. An IFG G is said to be strong IFG if μ_2(x, y) = μ_1(x) $\wedge$ μ_1 (y) and γ_2(x, y) = γ_1(x) $\vee$ γ_1(y) for all (vi, vj) $\in$ E.

Definition 2.9. The complement of an IFG G = (V, E) is denoted by $\bar{G} = (\bar{V}, \bar{E})$ and is defined as i) $\bar{\mu}_1(v) = \mu_1(v)$ and $\bar{\gamma}_1(v) = \gamma_1(v)$
ii) $\bar{\mu}_2$(u,v) = μ_1 (u) $\wedge$ μ_1 (v) - μ_2 (u,v) and $\bar{\gamma}_2$(u,v) = γ_1(u) $\vee$ γ_1(v) - γ_2(u,v) for u,v in V. $\bar{G}$ also denoted by G^c.

Definition 2.10. Let G = (V,E) be an IFG. The neighbourhood of any vertex v is defined as N(v)=(N_μ(v),N_γ(v)), where N_μ(v) = {$w \in V$; $\mu_2(v.w) = \mu_1(v) \wedge \mu_1(w)$} and N_γ(v) = {$w \in V$; $\gamma_2(v.w) = \gamma_1(v) \vee \gamma_1(w)$}. N[v] = N (v) $\cup${v} is called the closed neighbourhood of v.

Definition 2.11. The neighbourhood degree of a vertex is defined as
$d_N(v) = (d_{N\mu}(v), d_{N\gamma}(v))$, where $d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N\gamma}(v) = \sum_{w \in N(v)} \gamma_1(w)$.
The minimum neighbourhood degree is defined as $\delta_N(G) = (\delta_{N\mu}(v), \delta_{N\gamma}(v))$, where
$\delta_{N\mu}(v) = \Lambda \{ d_{N\mu}(v): v \in V \}$ and $\delta_{N\gamma}(v) = \Lambda \{ d_{N\gamma}(v): v \in V \}$.
The maximum neighbourhood degree is defined as $\Delta_N(G) = (\Delta_{N\mu}(v), \Delta_{N\gamma}(v))$, where
$\Delta_{N\mu}(v) = V\{ d_{N\mu}(v): v \in V \}$ and $\Delta_{N\gamma}(v) = V\{ d_{N\gamma}(v): v \in V \}$.

Definition 2.12. Let G= (V, E) be an IFG. Let u, v ∈ V, we say that u dominated v in G if there exist a strong arc between them. A subset D ⊆ V is said to be dominating set in G if for every v∈ V-D, there exist u in D such that u dominated v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by γ(G). The maximum scalar cardinality of a minimal domination set is called upper domination number and is denoted by the symbol Γ(G) .

Definition 2.13. An independent set of an intuitionistic fuzzy graph G = (V, E) is a subset S of V such that no two vertices of S are adjacent in G.

Definition 2.14. A Bipartite IFG, G= (V,E) is said to be complete Bipartite IFG, if $\mu_2(v_i, v_j) = \mu_1(v_i) \Lambda \mu_1(v_j)$ and $\gamma_2(v_i, v_j) = \gamma_1(v_i) V \gamma_1(v_j)$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{V1i, V2j}$.

Definition 2.15. Let G = (V, E) be a connected IFG with effective edges. G is said to be semi complete IFG, if every pair vertices have a common neighbor in G. An IFG-G is said to be purely semi-complete IFG if and only if G is semi complete IFG but not complete IFG.

