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# Different Algorithmic Approach for Type 2 Fuzzy Shortest Path Problem on a Network

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***Abstract.*** In this paper we present the fuzzy shortest path and shortest path length with Type-2 fuzzy number. To solve this problem we proposed an algorithm from a specified node to other nodes in a network and we have compared the results with other distance measures like Hamming, Normalized Hamming, Exponential type distance measures also. Our proposed algorithm is illustrated with the help of numerical example.

***Keywords:*** Type-2 fuzzy number, type-1 fuzzy number, distance measure, similarity measure, extension principle

***AMS Mathematics Subject Classification (2010):*** 94D05

## 1. Introduction

In network optimization, a large number of shortest path algorithms have been worked out more thoroughly than any other algorithm. Some of these algorithms are better than others, some are more suited for a particular structure than others and some are only minor variations of earlier algorithms. Some algorithms like the Dijkstra'salgorithm[3] can solve shortest path problems where there are no negative weights. The algorithms given by Bellman, Dijkstra [3] and Dreyfus [4] are referred to as the standard shortest path algorithms. The fuzzy shortest path problem was first analyzed by Dubois and Prade [5] in 1980. Klein [9] proposed a dynamical programming recursion-based fuzzy shortest path algorithm. Lin and Chen [11] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Okada and Soper [13] proposed a fuzzy shortest path algorithm based on multiple labeling methods. Chuang and Kung [2] found fuzzy shortest path length procedure. Kung and Chuang [10] proposed a new algorithm to deal shortest path problems with discrete fuzzy arc lengths. Zadeh proposed type-2 fuzzy sets as an extension of (type-1) fuzzy sets whose membership values are fuzzy sets on the interval [0,1]. Type reduction was proposed by Karnik and Mendel [6,7,8]. It is an 'extended version' [14] of type-1 defuzzification methods and is called type reduction because this operation takes us from the type-2 output sets of the fuzzy logic system to a type-1 fuzzy set that is called "type reduction set". Similarity is an important tool to provide the foundation for analogical reasoning between two fuzzy concepts and has widespread applications. The most obvious way of calculating similarity of fuzzy sets is based on their distance. Various distance measures are present in literature. In this paper we have proposed one distance measure and made comparison is made with other measures.This paper is organized as follows: In section 2, we have some basic concepts required for analysis. In section 3, an algorithm is proposed to find the fuzzy shortest path and shortest path length combined with distance based similarity measure. A comparative study is made with the help of various distances like Hamming, Normalized Hamming and Normalized exponential type distance. Section 4, gives the network terminology. To illustrative the proposed algorithm the numerical example is solved in section5.

## 2. Concepts

### 2.1. Type-2 fuzzy set

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$.

ie., $\tilde{A} = \{ ((x,u), \mu_{\tilde{A}}(x,u)) \,/\, \forall\, x \in X,\ \forall\, u \in J_x \subseteq [0,1] \}$ in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$.

$\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)\ J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

### 2.2. Discrete type-2 fuzzy number

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x\in X} \mu_{\tilde{A}}(x)/x$ where $\mu_{\tilde{A}}(x) = \sum_{u\in J_x} f_x(u)/u$ where $J_x$ is the primary membership.

### 2.3. Extension principle

Let $A_1, A_2, \ldots, A_r$ be type-1 fuzzy sets in $X_1, X_2, \ldots, X_r$, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, \ldots, A_r$ a type-1 fuzzy setB on Y, through f, i.e,B = $f(A_1, \ldots, A_r)$, such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1,x_2,\ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1),\ldots,\mu_{A_n}(x_n)\ iff^{-1}(y) \neq \phi \\ 0, \qquad\qquad f^{-1}(y) = \phi \end{cases}$$

### 2.4. Addition on type-2 fuzzy numbers

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A} = \sum \mu_{\tilde{A}}(x)/x$ and $\tilde{B} = \sum \mu_{\tilde{B}}(y)/y$ where $\mu_{\tilde{A}}(x) = \sum f_x(u)/u$ and $\mu_{\tilde{B}}(x) = \sum g_y(w)/w$. The addition of these two types-2 fuzzy numbers $\tilde{A} \oplus \tilde{B}$ is defined as

$$\mu_{\tilde{A}\oplus\tilde{B}}(z) = \bigcup_{z=x+y} (\mu_{\tilde{A}}(x) \bigcap \mu_{\tilde{B}}(y)) = \bigcup_{z=x+y} ((\sum_i f_x(u_i)/u_i) \bigcap (\sum_j g_y(w_j)/w_j))$$

$$\mu_{\tilde{A}\oplus\tilde{B}}(z) = \bigcup_{z=x+y} ((\sum_{i,j} (f_x(u_i) \wedge g_y(w_j))/(u_i \wedge w_j))$$