3. Point set domination in intuitionistic fuzzy graph

Definition 3.1. Let G = (V, E) be IFG. A set D ⊆ V is said to be intuitionistic fuzzy Point set dominating set(psd-set) of G if for every S⊆V- D there exists a node d ∈ D such that the intuitionistic fuzzy sub graph < S∪{d} > induced by S∪{d} is a connected strong intuitionistic fuzzy graph.The intuitionistic point set fuzzy domination number $\gamma_{ip}(G)$ of G is the minimum scalar cardinality of intuitionistic fuzzy Point set dominating sets.
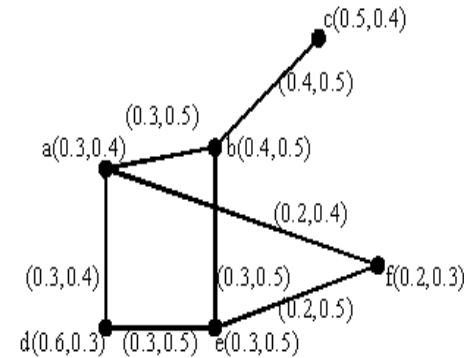
Example 3.1.

Figure 1: intuitionistic fuzzy graph

Here $|a| = 0.45$, $|b| = 0.45$, $|c| = 0.55$, $|d| = 0.65$, $|e| = 0.4$, $|f| = 0.45$ and γ_{ip} - set is $\{c,d,f\}$.Then $\gamma_{ip}(G) = 1.65$.

Theorem: 3.1. For any intuitionistic fuzzy graph G, Min $\{| vi| \} \leq \gamma_{ip}(G) < p$

Proof: If G is complete intuitionistic fuzzy graph then $\gamma_{ip}(G)$ –set contains only one vertex and we know that $\gamma_{ip}(G)$ –set have at least one vertex. That is Min $\{| vi|\} \leq \gamma_{ip}(G)$ Also, Suppose S be the intuitionistic fuzzy point set dominating set and V – S is not an empty set then S contain at most n- 1 vertices. That is the cardinality of S is less than p. Therefore $\gamma_{ip}(G) < p$. Hence we get Min $\{| vi| \} \leq \gamma_{ip}(G) < p$.

Remark 3.1.

 (i) $\gamma_{ip}(K_n) = \text{Min} \{ |v_i|\}$ where $v_i \in V$

 (ii) $\gamma_{ip} (K_{v1,vj}) = |v_1|$ where v_1 is centre vertex of IFG and $j = 1,2,.., n$

 (iii) $\gamma_{ip} (K_{V1i,V2j}) = \text{Min} \{ |v_{1i}|+ |v_{2j}| \}$ where $v_i \in V_1$ and $v_j \in V_2$

 (iv) If C_n is intuitionistic fuzzy graph and the induced crisp graph is cyclic graph with n vertices then $\gamma_{ip} (C_n) = \sum_{j=1}^{n-2} v_j$ where $v_j = \text{Min}\{|v_i|\}$, $i = 1,2,\dots,n$.

Theorem 3.2. Every intuitionistic fuzzy point set dominating set is intuitionistic fuzzy dominating set but not conversely.

Proof: Let S be intuitionistic fuzzy psd-set, By the definition it is always a dominating set of G but may not minimum. But the for the minimal dominating set D there exist a vertex $d \in D$, $<S \cup \{d\} >$ may not be connected where $S \subseteq V - D$. That is , converse need not true.

Example 3.2. In fig -1, γ-set is D = $\{a, b\}$ and $\gamma(G) = 0.9$. Also γ_{ip} - set is $\{c, d, f\}$. Here, γ-set is not $\gamma_{ip}(G)$ –set. Since, for the set S = $\{c, f\} \subseteq V - D$, both $< S \cup \{a\} >$ and $< S \cup \{b\} >$ are disconnected. Also $\gamma(G) \leq \gamma_{ip}(G)$

Theorem 3.3. Let G = (V, E) be the IFG and D the intuitionistic fuzzy psd-set of G . Then $<V - D>$ is a proper sub graph of a component H of G.

Proof: Suppose there exist vertices u and v belonging to two different components of G. Since D is an intuitionistic fuzzy psd-set of G, there much exist $w \in D$ such that $<\{u,v,w\}>$ is stongly connected IFG.Which is contradiction to our assumption, i.e. $V - D \subseteq V(H)$ for some component H of G. Further $D \cap V(H) \neq \emptyset$ which implied $<V - D >$ is a proper sub graph of H. Hence the proof.