**2.5. Minimum of two discrete type-2 fuzzy number**

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then minimum of two type-2 fuzzy sets is denoted as Min( $\tilde{A}$ , $\tilde{B}$ ) is given by

$$Min(\tilde{A},\tilde{B})(z) = \sup_{z=Min(x,y)} \left[ (f_x(u_i) \wedge g_y(w_j)) / (\mathrm{u}_i \wedge w_j) \right]$$

where $\tilde{A} = \sum f_x(\mathrm{u}) / \mathrm{u} / \mathrm{x}$ and $\tilde{B} = \sum g_y(\mathrm{w}) / \mathrm{w} / \mathrm{y}$ .

**2.6. Similarity measure**

If d is the distance measure between two fuzzy sets A and B on the universe X, then the following measures of similarity is presented respectively.

$S(A,B) = \dfrac{1}{1+d(A,B)}$; $S(A,B) = 1 - d^N(A,B)$; $S_e(A,B) = 1 - d_e(A,B)$

**2.7. Distance based similarity measures for fuzzy sets**

Various distance measures are available in literature. Here we are using the following distance measures for the proposed algorithm.

1. **The Hamming distance:** $d\ (A,B) = \sum_{i=1}^{n} \left| A(x_i) - B(x_i) \right|$
2. **Normalized Hamming distance:** $d^N(A,B) = d(A,B)/n$
3. **Normalized Exponential type distance:** $d_e(A,B) = \dfrac{1-\exp(-d^N(A,B)}{1-\exp(-1)}$
4. **Proposed distance:** $d\left(A,B\right)(\mathrm{z}) \ = \sup_{z=Min(x,\mathrm{y})} \left\{ \left| B\left(y_i\right) \text{ -} A\left(x_i\right) \right| \ \right\} \forall z \in$ 

**2.8. Centroid of type-2 fuzzy sets**

Suppose that $\tilde{A}$ is a type-2 fuzzy set in the discrete caase. The centroid of $\tilde{A}$ can be defined as follows:

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x_1}} \cdot \int_{\theta_2 \in J_{x_2}} \cdots\cdots \int_{\theta_R \in J_{x_R}} [\, f_{x_1}(\theta_1).f_{x_2}(\theta_2)\ldots\ldots f_{x_R}(\theta_R) \Bigg/ \frac{\sum_{j=1}^{R} x_j \mu_A(x_j)}{\sum_{j=1}^{R} \mu_A(x_j)}$$

, where $\tilde{A} = \sum_{j=1}^{R} \left[ \sum_{u \in J_{x_j}} f_{x_j}(\mathrm{u}) / \mathrm{u} \right] / \mathrm{x}_j$

**3. Algorithm**

**Algorithm for fuzzy shortest path length**

**Step 1:** Find the path length for the required paths.

**Step 2:** Reduce the Type-2 fuzzy path length to Type-1 fuzzy path length using typereduction method.

**Step 3:** If path length is single then that path length is the shortest path length $C_{\tilde{L}}$ andthat path is the shortest path $\tilde{p}$ . Stop the procedure. Otherwise go to step 4.

**Step 4:** Compute the minimum path length $C_{\tilde{L}}$ using def 2.8.

**Step 5:** Compute distance measure for all distance between the minimum path lengthand the remaining path lengths using def 2.7.

**Step 6:** Compute the similarity measure for all distance measures using def 2.6
**Step 7:** Choose the shortest path $\tilde{p}$ with the highest similarity measure.
**Step 8:** The shortest path length is $C_{\tilde{L}}$ and the shortest path is $\tilde{p}$ .