Theorem 3.4. Let G = (V, E) be a strongly connected IFG and S is the intuitionistic fuzzy dominating set of G. If for every $D \subseteq V - S$ and $<D>$ has an intuitionistic fuzzy dominating set itself, then S is intuitionistic fuzzy psd-set of G.

Proof: By the given hypothesis, $<D>$ is connected for every $D \subseteq V - S$. Since S is dominating set of G then there is a $u \in S$ such that u is adjacent with some vertex of D. Hence $<D \cup \{u\}>$ is connected. Thus S is an intuitionistic fuzzy psd-set of G.

Theorem 3.5. Let G = (V, E) be IFG. If S is intuitionistic fuzzy Point set dominating set and u, v $\in$ V – S then between u and v there are at most two strong arcs.

Proof: Let D = {u, v}.Let u and v are not adjacent and S is the intuitionistic fuzzy psd-set, Then there exist a vertex x in S such that the sub graph <{u, v, x}> is Strongly connected. This implies between the vertices u and v there are at most two strong arcs only.

Theorem 3.6. Let $G = (V, E)$ be a strongly connected IFG and S is the intuitionistic fuzzy dominating set of G which is not intuitionistic fuzzy psd-set in G then the minimal γ-set contains at least two vertices.

Proof: Since S is not an intuitionistic fuzzy psd-set then there is a D ⊆V-S such that < D ∪{v}> is not connected for any v ∈ S. Hence < D > is not connected and therefore has at least two components. So at least two vertices in S are not dominated by a single vertex from S, which implies γ-set contains at least two vertices.

Remark 3.2.
(i) In Fig-1, γ-set is {a,b} which is not intuitionistic fuzzy psd-set. γ-set has two vertices.
(ii) The Converse is not true. That is, S is γ-set which contains two vertices then S is also γ_{ip}-set may happen.

Theorem 3.7. Let $G = (V, E)$ be the purely semi complete IFG and has a unique path between any pair of vertices in G, then the intersection of all intuitionistic psd-sets in G is a singleton set.

Proof: Since G is purely semi complete IFG then G is a union of edge disjoint triangles having a common vertex, say u. It follows that any intuitionistic psd-set contains the vertex u. Also {u} itself is intuitionistic psd-set. Hence their intersection is {u}.

Proposition 3.1. The converse of the above theorem is false, i.e. the intersection all intuitionistic psd-sets in G is a singleton set but G need not be purely semi complete IFG with unique path between any pair of vertices in G.
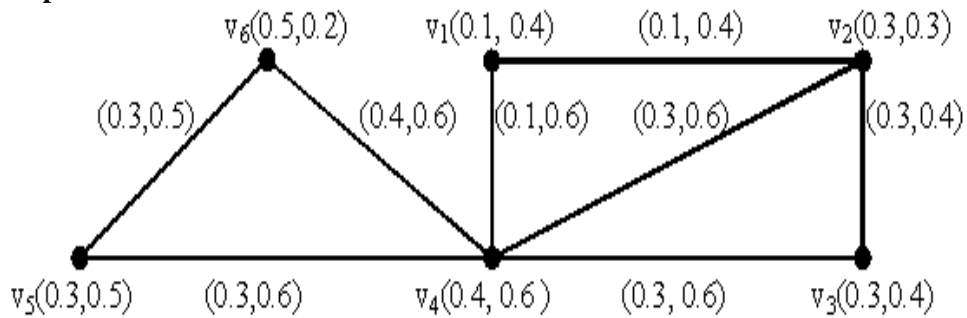
Example 3.3.