**Algorithm for fuzzy shortest path from source node to all other node in a network**
**Step 1:** Letnode 1 be the source node in the given network.
**Step 2:** Find the collection of nodes $S_1$ in the network which are adjacent to node 1.
**Step 3:** If $S_1$ is empty, then go to step 10. Otherwise go to the step 4.
**Step 4:** Compute the minimum path length at each node of $S_1$ from node 1 using path length algorithm.
**Step 5:** Find the collection of nodes $S_2$ in the network which are adjacent to $S_1$.
**Step 6:** If $S_2$ is empty, then go to step 10. Otherwise go to step 7.
**Step 7:** Compute the minimum path length at each node of $S_2$ from $n_0$ using result 1.
**Step 8:** Repeat step 2 to step 7 until to obtain the set of collection of nodes in the network which are adjacent to each of the shortest path node is empty.
**Step 9:** Compute the shortest path from node 1 to each of nodes in the network in step 8.Stop the procedure.
**Step 10:** There is no path from the node 1 to the specified node.

### 3.1. Network terminology

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$ . Each edge is denoted by an ordered pair (i,j), where i,j$\in$ V and i$\neq$j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path $P_{ij}$ as a sequence $P_{ij} = \{i = i_1, (i_1,i_2),i_2,. . . ., i_{l-1}, (i_{l-1},i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path $P_{si}$ in G(V,E) is assumed for every node i$\in$ V – {s}.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j\in P)} \tilde{d}_{ij}$

### 3.2. Numerical example

The problem is to find the shortest path and shortest path length from source node to all other nodes in the network having 6 vertices and 7 edges with the association of type-2 fuzzy number.

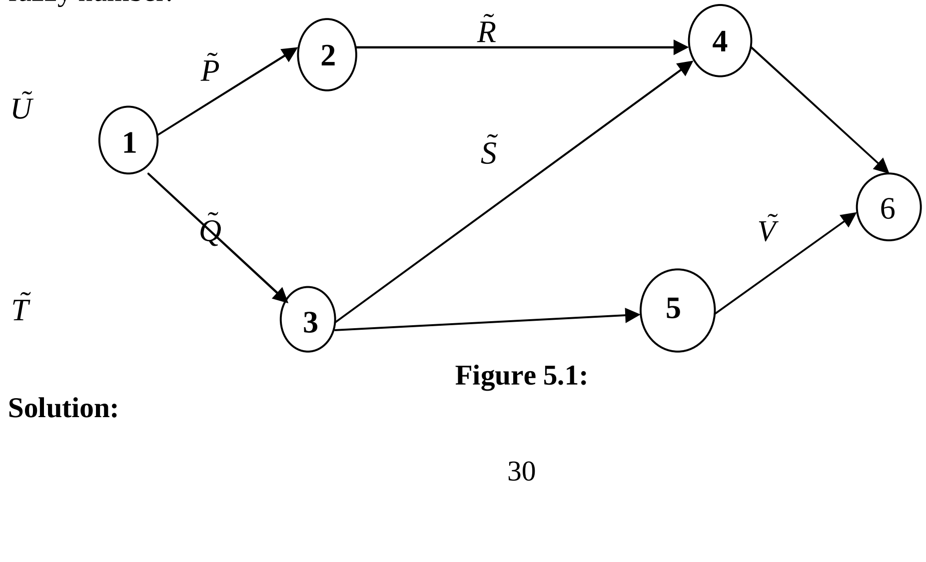


**Figure 5.1:**

**Solution:**

The edge weights are

$\tilde{P}$ = (0.5/0.2+0.4/0.3)/2 + (0.4/0.2)/3

$\tilde{Q}$ = (0.3/0.2 + 0.8/0.3)/1 + (0.2/0.8)/3

$\tilde{R}$ = (0.7/0.2)/2 + (0.9/0.4 +0.7/0.5)/4

$\tilde{S}$ = (0.6/0.2)/4

$\tilde{T}$ = (0.9/0.4 +0.7/0.5)/3 + (0.4/0.7)/5

$\tilde{U}$ = (0.8/0.3 + 0.4/0.5)/2

$\tilde{V}$ = (0.6/0.4)/2 +(0.7/0.5+ 0.5/0.6)/4

**Illustration to find shortest path**

**Step 1:** Let node 1 be the source node in the given network.

**Step 2:** $S_1$ = {2,3} and using step 3 and step 4 of the proposed algorithm we have the following:

| End node | Possible paths | Path Length | Minimum path Length | Shortest Path |
|---|---|---|---|---|
| 2 | 1-2 | (0.5/0.2+0.4/0.3)/2+(0.4/0.2)/3) 0 | (0.5/0.2+0.4/0.3)/2+(0.4/0.2)/3) | 1-2 |
| 3 | 1-3 | 0.3/0.2+0.8/0.3)/1+(0.2/0.8)/3 | 0.3/0.2+0.8/0.3)/1+(0.2/0.8)/3 | 1-3 |

**Table 5.1:**

**Step 5:** $S_2$ = {4,5} using step 6 and step 7 of the proposed algorithm we have the following:

| End node | Possible paths | Path Length | Minimum path Length | Shortest Path |
|---|---|---|---|---|
| 4 | 1-2-4<br><br>1-3-4 | (0.5/0.2)/4+(0.4/0.2)/5+(0.5/0.2+0.4/0.3)/6 + ( 0.4/0.2)/7<br><br>(0.6/0.2)/5 + (0.2/0.2)/7 | (0.6/0.2)/5+(0.2/0.2)/7 | 1-3 – 4 |
| 5 | 1-3-5 | 0.3/0.2+0.8/0.3)/4+(0.2/0.4)/6+(0.2/0.7)/8 | 0.3/0.2+0.8/0.3)/4+(0.2/0.4)/6+(0.2/0.7)/8 | 1-3-5 |

**Table 5.2:**

**Step 8 :**$S_3$ = {6} using step 6 and step 7 of the proposed algorithm we have the following:

| End node | Possible paths | Path Length | Minimum path Length | Shortest Path |
|---|---|---|---|---|
| 6 | 1-3-4-6<br><br>1-3-5-6 | (0.6/0.2)/7 + (0.2/0.2)/9<br><br>(0.3/0.2+0.6/0.3)/6+(0.2/0.4)/8+(0.2/0.4)/10+(0.2/0.5+0.2/0.6)/12 | (0.6/0.2)/7+ (0.2/0.2)/9 | 1-3 – 4-6 |

**Table 5.3:**

**By the proposed method, the fuzzy shortest path and shortest path length from the node 1 to each other nodes is given below:**

| End node | Possible paths | Path Length | Minimum path Length | Shortest Path |
|---|---|---|---|---|
| 2 | 1-2 | (0.5/0.2+0.4/0.3)/2+(0.4/0.2)/3) | (0.5/0.2+0.4/0.3)/2+(0.4/0.2)/3 | 1-2 |
| 3 | 1-3 | 0.3/0.2+0.8/0.3)/1+(0.2/0.8)/3 | 0.3/0.2+0.8/0.3)/1+(0.2/0.8)/3 | 1-3 |
| 4 | 1-2-4<br>1-3-4 | (0.5/0.2)/4+(0.4/0.2)/5+(0.5/0.2 + 0.4/0.3 )/6 + ( 0.4/0.2)/7<br>(0.6/0.2)/5 + (0.2/0.2)/7 | (0.6/0.2)/5+ (0.2/0.2)/7 | 1-3 – 4 |
| 5 | 1-3-5 | 0.3/0.2+0.8/0.3)/4+(0.2/0.4)/6+(0.2/0.7)/8 | (0.3/0.2+0.8/0.3)/4+(0.2/0.4)/6 + (0.2/0.7)/8 | 1-3-5 |
| 6 | 1-3-4-6<br>1-3-5-6 | (0.6/0.2)/7 + (0.2/0.2)/9<br>(0.3/0.2+0.6/0.3)/6+(0.2/0.4)/8+(0.2/0.4)/10+(0.2/0.5+0.2/0.6)/12 | (0.6/0.2)/7+ (0.2/0.2)/9 | 1-3 – 4-6 |

**Table 5.4:**

The Fuzzy Shortest path and the corresponding path length using proposed and existing distance measures are given below.