Figure 2: Purely semi-complete IFG

Here, all the psd-sets include v_4. But, between the vertices {v_2, v_4} there are two different shortest paths which are {v_4, v_1, v_2} and {v_4, v_3, v_2}

Theorem 3.8. Let G = (V, E) be the purely semi-complete IFG with n vertices and S be the intuitionistic fuzzy dominating set with number of vertices is greater than or equal to n - 2 , then S is intuitionistic fuzzy psd-set in G.

Proof: Let D ⊆ V-S and assume S has n-2 vertices then either D is singleton or contain two vertices. Let D = {v}, since S is intuitionistic fuzzy dominating set then there is a u∈ S such that v is adjacent to u. That is < D ∪ {u}> is connected.Now, let D = {v_1, v_2}. If v_1 and v_2 are adjacent then there is a u∈ S such that < D ∪{u}> is connected. Suppose v_1 and v_2 are not adjacent then by the nature of S, there is a u∈ S such that < D ∪ {u}> is connected. Thus S is intuitionistic fuzzy psd-set in G.

Theorem 3.9. Let G = (V, E) be the purely semi complete IFG and S is an intuitionistic fuzzy dominating set in G , then S is intuitionistic fuzzy psd-set if and only if for any independent vertex set B ⊆ V- S , there is a v ∈ S such that every vertex of B is strongly adjacent to v in G.

Proof: Given S is an intuitionistic fuzzy psd-set in G. Let B be any independent vertex set and B ⊆ V- S, then there is a v ∈ S such that < S ∪{v}> is strongly connected. Since no two elements in B are adjacent then by previous theorem between two vertices there are two strong arcs. Therefore G is purely semi complete IFG and S is an intuitionistic fuzzy dominating set in G.

Conversely, suppose S is an intuitionistic fuzzy dominating set in G and any independent vertex set B ⊆ V- S, there is a v ∈ S such that every vertex of B is strongly adjacent to v in G. Let D ⊆ V- S and <D> is connected then S is intuitionistic fuzzy psd-set. Otherwise let there are more than one components of D then B= {v_1, v_2, …,v_n} is an independent set in G.Now by hypothesis, since each v_i is adjacent to v in G then there exist v ∈ S such that< B ∪ {v}> is connected. Therefore, < D ∪ {v}> is connected. Hence, S is intuitionistic fuzzy psd-set in G.

Definition 3.10. Let G = (V, E) be IFG. A set D ⊆V is said to be intuitionistic fuzzy set-domination set if every set T⊆ V-D, there exists a non empty set S⊆D such that the intuitionistic fuzzy induced sub graph < S ∪ T > is connected. The minimum cardinality taken over all intuitionistic fuzzy set –domination sets is called intuitionistic fuzzy set domination number and denoted by γ_s(G).

Theorem 3.11. Let G=(V, E) be IFG with effective edges then, γ (G) $\leq$ γ_s(G) $\leq$ γ_{ip}(G).

Proof: Let D be a intuitionistic fuzzy Point set dominating set of IFG, for every set T ⊆V-D there exist a non empty singleton set {v}∈ S⊆D such that the sub graph <S∪T> is connected. Thus D is an intuitionistic fuzzy set –domination set.

Then γ_s(G) $\leq$ γ_{ip}(G).Suppose D be an intuitionistic fuzzy set –domination set of IFG, the for every set {x} = T ⊆V-D there exist a non empty singleton set {v}∈ S⊆D such that the sub graph <S∪T> is connected. Thus D is an intuitionistic fuzzy domination set. Then γ (G) $\leq \gamma_s$(G). Therefore we have γ (G) $\leq \gamma_s$(G) $\leq \gamma_{ip}$(G).

Remark: The converses are need not be true.

S.Yahya Mohamed, A.Nagoor Gani and R.Jahir Hussain

4. Conclusion
Here, we discussed some results on intuitionistic fuzzy points set domination and intuitionistic set domination on IFGs and derived some bounds in the standards IFGs and semi complete IFG. Also we established some theorems on intuitionistic psd-sets in purely semi complete IFG which is useful to solve communication network, medical diagnosis problems and transportation networks.

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