| End node | Possible paths | Path Length | Similarity Degree using | | | | Shortest Path | Shortest path Length |
|---|---|---|---|---|---|---|---|---|
| | | | Hamming Distance | Normalized Hamming Distance | Normalized Exponential type Distance | Proposed Distance | | |
| 2 | 1-2 | (0.5/0.2+0.4/0.3)/2+(0.4/0.2)/3) | __ | __ | __ | __ | 1-2 | (0.5/0.2+0.4/0.3)/2+(0.4/0.2)/3 |
| 3 | 1-3 | 0.3/0.2+0.8/0.3)/1+ (0.2/0.8)/3 | _ | __ | __ | __ | 1-3 | 0.3/0.2+0.8/0.3)/1+(0.2/0.8)/3 |
| 4 | 1-2-4<br>1-3-4 | (0.5/0.2)/4+(0.4/0.2)/5+(0.5/0.2+0.4/0.3)/6+(0.4/0.2)/7<br>(0.6/0.2)/5+(0.2/0.2)/7 | 0.839<br>1 | 0.936<br>1 | 0.902<br>1 | 0.893<br>1 | 1-3 – 4 | (0.6/0.2)/5+(0.2/0.2)/7 |
| 5 | 1-3-5 | 0.3/0.2+0.8/0.3)/4+(0.2/0.4)/6+(0.2/0.7)/8 | __ | __ | __ | __ | 1-3-5 | (0.3/0.2+0.8/0.3)/4+(0.2/0.4)/6+(0.2/0.7)/8 |
| 6 | 1-3-4-6<br>1-3-5-6 | (0.6/0.2)/7 + (0.2/0.2)/9<br>(0.3/0.2+0.6/0.3)/6+(0.2/0.4)/8+(0.2/0.4)/10+(0.2/0.5+0.2/0.6)/12 | 1<br>0.882 | 1<br>0.97 | 1<br>0.958 | 1<br>0.89 | 1-3– 4-6 | (0.6/0.2)/7+(0.2/0.2)/9 |

**Table 5.5:**

## 4. Conclusion

The Shortest path problem is a classical and important network optimization problem appearing in many real life applications. In this paper, we provide a new algorithm for solving shortest path problem on a network. In the proposed method, we are able to obtain all non-dominated paths from the specified node to all other nodes. Here we have compared the existing distance measures with our proposed distance measure. From our comparative study we conclude that the fuzzy shortest path obtained from a specified node to all other nodes, is same in the case of proposed method and all other existing methods.

## REFERENCES


1. V.Anusuya and R.Sathya, Type-2 fuzzy shortest path, *International Journal of fuzzy mathematical Archive*, 2 (2013) 36-42.
2. T.N.Chuang and J.Y.Kung, The Fuzzy shortest path length and the corresponding shortest path in a network, *Computers and Operations Research*, 32 (2005) 1409-1428.
3. E.W.Dijkstra, A note on two problems in connection with graphs, *Numerische Mathematik*, 1 (1959) 269-271.
4. S.Dreyfus, An Appraisal of some shortest paths algorithms, *Operations Research*, 17 (1969) 395-412.
5. D.Dubois and H.Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, 1980.
6. N.Karnik and M.Mendel, Type-2 fuzzy logic systems: Type – reduction, in *IEEE Syst., Man, cybern. Conf.,* San Diego, CA, oct. 1998.
7. N.Karnik and M.Mendel, An introduction to type-2 fuzzy logic systems, USC Report, http://sipi.usc.edu/~mendel/report, oct. 1998.
8. N.Karnik and M.Mendel, Type-2 fuzzy logic systems, *IEEE Trans. Fuzzy Syst.,* 7 (1999) 643-658.
9. C.M. Klein, Fuzzy Shortest Paths, *Fuzzy Sets and Systems*, 39 (1991) 27-41.
10. J.Y.Kung and T.N.Chuang, The shortest path problems with discrete fuzzy arc lengths, *Computers and mathematics with applications*, 49 (2005) 263-270.
11. K.Lin and M.Chen, The fuzzy shortest path problem and its most vital arcs, *Fuzzy Sets and Systems,* 58 (1994) 343 – 353.
12. S.Okada and T.Soper, A shortest path problem on a network with fuzzy arc lengths, *Fuzzy sets and systems*, 109 (2000) 129-140.
13. P.Pandian and P.Rajendran, A new algorithm for minimum path in a network, *Applied Mathematical Sciences*, 4(54) (2010) 2697-2710.
14. L.A.Zadeh, The concept of a linguistic variable and its application to approximate reasoning – 1, *Inform. Sci.*, 8 (1975) 199 – 249.